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# Capacitated Location-Allocation Problem in a Competitive Environment

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## Abstract

In this paper, we propose a game theoretic model to solve the Capacitated location-allocation problem in a competitive environment for a two level distribution Network. The problem is a variant of the TLCLAP problem. We study the location allocation decisions for two firms that wish to extend their distribution networks. Through the integration of several economic parameters, we propose an algorithm for finding Nash equilibrium in two-player games from the proposed mathematical model. Thus, we assumed the case of mixed strategies to establish the firms' best response functions. An optimization approach is proposed to find equilibrium under capacity constraint. To provide a better analysis of the problem, five scenarios were given. Computational results show the effectiveness of the proposed approach.

**Keywords:** Location allocation problem, Nash equilibrium, Game location, Distribution Network

## INTRODUCTION

Large-scale distribution is experiencing a booming in recent years despite a slower growth in economies, saturation of markets, a continues costs rising, fragmented markets and an increasing competitive environment [1].In this context, the opening of one or more stores is one of the key decisions that must be taken to deal with this situation. In addition, the market study for the implantation, the number of stores to be opened and the financial capacity of a distribution business must take into account the projected market share, which is impacted by the competition factor. Thus, the present paper aims to solve the problem of location-allocation in a competitive environment.

## LITERATURE REVIEW

The problems of facility location have been studied thoroughly during the last decades, due to the large variety of applications based on this kind of problems. The problem of location and allocation was introduced by Alfred Weber1 who focused in industrial factory site implantation.

Later, the notion of spatial competition in a situation of duopoly was introduced by [1]. In his book, he has highlighted the competitive interactions between two stores in a distribution network, based on the principle of homogeneous distribution of clients to determine the optimal location of two stores with similar characteristics.

In the same research focus, [2] introduced the competition with multi-stores in the original model [1] and shows that there is no equilibrium to a pure strategy. The study of the problem of multi-store location on a discrete space was the subject of the work of [3] that have generated a great interest among researchers and a large-scale distribution companies. Moreover, the assumption that transport costs are quadratic in distance, [4] show that companies tend to open one store. of [5] who first introduce the basic elements of location models in a competitive environment followed by the basic model introduced by [1].

Lately, the notion of competition in the problems of location/allocation has become increasingly studied. For the location of facilities in a logistics network, the approach adopted is to make a modeling in the form of a game whose solution is given by a Nash equilibrium [6]. In addition, it is important in any decision of location that the competitors review their strategies to constantly innovate in order to differentiate from the others players (competitors) [7]. The proximity of the client, the interactions with the market of the large distribution referring especially to the strategy of localization, and response to changes in demand offers an advantage for the company in the decision of localization [8].

### *Game theory and competition*

In general, the models of competition in economics are based on the application of the game theory. The objective is to study the interactions of the behaviors of several individuals,

[1] FRIEDRICH , ''Theory of the Location of Industries'', University of Chicago Press, 1929

called players, who are aware of these interactions. The "Nash equilibrium", is a fundamental notion in this theory.

In a game theory, a strategy can be reduced to an elementary decision, but can also be a complex action plan. The notions of Nash equilibrium and the best response function are fundamental. Indeed, they represent the solutions to the problems studied in game theory.

### *Duopoly and oligopoly*

In the context of competitive strategies, firms make decisions in response to competitors. This reaction creates a situation of strategic interactions that is accentuated by an atomistic demand in the market. In the case of two firms that compete with one another, we speak of a duopoly. we try to illustrate the interest of this theory in these competitive situations.

In the case of Bertrand's duopoly, firms compete by prices. We denote by $p_1$ and $p_2$, the prices that the firms F1 and F2 must apply. These prices define the requests addressed to F1 and F2. We denote by $D_i(p_1, p_2)$ the demand addressed to firm $i, i = 1,2$, D the aggregate demand on the market and $C_1$, $C_2$ are respectively the constant unit costs for firms F1 and F2. Each firm seeks to maximize its profit in the market. Therefore, the problem of the firm i is formulated as follows:

$$\max \pi_i\,(p_i - C_i) D_i(p_1, p_2), i = 1,2$$

We assume that the firms are symmetric in costs, $C_1 = C_2 = C$ and each firm serves the entire demand addressed to it. In attempting to analyze how price policy can affect the competitive situation, we can notice that:

- If $p_1 > p_2$, customers will turn to F2, which offers low prices for the same products offered by F1. Then the total demand will be satisfied by F2. We thus have: $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = D$;
- If $p_1 < p_2$, customers will go to the firm F1 which offers low prices for the same products offered by F2. Then the total demand will be satisfied by F1. We thus have: $D_1(p_1, p_2) = D$ and $D_2(p_1, p_2) = 0$;
- If $p_1 = p_2$, the demand will be shared equally between F1 and F2, so we have : $D_1(p_1, p_2) = D_2(p_1, p_2) = \frac{D}{2}$.

Thus, the unique equilibrium of Bertrand's duopoly with symmetric costs is characterized by: $p_1^* = p_2^* = C$, which gives: $\pi_i^* = (p_1^* - C) D_i(p_1, p_2) = 0$.

### *Location models in a competitive environment*

In the models of duopoly presented by Cournot, Bertrand, the decisions of the two competing companies are taken simultaneously. The optimal solutions for these concurrent decisions are called "Nash equilibrium" or sometimes called Cournot-Nash equilibrium .The preliminary work of locating facilities simultaneously has been proposed in [9]. Thus, since sequential decision-making leads to an asymmetry between decision-makers, we must differentiate the identity of decision-makers. In the case of a duopoly, the company that starts with making the location decision is called the leader and the other is called the follower. The Stackelberg model is based on three major assumptions: Decisions are made once and for all, Decisions are made sequentially, The leader and the follower have a complete knowledge of the localization game. An extension of the aforementioned model is given by the sequential localization model proposed by [10]. It deals with the case where a firm locates ***n*** installations, but the competitor firm locates only ***one*** installation.

Recently, a cooperative competition in facility location problems in which potential investors are in competition over acquiring suitable sites and clients is studied in [11]. An acceptance threshold constraint is applied to facility allocation that is based on a combination of distance between a facility and clients, and investors' product prices.

## PROBLEM DESCRIPTION

The problem considered in the paper is a two level supply chain network involving multiple demand points and two firms that have their own warehouses. Each firm tries to expand its distribution network by opening new stores. The objective for both firms is to have additional market shares.

Each firm in our paper faces demand from customers. In our problem, it is assumed that one of the two firms already has stores that are open to the market. The problem for every firm is to determine the exact location of the stores that will open. Then, these stores will have to supply a set of demand points. Similarly, we will have to determine for each store its warehouse that will supply it. Thus, the objective is to determine a distribution network that will provide maximum profit on the market.

### *Modeling problem*

In this section, we will formulate the Capacitated location-allocation problem in a competitive environment for a two level distribution Network. Indeed, it is a more realistic model than the model TLCLAP [12]. The problem is considered as localization game that is played between two players (frims) who want to locate stores among a predefined set of potential sites so as to satisfy demand points and maximize profit.

### *Formulation*

Now we define the notation used

n: Number of potential sites

m: Number of firm 1 pre-existing opened stores

N: Number of demand points

i: Index of potential sites i: $\{1,2,\ldots\ldots.., n\}$

j: Index of demand points $j = \{1,2,\ldots\ldots.., N\}$

f: Index of firms $f = \{1,2\}$

$w_{ij}$: Demand quantity expected by site i from demand point j

$C_f$ : Transportation cost paid by firm f to supply stores from warehouses

$Pr_f$ : The price fixed by the firm f

$F_{if}$ : Fixed cost associated with firm f for opening a store at site i

$\Gamma_{if}$ : Capacity available of store i , estimated by a firm f.

$A_{ij}$ : Attractiveness of demand point j with respect to store i

Decision variables:

$X_{ij}^f$ : Binary variable indicating whether demand point j is assigned to store i of firm f

$Z_i^f$ : Binary variable indicating whether store i is opened by firm f

Price policy is not defined by firms. We may consider that we are dealing with a factory price policy [13]since the price in each location is fixed and the customers insure their own transport. The price is independent of the chosen sites and is set by each firm. The fact that customers provide their own transport justifies the fact that demand will not increase with distance, and customers of a demand point will tend to frequent the store with greater attractiveness. Similarly, it is assumed that both firms have the same Transportation costing product from warehouses to their stores. For simplicity, this cost is assumed to be fixed.

As a game, we consider certain rules of the game. These rules are defined as follows:

- The pre-established stores of firm 1 remain open;
- Whenever the two firms decide to locate a service on a given site i, only firm 1 is able to do so. Thus, co-location is forbidden. This can be written as follows: $Z_i^1=(1-Z_i^2)$;
- A customer belonging to a demand point j will be served by a store with more attractiveness. This can be written as follows:

$$X_{ij}^f = \begin{cases} Z_i^f \text{ if } A_{ij} \geq A_{kj} \\ 0 \ otherwise \end{cases}$$

Firm 1 decides to open exactly p stores, this is expressed by the equation:

$$\sum_i^n Z_i^1 = p$$

It can also be seen that the total cost of opening stores can be written as follows:

$$\varphi(Z^1, Z^2) = \sum_i^n F_{i1} Z_i^1$$

To explain the problem, we define the profit equations for each firm. This profit corresponds to the difference between the revenues and operating cost of each firm. Taking the case of firm 1, we have its charge which corresponds to the Transportation costing the total quantity of products between an open node j and warehouse and the cost of opening node j. Moreover, the firm's total revenue corresponds to the product of the quantities and prices applied by the firm.

The profit formula is given as follow:

$$\pi_1(\sigma_1, \sigma_2) = (Pr_1 - C_1) \sum_j^N \sum_i^{n+m} w_{ij} X_{ij}^1 - \sum_i^n F_{i1} Z_i^1$$

With $\sigma_1$ and $\sigma_2$ are the location and allocation strategies respectively for firm 1 and firm 2. Thus, $\sigma_1$ can be expressed as a pair $(X_{ij}^1, Z_i^1)$ and $\sigma_2$ can be expressed as a pair $(X_{ij}^2, Z_i^2)$.

For firm 2, its profit equation is as follows:

$$\pi_2(\sigma_1, \sigma_2) = (Pr_2 - C_2) \sum_j^N \sum_i^n w_{ij} X_{ij}^2 - \sum_i^n F_{i2} Z_i^2$$

In the context of a game of Nash, $\sigma_1, \sigma_2$ are actions for firm 1 and firm 2, respectively, which we will call player 1 and player 2. These actions concern store locations and their assignments to a set of demand points.

We want to make predictions about the outcome of the game. In some cases we will be able to find an action for each player so that each player's action is the best response to the other's action. This means that even if one player can guess the other's action, he will not be able to change his chosen action, and this is valid for both players.

### *Calculating Nash Equilibrium*

Nash's theorem guarantees the existence of a Nash equilibrium, possibly involving mixed strategies for one or both actors, in each non-cooperative finite set such as the one considered in this paper. Moreover, it is important to note that, given the strategies, we do not assume that firms choose a random action, but only that the action of each firm can be seen as random by the other firm [14].

Based on the approach adopted by [15], a Nash equilibrium is calculated for the game. Thus, an algorithm based on the best responses of each player to the strategy of the other.

Let $\sigma_1^t$ and $\sigma_2^t$ be the vectors of strategy such that $\sigma_1^t$ is strategy t taken by the firm 1 and $\sigma_2^t$ by the firm 2. To simplify the implementation of the algorithm, we assume that we will start with the localization. Since we consider that the game is played simultaneously we suppose that the Player 1 always has the privilege of starting the game. This player starts by choosing, randomly, a location strategy $\sigma_1^1$. Then, it will have to build its distribution network by:

- Assign to each open site a set of demand points so as to respect the capacity constraint;
- Assign to each open site its supply warehouse

The second player looks for the best response for the strategy $\sigma_1^1$. We note this strategy $\sigma_2^1$. Player 2 have also to build his network distribution with the same procedure adopted by player 1.

Faced with this strategy, player 1 must find the best response that will be noted by $\sigma_1^2$. The game continues, until we find the Nash equilibrium.

To better understand how this game will enable us to seek the resolution of the problem, we present the matrix payoff as follows:

| | | Firm 1 | | | |
|---|---|---|---|---|---|
| Firm2 | | $\boldsymbol{\sigma}_1^1$ | $\boldsymbol{\sigma}_1^2$ | … | $\boldsymbol{\sigma}_1^s$ |
| | $\boldsymbol{\sigma}_2^1$ | $\pi_1^1, \pi_2^1$ | $\pi_1^2, \pi_2^1$ | … | $\pi_1^s, \pi_2^1$ |
| | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | $\boldsymbol{\sigma}_2^s$ | … | … | … | $\pi_1^s, \pi_2^s$ |

**Figure 1: M**atrix of game payoffs

Where,

$\pi_1^s = \pi_1(\boldsymbol{\sigma}_1^s, \boldsymbol{\sigma}_2^s)$ and $\pi_2^s = \pi_2(\boldsymbol{\sigma}_1^s, \boldsymbol{\sigma}_2^s)$

Moreover, by referring to the work of (Godinho and Dias 2010), when one begins with an action in which a given player does not open any store, he tends to lead the algorithm to reach a balance which slightly benefits the other player. In order to avoid this bias, in computational experiments we will always apply the algorithm twice: The first consists in starting with a zero action for firm 1 (ie the action in which the firm 1 does not open a store) and the second consists of a null action for firm 2.

As we can see in algorithm 1, a sequence of actions is played. A player chooses his strategy then the other player seeks the best response to this strategy. The game continues until reaching the maximum value S, which corresponds to the sequence where all the demand points are satisfied (assigned to a firm).

Algorithm 1: Location allocation game progress

**While (t<S)**

t=1,

- *Strategy ( $\boldsymbol{\sigma}_f^t$)*
- *Best response to Strategy ( $\boldsymbol{\sigma}_f^t$)*
- t=t+1

**End while**

The strategy of a player consists, firstly, to locate the sites and then to allocate them to the demand points and the warehouses. Suppose it is firm 1 that will start the game, then it will choose p candidate sites to open. This choice is made randomly. Then, the firm will try to allocate each selected site. The algorithm is based on the method of finding the demand points having a good attractiveness with respect to the chosen site. In addition, this allocation must take into account the capacity parameter of the installation.

However, it can be noted that in the algorithm the assignment of the sites to the warehouses was omitted. This is due to the fact that the transport cost is fixed for each firm which will change nothing in the calculation of the profit.

Algorithm 2 gives how a firm's strategy is constructed. Note that we have marked a firm by f, to say that it is possible to start the game either by firm 1 or firm 2.

Algorithm 2: Getting a strategy location allocation

*Strategy ( $\boldsymbol{\sigma}_f^t$)*

***Begin***

Let $E_p$ be a set of p potential sites, $E_p \subset \{1,2, \dots \dots .., n\}$;

Firm f: Choose randomly $E_p$

For each k in $E_p$ do

While ($\Gamma_{kf} \leq \sum_{j=1}^{N} w_{kj} X_{kj}$)

Begin

for j=1 to N do

If $A_{kj}$ is maximal then

$X_{kj}^f = 1$

End

End while

Return $\boldsymbol{\sigma}_f^t$

***End***

Against the strategy of firm 1, firm 2 must also take its strategy which will present the correct answer to that taken by firm 1. Indeed, the idea described in this paper consists in maximizing the profit of firm 2 by choosing sites with a low site establishment cost. This choice will concern a set of candidates' sites whose cardinality is r. This set corresponds to the sites to be opened by firm 2.

Algorithm to get a best response to a strategy $\sigma_f^t$

Algorithm 1: Best response to Strategy $\boldsymbol{\sigma}_f^t$

***Begin***

Choose the potential sites for which capacity is minimal $F_{if}$ is minimal

Let $E_r$ be a set of $r$ potential sites, $E_r \subset \{\{1,2,\ldots\ldots,n\} - E_p\}$;

For each k in $E_r$ do

While ($\Gamma_{kf} \leq \sum_{j=1}^{N} w_{kj} X_{kj}$)

Begin

For each j in Not Selected demand point {1,….,N}

If $A_{kj}$ is maximal then

$X_{kj}^{f} = 1$

End

End while

Return $Br(\boldsymbol{\sigma}_f^t)$

***End***

As this game is in a strategic form, the payoff matrix, established by the strategies given by the algorithms, provides all necessary information for finding the Nash equilibrium.

Algorithm 1: Nash equilibrium procedure

***Begin***

For t1=1 to S

For t2=1 to S

If ( $Br(\boldsymbol{\sigma}_1^{t1}) = \boldsymbol{\sigma}_2^{t2}$) and ( $Br(\boldsymbol{\sigma}_2^{t2}) = \boldsymbol{\sigma}_1^{t1}$)

Then ( $\boldsymbol{\sigma}_1^{t1}$, $\boldsymbol{\sigma}_2^{t2}$) is a Nash equilibrium

$\boldsymbol{\sigma}_1^{t1}$, $\boldsymbol{\sigma}_2^{t2}$ are the solutions for the problem

***End***

## PROBLEM RESOLUTION

To allow the model to be used as a tool of analysis, we developed an algorithm which solves several scenarios. It was then implemented in a computer via a programming language.

### *Experiments Design*

In order to provide better solutions for a good analysis, an Object-Oriented Programming approach was chosen. Indeed, we have defined several classes of objects that represent all the elements of the game. Which are as follows :

The Node class: It is used to manage node objects. It has all the properties for a node: node type (stores, warehouse or request point), node index, node setup cost if it is to be opened as a store, and finally the capacity. In addition to the properties, this class also includes the method of retrieving the index from the node, since the matrices related to the distribution network require it.

The Network class: It is used to manage the distribution network. It contains all the properties relating to a network: the list of objects of the node class, the number of nodes of the network, the matrix of requests for each node with respect to another, the attractiveness matrix for each node with respect to another. In addition, two methods are implemented: the method that allows to know the state of a node during the course of a game, and the method of initialization of the network. The latter consists in creating the network according to the number of fixed nodes, categorizing the nodes by type and inserting logistic data of each node and defining the nodes already opened by a given firm.

The Firms class: It allows to manage the firms that performs the allocation location set. It includes all the properties relative to a player: The player's index, the price he applies to the stores, the Transportation costing the merchandise the network object representing the distribution network of each firm, the list of the sites to be opened and the assignment matrix of each node. As for the methods, this class includes two essential functions that are to locate and allocate.

The Games class: This object class is used to manage the game. This class is responsible for generating the game instances related to stores to be opened by a firm, the number of nodes, logistics cost limits to allow generation of random values. Consequently, it implements the algorithm, and makes it possible to recover the results of each game. Similarly, this class allows you to calculate the profits for each player in a given experiment of the game.

### *Instantiation of the problem*

The procedure for generating each problem instance runs as follows:

A given number of candidate sites are chooses among the nodes of the network. This choice is made according to the capacity of each site. Thus, only those nodes with a large capacity are retained. This criterion allows to satisfy as much as possible the capacity constraint.

A firm defines the number of sites to open. Through a random function, we choose arbitrarily, among the candidate sites, those to be opened for each firm.

For allocation, a site is assigned to a node with the greatest attractiveness. This operation is repeated until the maximum of the network is covered.

***Program sequence***

The program was implemented with "C #" language, on a 2.53 Intel i5 machine with 4 GB of RAM. A random function was used to generate the problem instances. During the initialization phase,we define the global logistic network with the nodes and the different logistic parameters, we create the players through their parameters, and then the game starts with an arbitrary chosen player. He performs two operations: selecting sites to open and allocating each site to demand points. The other player performs the same approach until finding the best response to the choices made by the first player.

***Numerical tests***

We conducted numerical tests to evaluate the relevance of the chosen approach to determine the equilibrium of the game. We start by the instantiation of the tests objects. Indeed, a set of instances based on a distribution network has been generated in a random manner that include the number $\boldsymbol{m}$ and $\boldsymbol{n}$.

Those data are related to potential sites, logistics values, warehouse locations and demand points. Similarly, an attractiveness value has been assigned for each potential site with respect the nodes.

Furthermore, in order to evaluate the variation of the results according to particular parameters of the problem, we defined 19 sets of experiments. Each set includes instances.

On the other hand, because the objective of the problem presented is to look for the localization and allocation solutions in the case of competition, we will then consider how to look for possible Nash equilibrium. For this purpose, we will consider instances of small size so that we can use a simple algorithm without facing the constraint of the execution time. Therefore, we defined a network with 200 nodes.

For each experiment, we retains the profit of the firm 1 and the profit of the firm 2 which corresponds to the best response to the strategy of location and allocation of the firm 1. We will also be interested in the relative advantage of the Firm 1 on firm 2, which we measure as the ratio between the two profits.

To better analyze the established model, we have assumed 5 scenarios which are as follows:

Scenario 1: Both firms opt for the same price and the same freight cost. Firm 1 opens p stores and firm 2 opens $r$ stores with $\frac{\mathrm{p}}{\mathrm{r}} \leq 0{,}25$;

Scenario 2: Both firms opt for the same price and for the same freight cost. Firm 1 opens p stores and firm 2 opens r stores with $0{,}75 \leq \frac{\mathrm{p}}{\mathrm{r}} \leq 1$;

Scenario 3: Firm 2 adopts a double price compared to firm 1. Firm 1 opens p stores and firm 2 opens r stores with $0{,}75 \leq \frac{\mathrm{p}}{\mathrm{r}} \leq 1$ ;

Scenario 4: Firm 2 which adopted a double price compared to firm 1. Firm 1 opens p magazines and firm 2 opens r magazines with $0{,}75 \leq \frac{\mathrm{r}}{\mathrm{p}} \leq 1$ ;

Scenario 5: Firm 2 applies the same price as firm 1. Firm 1 opens p magazines and firm 2 opens r magazines with $0{,}75 \leq \frac{\mathrm{p}}{\mathrm{r}} \leq 1$ with advantage to firm 2.

***Results and analysis***

In this section, we will analyze the results obtained through the various pre-established scenarios in order to seek not only the solutions of the problem but also to examine the impact of the important parameters defining the problem. Thus, we will study the impact of the number of stores to be opened and the prices applied in the firms' distribution network.

Table 1 contains the results obtained for the case of the scenario1. We note that in all the strategies considered for firm1, profit is always lower than that of firm 2. This is explained by the number of open nodes. Indeed, increasing the number of sites to be opened for firm 2 enhance its profit advantage over that of firm 1 even if the preferential advantage has been granted to firm 1. This experiment has been repeated several times and a profit ratio of less than 1 is always obtained and the average of the profit ratios always approximates to 0.2.

In

Table 2, a similar experience to that of scenario 1 was carried out except that this time we assign 6 sites to be opened by firm 1 and 8 sites by firm 2. It is noted that Always firm 2 has a competitive advantage since only for strategies 4 and 8 it is surpassed by firm 1. In addition, there is always an average profit ratio of 0.2.

**Table 1:** Scenario 1 results

- **Number of sites to be opened: (Firm1: 2, Firm2: 10)**
- **Transportation cost: (Firm1: 200, Firm2: 200)**
- **Applied Price: (Firm1: 412, Firm2: 412)**

| Strategies | Profit Firm 1 | Profit best response Firm 2 | Profit1/Profit2 |
|---|---|---|---|
| **1** | 12210,85997 | 66231,90342 | 0,18436523 |
| **2** | 12661,52826 | 76844,37974 | 0,16476844 |
| **3** | 13387,70493 | 57447,30732 | 0,23304321 |
| **4** | 10038,09483 | 66249,28513 | 0,15152005 |
| **5** | 16807,92038 | 69318,49872 | 0,24247381 |
| **6** | 9501,558347 | 71442,56607 | 0,13299576 |
| **7** | 15686,27321 | 72461,12702 | 0,21647846 |
| **8** | 9831,182135 | 59829,70603 | 0,16431941 |
| **9** | 14717,95583 | 60494,09202 | 0,24329576 |
| **10** | 11609,06846 | 67183,25244 | 0,17279706 |
| **11** | 8598,234715 | 57007,63865 | 0,15082601 |
| **12** | 18218,65645 | 69520,36775 | 0,26206214 |
| **13** | 15593,0393 | 70218,24445 | 0,22206535 |
| **14** | 12042,67023 | 85478,55871 | 0,14088527 |
| **15** | 16597,22273 | 77033,59373 | 0,21545435 |
| **16** | 13666,37096 | 79599,63442 | 0,17168887 |
| **17** | 11896,1166 | 56543,55659 | 0,21038854 |
| **18** | 12049,16269 | 63474,76616 | 0,18982603 |
| **19** | 19547,07188 | 64593,16523 | 0,30261827 |

**Table 2:** Scenario 2 results

- **Number of sites to be opened: (Firm1: 6, Firm2: 8)**
- **Transportation cost: (Firm1: 200, Firm2: 200)**
- **Applied Price: (Firm1: 412, Firm2: 412)**

| Strategies | Profit Firm 1 | Profit best response Firm 2 | Profit1/Profit2 |
|---|---|---|---|
| **1** | 30317,71286 | 50201,7134 | 0,60391789 |
| **2** | 40504,69928 | 46931,34804 | 0,86306277 |
| **3** | 31708,7325 | 46589,32164 | 0,68060086 |
| **4** | 40529,44123 | 37609,8614 | 1,07762804 |
| **5** | 34419,56309 | 40976,22004 | 0,83998873 |

| | | | |
|---|---|---|---|
| **6** | 32652,48806 | 49951,6996 | 0,65368122 |
| **7** | 39116,06858 | 45349,90653 | 0,86253912 |
| **8** | 27558,62228 | 43012,38673 | 0,64071363 |
| **9** | 36317,03321 | 45306,8938 | 0,80157853 |
| **10** | 33242,62714 | 41397,9054 | 0,80300264 |
| **11** | 31711,29231 | 43088,78969 | 0,73595226 |
| **12** | 24843,09717 | 42134,80748 | 0,58960984 |
| **13** | 36808,94592 | 39350,10409 | 0,93542182 |
| **14** | 33541,87079 | 41043,89092 | 0,81721957 |
| **15** | 40342,91181 | 39174,34391 | 1,02982993 |
| **16** | 41718,07641 | 47318,96406 | 0,88163545 |
| **17** | 35512,212 | 42788,84192 | 0,82994095 |
| **18** | 37135,35132 | 44509,6785 | 0,83432082 |
| **19** | 27124,21446 | 49042,29315 | 0,55307802 |

In Experiment 3, the objective is to see the impact of price on equilibrium. Values given in

Table 3 indicate that the price has a certain impact on the profit of the firms. As shown in this table, the firm 2 which has adopted a double price compared to firm 1 have a high profit for all strategies. But looking at the previous experiments this policy will be in vain in case the firm 1 adjusts its opened nodes number. It is interesting to study this case and analyze the impact of appropriate number of nodes to reach a game equilibrium against a high pricing policy of the competitor.

To confirm the remark raised in scenario 3, we conducted an experiment in which we keep the same assumptions as the previous scenario, except for the number of magazines to open (

Table 2).. Indeed, we opted for 80 stores for firm 1 with a low price compared to that of firm 2. We note that in strategies 8 and 13, the gain of firm 1 is clearly higher than that of the firm 2. Similarly, it has been observed that, on average, the profit ratio is close to 0.93. This means that against an unfavorable price policy, it is possible to review the number of stores to reverse the situation to its benefit and reach equilibrium of close profits. In this way, the sites to open are qualified as a best strategy for a firm to deal with unfair pricing policy of competitors.

**Table 3:** Scenario 3 results

- **Number of sites to be opened: (Firm1: 80, Firm2: 40)**
- **Transportation cost: (Firm1: 200, Firm2: 200)**
- **Applied Price: (Firm1: 400, Firm2: 800)**

| **Strategies** | **Profit Firm 1** | **Profit best response Firm 2** | **Profit1/Profit2** |
|---|---|---|---|
| **1** | 43510,81567 | 111928,818 | 0,38873649 |
| **2** | 45504,92074 | 98168,97117 | 0,4635367 |
| **3** | 43363,95078 | 94888,1237 | 0,45700082 |
| **4** | 47380,59183 | 114135,1827 | 0,41512696 |
| **5** | 46838,79318 | 108148,3729 | 0,43309753 |

| | | | |
|---|---|---|---|
| **6** | 48487,41365 | 106570,4164 | 0,45498005 |
| **7** | 44820,6032 | 102236,5785 | 0,43840085 |
| **8** | 42541,90509 | 103514,6757 | 0,41097463 |
| **9** | 48482,04164 | 101643,4162 | 0,47698162 |
| **10** | 47196,69936 | 104430,842 | 0,45194215 |
| **11** | 42563,86026 | 99532,34249 | 0,42763849 |
| **12** | 45896,38975 | 106584,179 | 0,43061166 |
| **13** | 41853,7029 | 102239,281 | 0,40937008 |
| **14** | 53709,80043 | 101969,0108 | 0,5267267 |
| **15** | 45211,53387 | 102871,1546 | 0,43949671 |
| **16** | 48692,83301 | 106801,6229 | 0,45591847 |
| **17** | 42364,49113 | 109835,2352 | 0,38570948 |
| **18** | 45501,31425 | 109118,543 | 0,41698975 |
| **19** | 43460,40703 | 85314,97802 | 0,50941122 |

In Scenario 5, it is assumed that both firms decide to open the same number of stores, to use the same price, and they have the same transport cost. Table 5, shows the results obtained for this scenario. It can be noted that the firm 1 always has the advantage and it has a considerable gain compared to the firm 2. Thus, the firm 2 has no interest in keeping the same prices as those applied by the firm 1, or to downsize the number of opened stores compared to the competitor firm.

**Table 4:** Scenario 4 results

| - Number of sites to be opened: (Firm1: 80, Firm2: 40)<br>- Transportation cost: (Firm1 :200, Firm2 :200)<br>- Applied Price: (Firm1 : 400, Firm2 : 800) | | | |
|---|---|---|---|
| **Strategies** | **Profit Firm 1** | **Profit best response Firm 2** | **Profit1/Profit2** |
| **1** | 56409,53223 | 61718,04701 | 0,91398764 |
| **2** | 59381,88675 | 65395,19657 | 0,90804661 |
| **3** | 56803,64857 | 64981,02285 | 0,8741575 |
| **4** | 56116,00716 | 64900,38559 | 0,86464829 |
| **5** | 59145,39419 | 61094,52599 | 0,96809646 |
| **6** | 58200,5991 | 64906,16871 | 0,89668825 |
| **7** | 59291,63114 | 57740,72382 | 1,02685985 |
| **8** | 58278,38024 | 58204,62318 | 1,0012672 |
| **9** | 59389,95386 | 71328,67667 | 0,8326238 |
| **10** | 59292,63053 | 62858,68045 | 0,94326878 |
| **11** | 56150,65176 | 56867,19922 | 0,98739964 |
| **12** | 58193,61405 | 66053,22029 | 0,88101101 |
| **13** | 61966,23709 | 59043,67786 | 1,04949826 |
| **14** | 59690,69943 | 60943,32551 | 0,97944605 |

| | | | |
|---|---|---|---|
| **15** | 59903,61373 | 66444,14086 | 0,90156352 |
| **16** | 57973,50102 | 63012,76413 | 0,9200279 |
| **17** | 59014,24517 | 62438,64038 | 0,94515583 |
| **18** | 58324,74033 | 64707,99718 | 0,90135289 |
| **19** | 58432,84624 | 65057,46209 | 0,89817285 |

**Table 5:** Scenario5 results

- **Number of sites to be opened: (Firm1: 35, Firm2: 40)**
- **Transportation cost: (Firme1 :200, Firme2 :200)**
- **Applied Price: (Firme1 : 400, Firme2 : 400)**

| Strategies | Profit Firm 1 | Profit best response Firm 2 | Profit1/Profit2 |
|---|---|---|---|
| **1** | 43300,60677 | 31806,50483 | 1,361375825 |
| **2** | 44599,09582 | 32029,02941 | 1,392458549 |
| **3** | 49318,24856 | 32176,12233 | 1,532759232 |
| **4** | 44642,98259 | 32766,70393 | 1,362449598 |
| **5** | 45868,27374 | 32844,62195 | 1,396523114 |
| **6** | 43111,39625 | 31178,86031 | 1,382712383 |
| **7** | 46431,24162 | 31313,62397 | 1,482780839 |
| **8** | 49987,59868 | 32259,42411 | 1,549550249 |
| **9** | 46043,35848 | 33000,94388 | 1,395213381 |
| **10** | 44569,70096 | 32121,96873 | 1,387514611 |
| **11** | 43088,68356 | 31981,0049 | 1,347321127 |
| **12** | 42908,0734 | 35212,04621 | 1,218562339 |
| **13** | 44653,70051 | 30427,65835 | 1,467536542 |
| **14** | 44684,48901 | 28937,00393 | 1,544198879 |
| **15** | 45155,66552 | 36048,58845 | 1,252633389 |
| **16** | 45676,09503 | 30157,06476 | 1,514606789 |
| **17** | 45373,8374 | 30064,99899 | 1,509191383 |
| **18** | 42471,90866 | 34610,39079 | 1,227143285 |
| **19** | 49021,34863 | 33783,2559 | 1,451054593 |

It should be pointed out that beside the studied scenarios; we analyze through other experiments the impact of transport costs in the first level of the distribution network. It appears that this parameter has little impact on the profits of firms in relation to the price and the number of sites to be opened.

As a matter of fact, in a problem of location allocation in a competitive environment, the price to apply remains an important parameter that gives a real competitive advantage. Nevertheless, this parameter must be studied according to the number of stores to be opened by the competitor. But in reality, this number depends closely on the investment budget of the firm. This budgetary constraint limits the leeway of firms in setting the number of stores to open. Therefore, emphasis should be placed on the pricing policy to be adopted.

## CONCLUSION

The model we have developed deals with the problem of localization of new stores and proposes a modeling of the distribution network, an enumeration of the parameters and logistic constraints linked to the distribution activity as well as a resolution approach by the game theory. The purpose is to determine the optimal location of the extensions to operate on the distribution network that leads to a minimization of the overall cost. The choice of location in an existing distribution network obviously takes in account a customer demand and service objectives in the relevant geographical area and it is concerned also by internal and external logistics and technical parameters related to competition.

The resolution of the model requires an analysis of the location and allocation strategy. Thus, face to a strategy of a given firm, the competitor must develop the best response that lead to the Nash equilibrium of the game.

This model, which can be used to search for investment opportunities in optimal areas, also proposes an analysis of the commercial activity and the detection of geographical location possibilities, based on logistic performance criteria derived from the strategy of the company and taking into account those of the competitors.

The game theory used for this problem takes into account interactions with stores through the attractiveness parameter which has an important role in any location and allocation strategy.

However, this work can be improved in the future by other work including the selection of other specific parameters, testing for large problems and comparison of results with other methods within the spatial approach.

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