

# A Design Method for the Prediction of the Behavior of Moorings

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## Abstract

The ships are constrained mainly by mooring lines and fenders to rigid structures such as jetties and piers. The evaluation of the forces that affected the mooring lines and fenders is essential for proper design of all components of the plan such as the chocks, the cables and the bollards. A calculation software that can estimate the loads acting and how they are distributed is essential to be able to perform these calculations with accuracy. The article describes the design phases of the program and the theoretical aspects discussed and is a guide for designers who want to predict the behavior of the mooring plan.

**Keywords:** Moorings optimization, berthing, ship design, safety, environmental forces

## INTRODUCTION

The mooring of a berthed ship is a complex theme and it involves different aspects. A moored ship is constrained with different methodologies and it is free to move itself in response to many external factors such as: loading and unloading cargo, waves, wind, tides and currents. These movements are bounded inside limits dependent on the response of the mooring equipment. Therefore they are depending on the method used for constraining the ship [1]. Cables and fenders are the principal elements of a mooring equipment. The first responses exclusively to tensile forces, the second reacts only to compression forces. One of the first analytical method for solving the mooring problem of a berth ship was proposed by Chernjawski [2]. This method involves the reduction of the non-linearity equations that govern the physic of the problem in an algebraic one and the solution is founded with a matrix approach for a particular condition of environmental effects. Natarajan et al. [3] proposed an alternative analytical method with a more one step: each external condition is evaluated with empirical formulations in order to account the actual condition of the ship under investigation. This methodology helps to provide a good mooring design for each environmental condition with fast and accurate results. The empirical formulations are very good for preliminary design but it is clear

that more accurate results can be founded with the new technologies such as : Computational Fluid Dynamics for the interaction of the ship with the external forces [4–7] and structural analysis for the evaluation of the final resistance of the cables and fenders [8–13]. Naturally, these methods are too much complex and time consuming and usually the choice for solving the mooring problem is oriented to analytical methods with empirical formulations. A fast and accurate assessment of the forces involved during the mooring in each environmental condition allows also to evaluate the possible failure of the cables, so reducing problems such as collisions with the port quay [14] or unexpected movements of the ship during the loading and unloading operations. These events could lead to dangerous conditions in term of safe for people and for environmental pollution [15, 16] and so it's very important to conduct this calculation rapidly in order to follow the fast changes of the external environmental conditions. The mooring problem of a berthed ship can be studied with the aid of matrix methods taking in account the unilaterality of the constraints. In this study the basis of the analytical approach follow the studies proposed in technical literature but new improvements are proposed with particular focus on the accuracy, rapidly and simplicity of the methodology.

## NOMENCLATURE

Definition	Symbol	Formula	Unit
<b>Cross-Sectional Area of i-th mooring line</b>	$A_i$	-	$m^2$
<b>Wind angle direction</b>	$\eta$		
<b>Current angle direction</b>	$\varepsilon$		
<b>Direction of the wave</b>	$\mu$		
<b>Waterplane area of the Ship</b>	$A_w$	-	$m^2$

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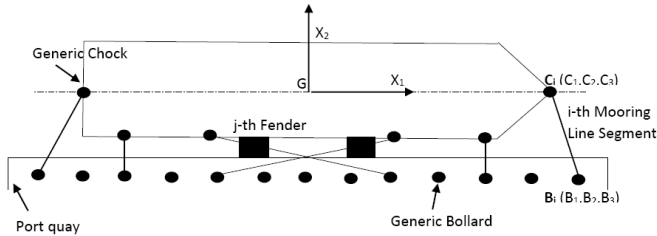
<b>Wind exposed projected area in i-th direction</b>	$A_{pi}$		$m^2$	<b>line</b>			
<b>Current exposed projected area in i-th direction</b>	$A_{ci}$		$m^2$	<b>Vector of external moments</b>	$M$		
<b>Breadth of the Ship</b>	$B$	-	$m$	<b>Number of mooring lines</b>	$n$		
<b>Coordinates of the ith bollard</b>	$(B_1, B_2, B_3)$			<b>Number of fenders</b>	$N$		
<b>Coordinates of the ith chock</b>	$C_i$	-	$m$	<b>Vector of the external forces</b>	$Q$		
<b>Matrix of i-th chock coordinates</b>	$[C]_i$		$m$	<b>Axis centered to Gravity of the Ship</b>	$X_{1,2,3}$	-	-
<b>Wind drag coefficient along the <math>\zeta</math> direction</b>	$C_w(\zeta)$			<b>Axis centered on the generic chock</b>	$X_{1i,2i,3i}$		
<b>Current drag coefficient along the <math>\varepsilon</math> direction</b>	$C_C(\varepsilon)$			<b>Stiffness of i-th mooring line along its own axis</b>	$k_i$		$N/m$
<b>Added mass coefficient of the ship</b>	$C_m$			<b>Stiffness of jth fender</b>	$k_j$		$N/m$
<b>Depth of the water</b>	$D$		$m$	<b>Displacement vector of the ship</b>	$U(U_1, U_2, U_3)$	-	$m$
<b>j-th fender reaction force</b>	$D_j$		$N$	<b>Displacement vector of the chock</b>	$U_{CI}$		$m$
<b>Deflection of the j-th fender</b>	$d_j$		$m$	<b>Velocity of the Wind</b>	$V_{Wind}$		$m/s$
<b>Draught of the ship</b>	$d$	-	$m$	<b>Velocity of the current</b>	$V_{Current}$		$m/s$
<b>Young's modulus of the mooring lines</b>	$E$	-	$MPa$	<b>Displacement of the Ship</b>	$\Delta$	-	$t$
<b>Stiffness of j-th fender</b>	$f_j$		$N/m$	<b>Angular vector of the ship</b>	$\theta(\theta_1, \theta_2, \theta_3)$	-	$rad$
<b>Centre of Gravity of the Ship</b>	$G_i$	-	$m$	<b>Wave number</b>	$k$		
<b>Gravity acceleration</b>	$g$		$m/s^2$	<b>Matrix of projected stiffness</b>	$[K]_i$		
<b>Longitudinal metacentric</b>	$GM_L$		$m$	<b>Mass density of the air</b>	$\rho_a$		
<b>Transversal Metacentric</b>	$GM_T$		$m$	<b>Mass density of the water</b>	$\rho_w$		
<b>Incident wave height</b>	$H$		$m$	<b>Angle between Segment mooring Line and plane</b>	$\alpha_i$		
<b>Length of the mooring line</b>	$l$		$m$	<b>Angle between Segment mooring Line and plane</b>	$\beta_i$		
<b>Unit vector of the generic mooring line</b>	$l_i$			<b>Angle between the mooring lines and its local axis</b>	$\varphi(\varphi_{1i}, \varphi_{2i}, \varphi_{3i})$		
<b>Reaction moments of i-th mooring</b>	$m_i$		$N\ m$				

In the equations, the vectors are identified by bold fonts and the matrixes by square brackets.

**MATERIALS AND METHODS**

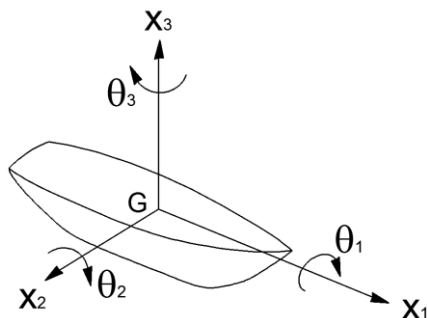
**The mooring problem - Reaction forces:**

The mooring lines that constrain a ship have a generic spatial orientation and assume, in function of the force with which they are put in tension, a form of catenary.



**Figure 1.** Generic mooring for a berthed ship

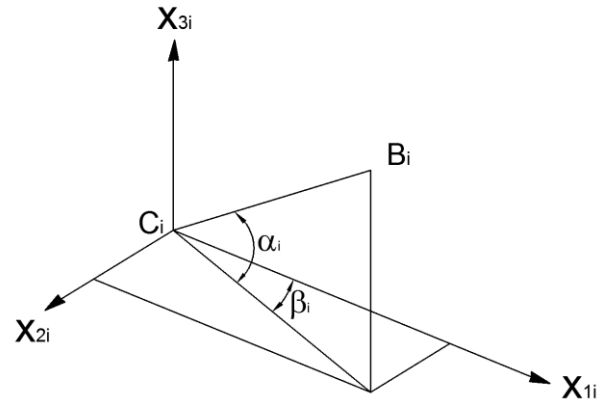
There are different lines during the mooring of a berthed ship and the name of each one of them is influenced by the constrain that they apply on the ship (bow lines, spring lines, breast lines and stern lines). In general, each line can be described, with an acceptable error, like a straight-line segment that starts on the ship in a chock and ends on the ground in a bollard. In this way, the catenary is approximated as a straight line but the error is very small when, as it is normally, the cable is sufficiently tense. Similarly, the contact between ship and fender can be taken as punctual, rather than superficial as in reality. Even this error can be considered acceptable in most cases because the size of the fenders is very small compared to the size of the ship. In the case in which the approximation is not acceptable it is possible to use a discretization of the fender in more points of contact neighbors. The system studied in fact has no limits in the number of mooring lines and fenders. The general reference used is a fixed reference system centered on the center of gravity of the ship and with the longitudinal axis  $X_1$ , positive towards the bow,  $X_2$  positive to starboard and  $X_3$  positive in vertical direction, as shown in Figure 1. In the picture are shown also the three orientation angles of ship respect the three axis. The position of the ship is identified by the displacement vector  $\mathbf{U}(\mathbf{u} \mathbf{1} \mathbf{0})$ , with the generic displacement  $u_1, u_2, u_3$  and the angles  $\theta_1, \theta_2$  e  $\theta_3$ .



**Figure 1.** Reference system and relative angles for motion of ship

*The mooring lines:*

A local reference system, parallel to the general one, for each mooring line and centered on the  $i$ -th chock, with the axes  $X_{1i}$ ,  $X_{2i}$  and  $X_{3i}$ , is defined in order to treat the generic mooring line such as segment. The origin is in the chock position defined by the vector  $\mathbf{C}_i(C_{1i}, C_{2i}, C_{3i})$  respect to the general reference frame, while the end of this segment is in the relative  $i$ -th bollard with position  $\mathbf{B}_i(B_{1i}, B_{2i}, B_{3i})$ . The line is geometrically defined by these points and by the angle  $\alpha_i$  (angle between the segment line and the Plane  $X_{1i}X_{2i}$ ) and the angle  $\beta_i$  (angle between the Segment Line and the Plane  $X_{1i}X_{3i}$ ) as shown in Figure 2.



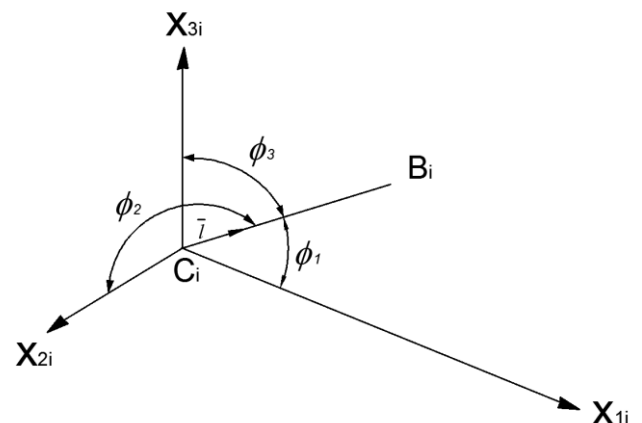
**Figure 2.** Segment mooring line and angles with Local Reference system

The unit vector  $\mathbf{l}$  is parallel to the  $i$ -th segment mooring line, it is defined with the  $\varphi$  vector angle with the components  $(\varphi_{xi}, \varphi_{yi}, \varphi_{zi})$  solved with the Equations (1-3) and shown in Figure 3.

$$(\cos \varphi_1)_i = \cos \alpha_i \cos \beta_i \tag{1}$$

$$(\cos \varphi_2)_i = \cos \alpha_i \sin \beta_i \tag{2}$$

$$(\cos \varphi_3)_i = \sin \alpha_i \tag{3}$$



**Figure 3.** The angles of the Segment mooring line with the local reference system

Each mooring lines is considerable as a spring which has its

elastic stiffness along its axis of work. The generic ropes reacts with a force only if it is stressed along the direction of tension. The i-th mooring line has a stiffness defined by the Equation (4) in case of ropes stressed with tension force, and Equation (5) in case of compression condition. The relation depends by the Young Modulus (E), the cross-sectional area (A) and by the length of the ropes between the chock position and the bollard one (l). This stiffness is the one along the axis of the i-th rope.

$$k_i = \left( \frac{E(T)A}{l} \right)_i \quad (4)$$

$$k_i = 0 \quad (5)$$

The movement of the ship produced by the external forces is defined by the displacement vector  $\mathbf{U}$ , with the its translational components ( $u_1, u_2, u_3$ ) and with its angular components ( $\theta_1, \theta_2, \theta_3$ ). This displacement of the center of gravity produces a stretching of the ropes along its own axis (6) and this deformation produces a tension (7) in each mooring line.

$$\delta \mathbf{l}_i = \mathbf{u} \cdot \mathbf{l}_i \quad (6)$$

$$\delta \mathbf{t}_i = k_i \cdot \delta \mathbf{l}_i \quad (7)$$

The i-th mooring line, in response to the displacement of the chock position (9-10), reacts with a force (11) in function of the projected stiffness defined by the Eq. (8).

$$[\mathbf{K}]_i = k_i \begin{bmatrix} \cos^2 \phi_1 & \cos \phi_1 \cos \phi_2 & \cos \phi_1 \cos \phi_3 \\ \cos \phi_1 \cos \phi_2 & \cos^2 \phi_2 & \cos \phi_2 \cos \phi_3 \\ \cos \phi_1 \cos \phi_3 & \cos \phi_2 \cos \phi_3 & \cos^2 \phi_3 \end{bmatrix} \quad (8)$$

$$\mathbf{U}_{ci} = \mathbf{u} - [\mathbf{C}]_i \boldsymbol{\theta} \quad (9)$$

$$[\mathbf{C}]_i = \begin{bmatrix} 0 & -C_3 & C_2 \\ C_3 & 0 & -C_1 \\ -C_2 & C_1 & 0 \end{bmatrix} \quad (10)$$

$$\mathbf{f}_i = [\mathbf{K}]_i \cdot \mathbf{U}_{ci} = [\mathbf{K}]_i \cdot \mathbf{u} - [\mathbf{K}]_i \cdot [\mathbf{C}]_i \boldsymbol{\theta} \quad (11)$$

Each reaction force produced by the mooring line is not applied in the center of gravity of the ship, so there are also moments (12) that depend by the position of the chock respect to the gravity of ship.

$$\mathbf{m}_i = [\mathbf{C}]_i \cdot \mathbf{f}_i = [\mathbf{C}]_i \cdot [\mathbf{K}]_i \cdot \mathbf{u} - [\mathbf{C}]_i \cdot [\mathbf{K}]_i \cdot [\mathbf{C}]_i \boldsymbol{\theta} \quad (12)$$

$$[\mathbf{A}]_i = -[\mathbf{K}]_i \cdot [\mathbf{C}]_i \quad (13)$$

$$[\mathbf{B}]_i = [\mathbf{C}]_i \cdot [\mathbf{A}]_i \quad (14)$$

Thanks to the simplification (13-14) is possible to resume the total effect of each mooring line (15).

$$\mathbf{F}_i = \begin{bmatrix} [\mathbf{K}]_i & [\mathbf{A}]_i \\ [\mathbf{K}]_i^t & [\mathbf{B}]_i \end{bmatrix} \cdot \mathbf{U} \quad (15)$$

The complete effects of all mooring lines can be summarized with a unique 6x6 matrix that is the sum of the single effects of

each mooring line. This matrix assumes that the mooring lines are spring with a bilateral behavior so it is symmetric and it does not consider the unilateral behavior of the ropes.

$$[\mathbf{K}_{Mooring}] = \begin{bmatrix} \sum_{i=1}^n [\mathbf{K}]_i & \sum_{i=1}^n [\mathbf{A}]_i \\ \sum_{i=1}^n [\mathbf{K}]_i^t & \sum_{i=1}^n [\mathbf{B}]_i \end{bmatrix} \quad (16)$$

*The fenders:*

In addition to the mooring lines, the ship is constrained with fenders in order to avoid impact against the quay and so protecting the ship during the rest in port. The fender works when it is compressed and the reaction force ( $D_j$ ) depend by the correspondent deflection ( $d_j$ ) by means of the stiffness of the fender ( $k_j$ )

$$D_j = k_j d_j \quad (17)$$

$$[\mathbf{K}_{fender}]_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_j & 0 & k_j r_{3j} & 0 & -k_j r_{1j} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_j r_{3j} & 0 & -k_j r_{3j}^2 & 0 & k_j r_{1j} r_{3j} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_j r_{1j} & 0 & k_j r_{1j} r_{3j} & 0 & -k_j r_{1j}^2 \end{bmatrix} \quad (21)$$

The fender is comparable to a spring with a stiffness that depends by the characteristics of the material. This quantity is equal to zero if the fender is working in tension condition so  $k_j = 0$  if the fender is in tension;  $k_j = f_j$  if the fender is in compression (material characteristic). The columns 1, 3 and 5 are null because, for longitudinal and vertical translation and trim rotation, the fenders can not react.

*The ship restoring forces:*

The ship has the capacity to response to the displacement caused by external forces with restoring forces and moments. These quantities are dependent by hydrostatic characteristics of the ship and they work with a force reaction along axis  $X_3$  and with two moments respect to the axis  $X_1$  and  $X_2$ .

$$F_{REST3} = \rho_w g A_w u_3 \quad (18)$$

$$L_{REST1} = \Delta G M_T \theta_1 \quad (19)$$

$$L_{REST2} = \Delta G M_L \theta_2 \quad (20)$$

The effects of these restoring forces can be resumed with a ship stiffness matrix (21) and all reactions of the ship are bilateral.

$$[\mathbf{K}_{Rest}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_w g A_w & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta G M_T & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta G M_L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

### The mooring problem - External forces

There are several kinds of external actions that can cause displacement of the ship during the rest in port. There are forces caused by the weather, such as wind or currents or waves. There are forces caused by internal operations like moving cargo or loading and unloading of weights. Usually all these kinds of forces acting in a dynamic manner, but it is acceptable to consider the hypothesis of quasi-static condition and apply this forces in static way.

#### Wind and current force:

The force produced by the wind is dependent by the exposed surface of the ship and the velocity of the wind. The most accurate way to know this quantity is the fluid-dynamics simulation, but it takes a long time so there are empirical formulations that help to know this quantity with sufficient precision and in very little time.

$$F_{wind}(\eta) = \frac{1}{2} \rho_{air} C_w(\eta) V_w^2 (A_{p1} \cos^2(\eta) + (A_{p2} \sin^2(\eta))) \quad (21)$$

The Eq. 21 helps to know the force of the wind in each direction of the wind. The exposed area is projected along the normal of the wind direction. The projection of this force along the longitudinal and transversal axis allows to know the external forces produced by the wind. These forces are not applied in the center of the gravity of the ship so they produce also moments. Usually the values of the  $C_w$  are between 0.4 and 1.2 for the case with wind along the longitudinal axis and between 0.6 and 1.4 for wind with transversal direction (the higher values are for tugs and trawlers, the lower values are for ferries).

In the same way is possible to calculate the force caused by the current (22). In this case the exposed surface is the immersed one. Also in this case there are moments produced by the force.

$$F_{current}(\varepsilon) = \frac{1}{2} \rho_{water} C_c(\varepsilon) V_c^2 (A_{c1} \cos^2(\varepsilon) + (A_{c2} \sin^2(\varepsilon))) \quad (22)$$

The drag coefficient  $C_c$  can vary between 0.1 and 0.4 for the longitudinal direction and between 0.7 and 1.0 for transversal direction in deep water. This coefficient is highly influenced by the depth of the water, in particular in shallow water, where the  $d/D$  ratio is less than 6, this coefficient can increase up to 6 times respect cases where the  $d/D$  is equal to 1.1.

#### Wave force

The wave load can be statically calculated thanks to the Eq. (23).

$$F_{wave}(\mu) = \rho_w C_m \mu g H D_0^2 \frac{\pi^2}{8} \left( \frac{\sinh kD - \sinh k(D-d)}{\cosh kd} \right) \quad (23)$$

$$D_0 = (L_s - B) \sin \mu + B \quad (24)$$

Generally  $C_m$  varies between 0.13 and 0.25 in longitudinal direction and between 1.45 and 1.75 in transversal direction.

A vector of external forces can be defined with a vector matrix (25) where there are the external forces components and the external moments components  $\mathbf{Q} = [\mathbf{F}_{ext} \mathbf{M}_{ext}]$ .

$$\mathbf{Q} = \begin{bmatrix} F_{wind1} + F_{current1} + F_{wave1} \\ F_{wind2} + F_{current2} + F_{wave2} \\ F_{tide3} + F_{load3} + F_{wave3} \\ M_{wind1} + M_{current1} + M_{wave1} + M_{load/unload1} \\ M_{wind2} + M_{current2} + M_{wave2} + M_{load/unload2} \\ M_{wind3} + M_{current3} + M_{wave3} \end{bmatrix} \quad (25)$$

### Unilaterality constraints problem

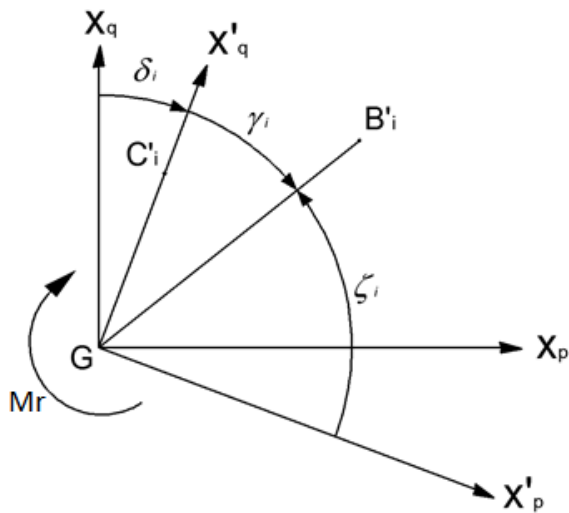
The problem of the unilaterality of the constraints must be taken in account in order to evaluate in correct way the reaction forces of the mooring lines and fenders. These two elements work only, respectively, in tension condition and compression condition. To solve this problem it is necessary, in the stiffness matrices of the mooring lines and fenders, to annul those terms related to a compression work of the mooring lines and those related to a pull work of the fender. The new stiffness matrices change from time to time in function of the external loads and they have an asymmetric shape.

#### Unilaterality of the mooring lines:

In order to understand if the generic mooring line is in compression condition we need to evaluate the sign of the external forces and moments. There is compression condition if the sign of the external forces are in agreement with the sign of the cosine director of the mooring line. In order to evaluate it, the fastest way is the sign of the product, element by element, between the  $\mathbf{F}_{ext}$  and the unit vector of the  $i$ -th mooring line  $\mathbf{i}_i$ . If the sign is positive, it indicates that the rope is in compression condition and so it is not able to react in that direction.

In the case of the moment, it useful to add three bidimensional reference local frame for each rope. Each defined local reference frame is centered in the vessel center of gravity. The first step is the projection of the chock point and bollard point in each plane of the fixed reference system  $X_p X_q$  ( $p = 1,2,3; q = 1,2,3; p \neq q$ ). With this projection is possible to define a local reference system  $X'_p X'_q$ . The axis of this local reference system are defined thanks to the angles  $\delta_i$  (26) for the projection of the chock point and  $\gamma_i$  (27) for the bollard point (Figure 4).

$$\cos \delta = \frac{C_q}{\sqrt{C_p^2 + C_q^2}} \quad \sin \delta = \frac{C_p}{\sqrt{C_p^2 + C_q^2}} \quad (26)$$



**Figure 4.** The moving and fixed bidimensional reference system

In the same figure it is also visible an external moment  $M_r$  ( $r = 1,2,3; r \neq p \neq q$ ) that acts around the perpendicular axis to the plane and the angle  $\zeta_i$  that is complementary to  $\gamma_i$ . Thanks to the Carnot theorem is possible to evaluate the angle between the projection of the chock point and the bollard point and so knowing the value of  $\gamma_i$  (27).

$$\gamma_i = \arccos \frac{\overline{GB_i^2} + \overline{GC_i^2} - \overline{C_iB_i^2}}{2\overline{GB_i}\overline{GC_i}} \quad (27)$$

In consequence of a rotation of the ship as a result of an external moment  $M_r$ , the angle  $\gamma_i$  will change, in the case of small displacements of a quantity  $\delta\gamma_i$  (29) and it will become equal to  $\gamma_i'$  (28)

$$\gamma' = \gamma - \delta\gamma \quad (28)$$

And the

$$\delta\gamma_i = \Psi \frac{M_r}{|M_r|} \cdot \frac{\cos \zeta_i}{|\cos \zeta_i|} \quad (29)$$

The  $\Psi$  is an arbitrarily small positive quantity. The only important thing of this small angle displacement is the sign. This sign is influenced by the plane of projection with the angle  $\zeta_i$  and by the sign of the moment. If the  $\delta\gamma > 0$ , the mooring line is not in tension and it does not work. If instead  $\delta\gamma < 0$  then the mooring line works and reacts with a force. In order to take account of this process, in the matrix of the chock points the one that does not work is deleted and the matrix of stiffness is recalculated.

#### Unilaterality of the fenders

A linearization of the problem it is necessary in order to take account about the unilaterality of the fenders. The evaluation of the unilaterality can be made with the use of a virtual

displacement of the ship as if it was constrained solely by the ropes. In this case is possible to find a virtual displacement vector  $U'$  (30)

$$U' = [K_{moor}]^{-1}Q \quad (30)$$

Because the matrix  $[K_{fender}]_j$ , related to the j-th fender, has only the columns 2, 4 and 6 non-zero, this columns are that ones have to be verified, in particular, exploiting the sign of the elements:

- the column 2 must be canceled if  $u'_2 R_{2j} < 0$ ;
- the column 4 must be canceled if  $\theta'_1 R_{2j} R_{3j} > 0$ ;
- the column 6 must be canceled if  $\theta'_3 R_{1j} R_{2j} < 0$ ;

Consequently, it is possible to build the matrix

$$[K_{fender}] = \sum_j [K_{fender}]_j$$

#### SOLUTION OF THE PROBLEM

The problem involves an algebraic system of equations. A global stiffness (31) matrix is obtained, with relative corrections in order to take in account the unilaterality characteristics of mooring lines and fenders.

$$[K] = [K_{moor}] + [K_{fender}] + [K_{Rest}] \quad (31)$$

An external vector force is defined thanks to empirical formulations. The solutions of the problem is obtained with the relation in Eq. (32) in terms of displacements.

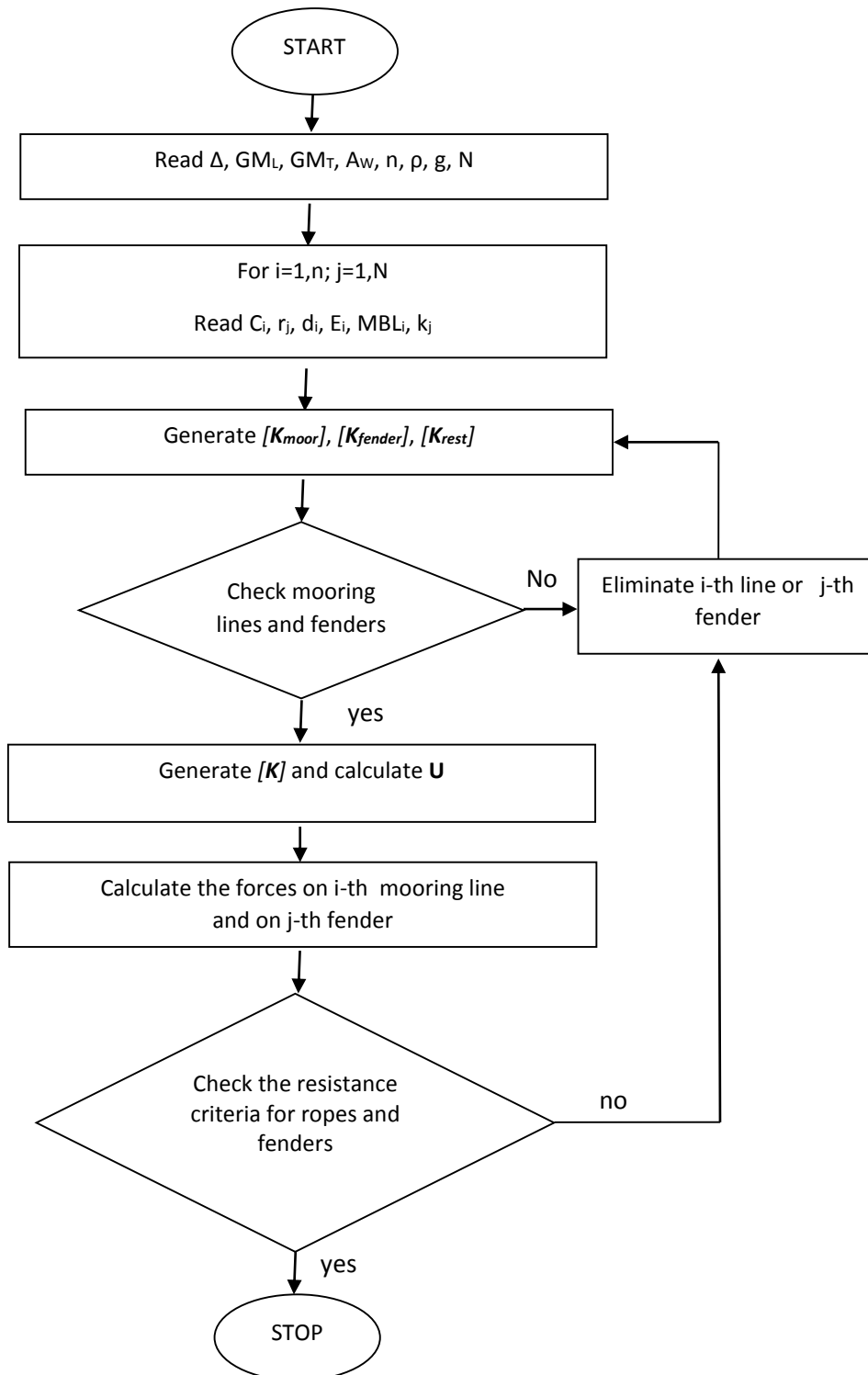
$$U = [K]^{-1}Q \quad (32)$$

With the Eq. (11) is possible to know the force on the i-th mooring line. In order to verify the is with the comparison of the i-th force with the maximum breaking load of the rope  $MBL_i$  multiplied for a coefficient ( $\chi$ ) that takes in account the limit of the breaks of the winches (generally calibrated to 70% of the  $MBL_i$ ). This condition on the winches allows working always under the hypothesis of small displacement.

$$|f_i| \leq MBL_i \chi \quad (33)$$

With the Eq. (17) it is possible to know the force on the j-th fender. Also in this case, the safe condition is evaluable with the condition defined above. When the force exceed the value of breakage, the hypothesis of small displacement will be no more valid.

The method here presented can be resumed with a flow-chart shown in Figure 5.



**Figure 5.** Flow chart of the entire process

### CONCLUSIONS

The presented method is simple to implement and it can be used for solving the problem of mooring of a berthed ship. The work is based on literature but new improvement has been added in order to take in account the unilaterality of the constraints. The method involves different aspects such as evaluation of geometrical characteristics of the mooring elements (ropes and fenders), the evaluation with empirical

formula of the external forces and the evaluation of the tensional state of the ropes and fenders. Each force is applied during the process with a quasi-static approach and the entire solution process is based on the hypothesis of small displacements. An important evolution of the method respect to precedent papers is the evaluation of the unilaterality behavior of the constraints, both for the mooring lines and fenders. For each condition, a global stiffness matrix is

constructed ad-hoc and in most of the cases this matrix is not symmetric. Thanks to this approach it is possible to simulate many external conditions in order to find the worst condition for the integrity of the ropes and of the fenders. It is also possible to construct the polar curves that indicate, in function of the direction and intensity of the wind, the moment in which the ropes reach the breaking force. The simulation run very fast and in few seconds is possible to have a complete panoramic of the all mooring conditions.

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