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Optimization of Thermal Calculation of Buildings on Permafrost Soils to Reduce Boundary Effects

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Abstract

Justifying calculations of engineering solutions in the construction of buildings in the conditions of permafrost requires solving the optimization task of the foundation construction, which provided sufficient reliability and low cost. The foundations calculations in the conditions of cryolite zone necessarily include the calculation of temperature fields to determine the depth of permafrost thawing zone, and vertical settlement. Currently these thermal tasks can be solved by using numerical methods, which allow taking into account the complexity of the foundation construction. These methods include the finite element method and finite difference method. The disadvantages of these methods include the impossibility of the exact solution with the boundary conditions at infinity, which leads to boundary effects. To reduce the time for solving optimization tasks it is necessary to approximate the infinitely large computational domain with such form of computational domain with limited sizes, to ensure the highest accuracy and with the smallest area. The authors set the task of a building thermal interaction with permafrost in bounded and unbounded domains in the stationary case and obtained an analytical solution for both cases. To eliminate the Gibbs phenomenon, negatively affecting the estimation of boundary effects, the solution for a limited area has been transformed from a Fourier series to the integral form. The solution for an unbounded domain was obtained by methods of the theory of the Green's function for the half-space. Numerical analysis of boundary effects on the expression obtained allowed us to determine the optimal ratio of the width and depth of the computational domain, equal to 2.8: 1. For ease of use results in engineering practice, nomograms and a technique were made, allowing determining the optimal size of the computational domain, which can reduce the amount of machine computations.

Keywords: permafrost, boundary effects, Fourier series, Gibbs phenomenon, the Green's function for the half-space

INTRODUCTION

Buildings construction in the engineering-geological conditions of cryolite zone requires calculation justification of the choice of foundation construction to ensure its stability [1-3]. The most important calculation is the thermal one, which shows the size of the thawed ground zone under foundation base [4-5]. Further, this size is used in the ground sediments calculation. In process of choosing foundation construction, it is necessary to conduct calculations of many structural designs options and choose the best one both from engineering and economic point of view.

Today, thermal problems are solved based on the differential heat conduction equation by numerical methods that allow considering the complexity of structural design [6, 7]. However, unlike the analytical, numerical solution is always limited by accuracy [8, 9]. For example, a two-fold increase in accuracy of an explicit difference scheme of the first order approximation in time is achieved by a twofold increase in grid density for each coordinate axis. This leads to a sixteen-fold increase in the volume of machine operations for three-dimensional problems, and eightfold - for two-dimensional. Reducing the span time on variative study can be achieved by reducing the size of the simulated soil array at a constant grid density. But the temperature field is distorted in this case under the influence of the so-called boundary effects. Therefore, when using numerical methods one always have to search for balance between speed and accuracy of the calculation.

Since the dependence of boundary effects on the parameters of the design scheme is investigated insufficiently, a problem of choosing the optimal size of the computational domain for a given solution accuracy for warm buildings on the surface of permafrost was solved by methods of the theory of convergent trigonometric Fourier series and the Green's function for the half-space. For convenience of work results practical use, a technique is described and a nomogram is compiled.

METHODS

A thawing bulb under the buildings is at its deepest level in the end of its expected lifetime. At this point, the temperature mode of permafrost becomes close to stationary, thermal effect of construction reaches boundaries of the computational domain, boundary effects take their maximum values. Therefore, while solving the problem, we will use heat conduction equation in a stationary setting:

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dz^2} = 0, \quad (1)$$

where V – temperature distribution, K; x, z – coordinates, m.

Boundary effects is the distortion of temperature field near the borders, simulating an infinitely large environment. Based on the determination, to estimate their value is possible by comparing the analytical solutions of the problem in a limited and unlimited domain.

First, consider the limited computational domain (Fig. 1)

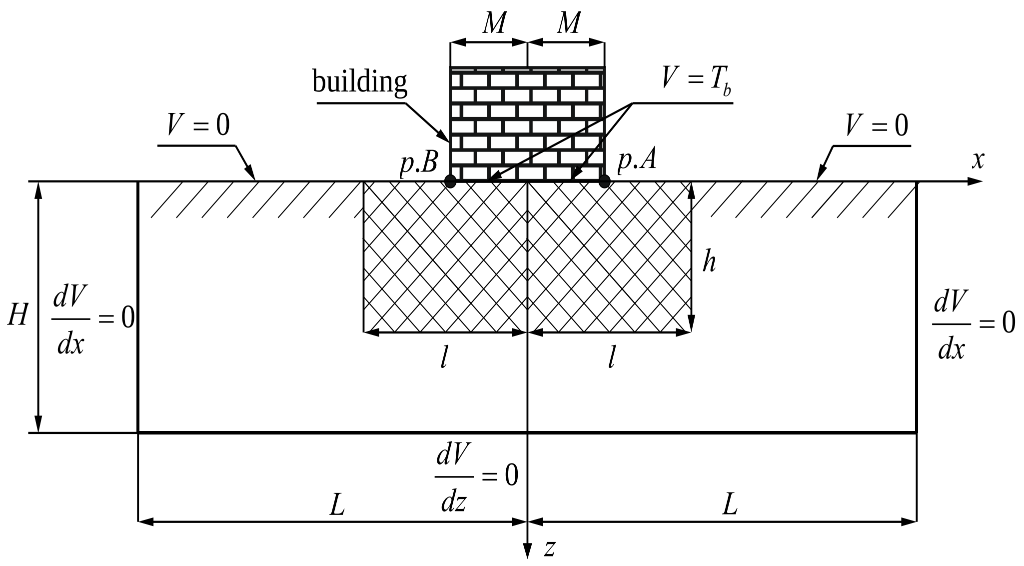

Figure 1. Design scheme for buildings with shallow foundations: T_b- temperature on the border of the contact warm building - ground, ° C; 2M - linear dimension of warm construction, m; H - The depth of the simulated ground array, m; 2L- width of the simulated ground array, m; h - the estimated depth of thawing halo; 2l - the estimated width of the thawing halo.

The boundary conditions for the calculation scheme in Figure 1:

$$-\lambda \frac{dV_1(x,z)}{dz}\bigg|_{z=H} = 0;\ -\lambda \frac{dV_1(x,z)}{dx}\bigg|_{x=0} = -\lambda \frac{dV_1(x,z)}{dx}\bigg|_{x=L} = 0; \quad (2)$$

$$V_1(x,0) = \begin{cases} 0, & -L < x < -M; \\ T_b, & -M \le x \le M; \\ 0, & L > x > \mathrm{M}. \end{cases} \quad (3)$$

The analytical solution of the equation (1) under the boundary conditions (2,3) in the form of Fourier series is well known [10]:

$$V_1(x,z) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{\pi n}{L}x\right) \cosh\left(\frac{\pi n}{L}(H-z)\right). \quad (4)$$

From (4) it is clear that the existence of functions $cos\left(\frac{\pi n}{L}x\right)$ and $cosh\left(\frac{\pi n}{L}(H-z)\right)$ allows satisfying boundary conditions (2). To determine the constants $\mathrm{A_n}$ and $\mathrm{A_0}$ you must use boundary condition (3) and the theory of trigonometric Fourier series:

$$A_0 = \frac{1}{2L}\int_{-L}^{L} V_1(x,0)\,dx; \quad A_n = \frac{1}{L}\frac{1}{\cosh\left(\frac{\pi n}{L}(H)\right)}\int_{-\mathrm{L}}^{L} V_1(x,0)\cos\left(\frac{\pi n x}{L}\right)dx. \quad (5)$$

We obtained expressions (4,5), determining the temperature in a limited ground array with a warm construction on the surface. However, this expression has a certain disadvantage. The presence of discontinuity points of the first kind (p.A and p.V) in the boundary conditions on the ground surface leads to a deviation of the Fourier series values from an approximating function by up to 18%. This deviation is known as the Gibbs phenomenon [11].

Due to the fact that the depth of penetration of distortions into the ground array caused by the Gibbs phenomenon is unknown beforehand and it is impossible to estimate the their distribution in the computational domain, the formula (4) was converted to integral form.

$$V_1(x,z) = \sum_{m=-\infty}^{m=+\infty} \int_0^{\infty} \frac{2T_b}{\pi} \frac{\sin(N_n M)\cos(N_n(x-2Lm))\cosh(N_n(H-z))}{N_n \cosh(N_n H)} dN_n. \quad (6)$$

Now it is necessary to accomplish the same task for an unlimited ground array ($L \to \infty$, $H \to \infty$). For this purpose, we use the methods of the Green's function theory for the half- space $z > 0$ [12]:

$$V_1(x,y,z) = \frac{z}{2\pi}\int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} \frac{V_2(u,0)}{((u-x)^2+(v-y)^2+(z)^2)^{3/2}} dv. \quad (7)$$

The equation (14) can be easily integrated, considering the on-surface ground temperature independence from the coordinate y:

$$V_2(x,z) = T_b \frac{atan\left(\frac{M-x}{z}\right)+atan\left(\frac{M+x}{z}\right)}{\pi}. \quad (8)$$

Now, to determine the value of boundary effects you only compare the results of calculations using formulas (13) and (15), and find in the considered ground array the deviation value, maximum in modulus:

$$\Delta U = |V_2 - V_1|. \quad (9)$$

RESULTS

Figure 2 shows a graph of given value of the temperature deviation $\Delta U/T_b$, calculated according to the formulas (6,8,9).

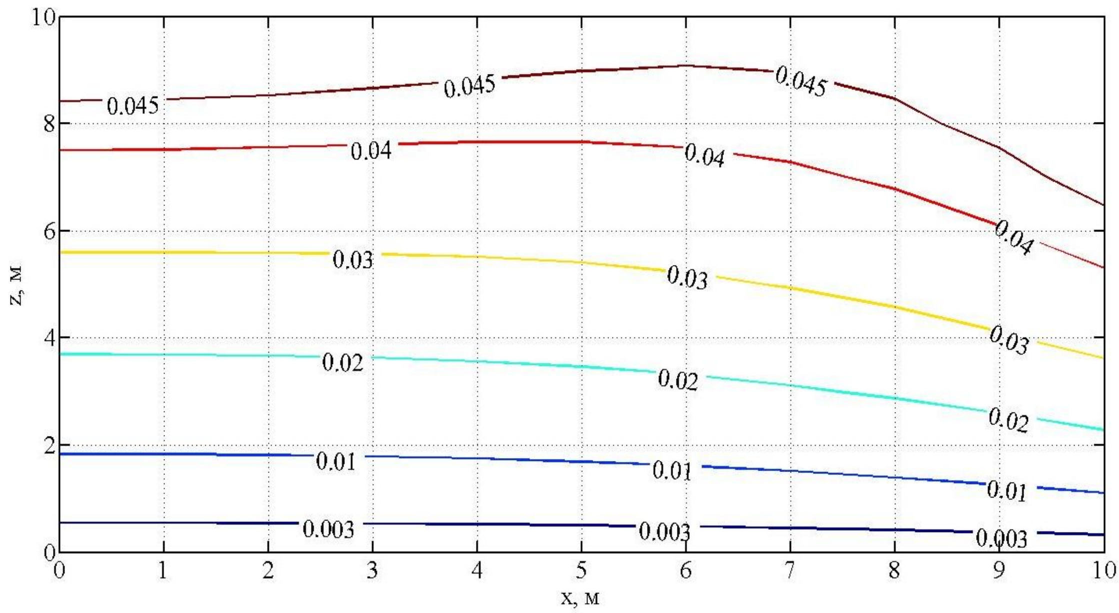

Figure 2. Isometric lines value. The calculation is performed for the case H=10M, L=10M

Distribution of boundary effects shown in Figure 2 is the most typical for the tasks with geometry shown in Figure 1. The graph shows that the magnitude of deviation $\Delta U/T_b$ increases with depth, whereas there is no deviation on the surface. This fact can be explained logically. At the upper boundary, we require meeting accurately the boundary condition - "temperature" and, consequently, the deviation on the surface is zero. By increasing the depth, boundary effects are growing and taking the greatest value at the lower boundary. Therefore, checking the deviation at the lower boundary is enough for calculation of the value of boundary effects.

DISCUSSION

The analysis results of calculations of boundary effects according to the formula (16) revealed the following:

1. Dependence $HL = f(H)$ at $\Delta U/T_b = const$ has a minimum. Consequently, there is such a relation of the sides L to H, at which the simulated ground array area has a minimum value for a given value of boundary effects. We have found that this ratio can be taken as: L/H=1,4.
2. The constancy of L/H allows us to create a simple and easy-to-use technique of choosing the optimal size of the computational domain for a given solution accuracy.

Following is the sequence of actions when using nomograms for selection of the optimal sizes of computational domain:

1. Collecting initial data: ΔT_{max} - required solution accuracy; °C;T_b – construction temperature difference module and ground temperature at a depth of zero annual amplitude; °C;h_{max} – the maximum depth, at which it is necessary achieve a desired solution accuracy ΔT_{max}, м; l_{max} – the maximum distance from the central axis of the building to the point in which it is necessary to achieve a desired solution accuracy ΔT_{max}, м; $2M$ – the smallest in terms of construction size, m.
2. Calculation of the dimensionless values:

   $q = \frac{\Delta T_{max}}{T_b}$ – given calculation accuracy, dimensionless;

   $h = \frac{h_{max}}{M}$ – given maximum depth to which it is necessary provide a desired solution accuracy q, dimensionless;

   $l = \frac{l_{max}}{M}$ – given distance from central axis of the building to the point in which it is necessary to provide a desired solution accuracy, dimensionless.
3. Finding the dimensionless depths of the ground array, which will provide the given solution accuracy. It is necessary: to find on the nomogram (Figure 3 - 6.) values h, l and q; lay off perpendicular to the axis h straight up to the intersection point with the isoline q; lay off a perpendicular from the found point to the left to the intersection with the axis $ln(H)$.

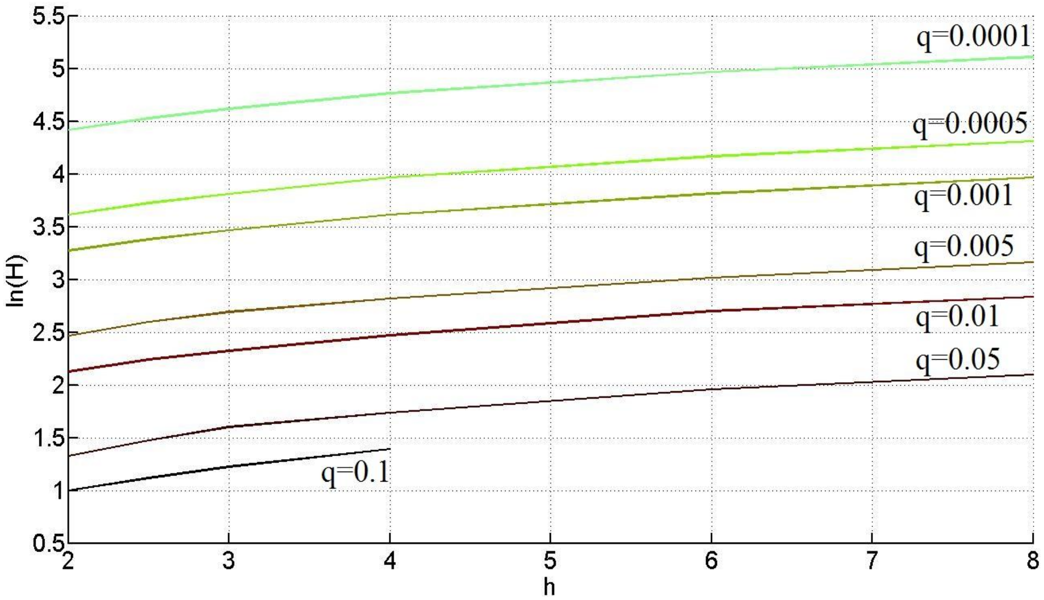

Figure 3. The nomogram for determining the optimum depth of the computational domain H for the case $h \in [2; 8]$, $l = 1$

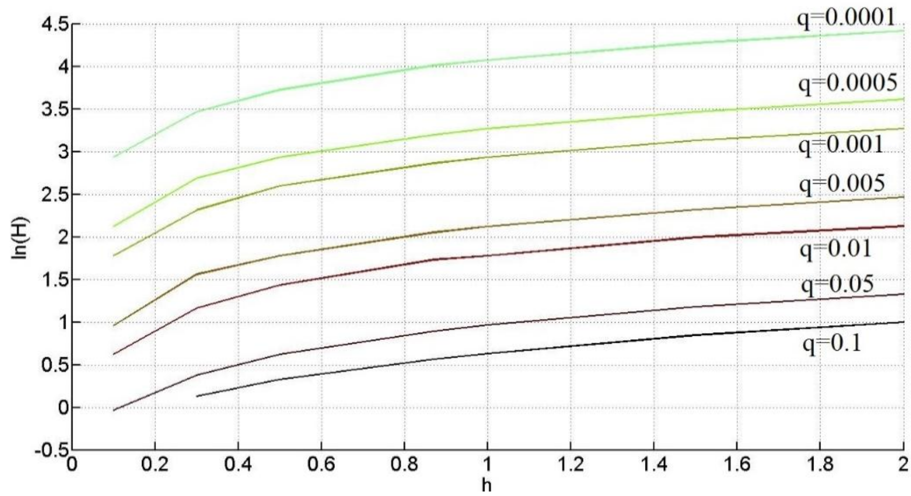

Figure 4. The nomogram for determining the optimum depth of the computational domain H for the case $h \in [0.1; 2]$, $l = 1$

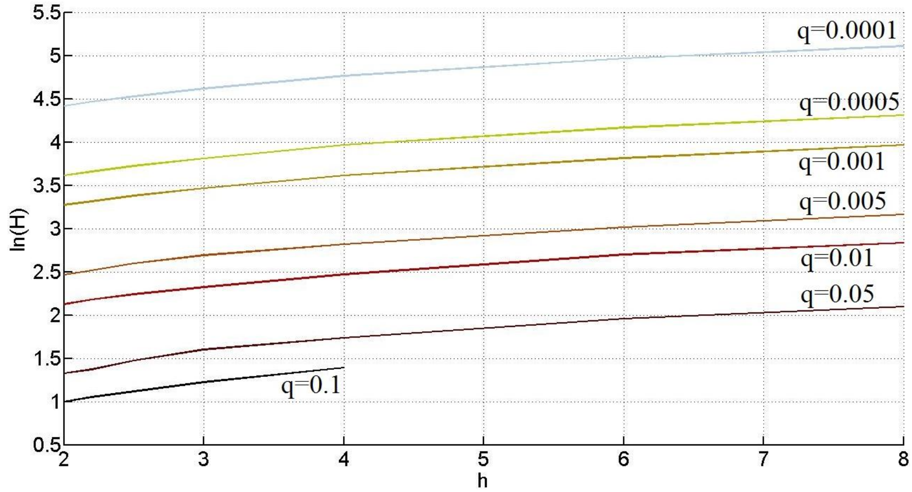

Figure 5. The nomogram for determining the optimum depth of the computational domain H for the case $h \in [2; 8]$, $l = 3$

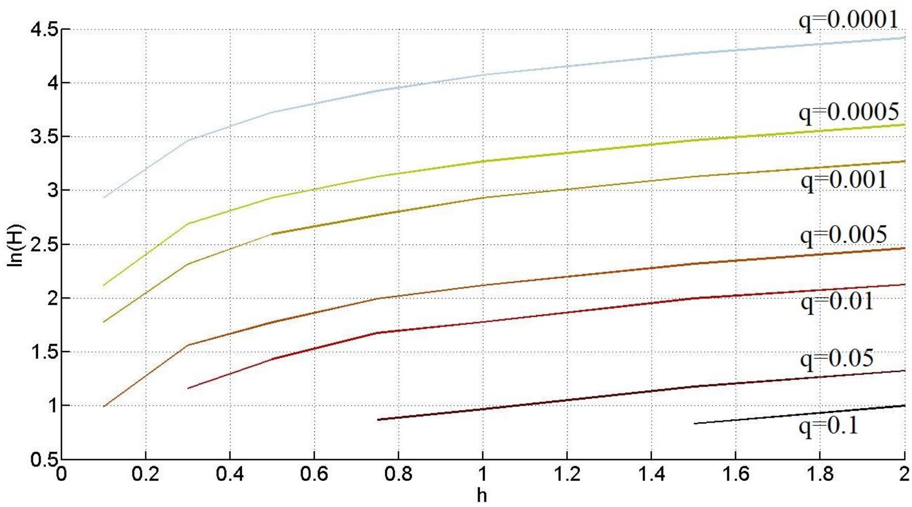

Figure 6. The nomogram for determining the optimum depth of the computational domain H for the case $h \in [0.1; 2]$, $l = 3$

4. Transfer of given depth to the absolute values:
   $H_{abs} = H \cdot M$ – the minimum depth of the ground array, which will provide given solution accuracy ΔT_{max}, m.
5. Calculation of ground array width:
   $2L_{abs} = 2{,}8 \cdot H_{abs}$– the width of the ground array, which will provide given solution accuracy q, m.

CONCLUSION

The problem of the building thermal interaction with permafrost in bounded and unbounded domains in the stationary case was solved. To eliminate the Gibbs phenomenon affecting negatively on estimation of boundary effects, the solution for a limited area was transformed from a Fourier series in integral form.

It was found that the optimal ratio of the width and depth of the calculated area is equal to 2.8: 1. For ease of use, the results in engineering practice, nomograms and a technique were made for determining the optimal size of computational domain, which ensures the highest accuracy of the calculation with the least time-consuming.

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