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Optimization of Discrete Problems using Artificially Smoothed Objective Function and Sectioning Method

Young-DooKwon[1*], Seong-Su Kim[2], and Jin-SikHan[2]

[1]School of Mechanical Engineering & IEDT, Kyungpook National University, Daegu, 41566, Korea.

[2]Department of Mechanical Engineering, The Graduate School of Kyungpook National University, Daegu, 41566, Korea.

[2]Department of Mechanical Engineering, The Graduate School of Kyungpook National University, Daegu, 41566, Korea.

Abstract
One may apply a global search algorithm like GA (genetic algorithm) in stepwise problems; however, the gradient method is difficult to apply because of the jump in the objective function at the boundary between each section. For rapid computation through the adaptation of the gradient method in a stepwise problem, the objective function could be modified to an artificially continuous function for a large-sized problem accepting the risk of local convergence. The artificially continuous objective function exhibits poor performance with a global search algorithm like GA.
In this study, a new method of modifying the objective function to an artificially smoothed function is proposed, and its effectiveness is tested. These artificially smoothed objective functions have no discontinuity with respect to the continuous variable of discrete condition; thus, they can be readily applied to gradient methods. The method of an artificially smoothed objective function does not exhibit any certain defect occurring in the method of the artificially continuous objective function. Its performance is almost the same as that of the original stepwise objective function, which results successfully for any global search.
Finally, the sectioning method is proposed which can search in different densities along variables according to the behavior of objective function to reduce computational time with the order 10 or 100.

Keyword: Discrete optimization, Artificially smoothed objective function, sectioning method, SZGA, hybrid method

INTRODUCTION

Discrete optimization is a type of optimization where the solution vector contains discrete variables [1-3]. There can be a couple of ways to handle them, one of which is discrete, as they are similar to the integer program. The other way is a continuous way that maps the discrete variables to a continuous variable. A variable representing the discrete variable is divided into many sections, and each section corresponds to a set of discrete variables. The objective function is mostly continuous and smooth; however, it is stepwise with respect to the representative continuous variable.

An objective function that is stepwise with respect to the representative continuous variable behaves suitably for a global search algorithm like GA (genetic algorithm) [4]; however, it will exhibit some divergence for a gradient search algorithm like the steepest descent algorithm [5]. Actually, GA works very ineffectively for a large computation problem such as FEM (finite element method) based on the optimization of a large structure. Thus, a method that modifies the continuous objective function artificially has been proposed to be applied to a large computational problem optimizing a rising sector gate [6]. The AC (artificially continuous) objective function was not as effective in the global search algorithm as it was in the gradient search.

In this concern, a method that modifies the stepwise objective function to an AS (artificially smoothed) objective function is proposed. This modification results in an objective function similar to the stepwise function, but the sudden jumps are smoothed by adding a modifying function. Therefore, the AS objective function acts as effectively as the original stepwise function for the global search algorithm. The AS objective function is smooth owing to the added modifying function. Thus, it is applicable to the gradient search algorithm without exhibiting any divergence. With this AS objective function, one can adopt any type of method, gradient search or global search, without any difficulty or inefficiency.

Including the kind of discrete problem mapped onto a continuous variable, there may be many problems where the objective function behaves differently along each continuous variables. In accordance with the complexity of objective function along each variable, the search density needs to be different. The sectioning method divides to searching domain into many sections along the variable of complex objective function. The searching points are sampled one by one in each section in turn, the densities of points are different along each variable then for effective searching algorithm.

DISCRETE OPTIMIZATION PROBLEM

In this study, a discrete variable optimization was attempted in a manner that the design variables get generated as continuous variables. Then, the objective function is stepwise along the representative continuous variable. This type of objective function is good for genetic algorithm, however, poor for a gradient method because of jumps. The artificially continuous objective function can serve effectively for the gradient method which has fast solution for time consuming objective function by FEM with possibility of local optimum. To resolve these problems artificially smoothed objective function is proposed that can be applied to any types of problem effectively.

Objective function

We set an arbitrary discrete variable, which is not continuous. The continuous variable is divided into many sections, and each section corresponds to a set of discrete variables. In other words, this objective function is the same as the stepwise objective function with respect to the representative continuous variable, as shown in Fig 1.

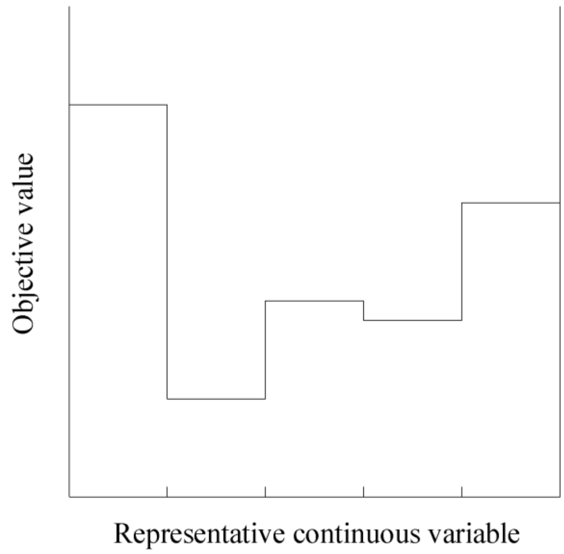

Figure 1. The stepwise objective function

For example, girder problem is a discrete problem. The conditions are as shown in Table 1.

Table 1. Conditions of discrete problem selecting best girder in each position

Discrete problem for girder	position 1	one from (1, 2, 3) girders
	position 2	one from (4, 5, 6) girders
	position 3	one from (7, 8, 9) girders
Find best set	set(1, 4, 7)	0 ~ 1/27 mapping
	set (1, 4, 8)	1/27~2/27 mapping
	set (1, 4, 9)	2/27~3/27 mapping
	⋮	⋮
	set (3, 6, 8)	25/27~26/27 mapping
	set (3, 6, 9)	26/27~27/27 mapping

Three girder positions should be determined as well as the optimal thicknesses satisfying assigned constraints. There are three possibilities in each position. Thus, there are 27 position sets. These sets are mapped in 27 sections of a continuous variable. Thus, objective function of girder problem becomes as shown in Fig. 1, where only 5 sections are displayed.

Next, we proposed an artificially continuous objective function, which is interpolated from the stepwise function. This artificially continuous objective function gives a gradient at any point, and a gradient method is applied to obtain the exact optimal solution in a shorter time, as shown Fig. 2.

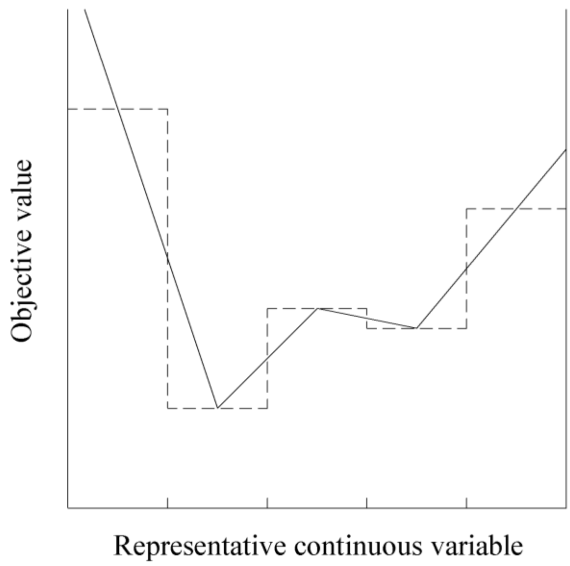

Figure 2. The AC objective function

Then, we proposed an artificially smooth objective function. This modification results in an objective function similar to the stepwise function, but the sudden jumps are smoothed by adding a modifying function. The artificially smooth objective function uses the delta function, as shown in Fig. 3.

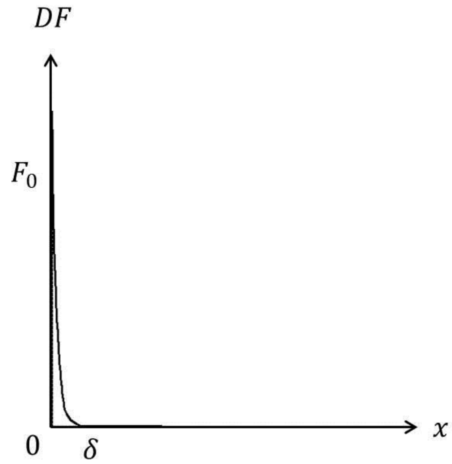

Figure 3. The delta function

The delta function is expressed in

$$\mathrm{DF} = \begin{cases} \mathrm{F_0}\left(1 - \frac{\mathrm{x}}{\delta}\right)^2 & (0 < \mathrm{x} < \delta) \\ 0, & (\mathrm{x} \geq \delta) \end{cases} \quad (1)$$

where,

$$\mathrm{DF}'(0) \neq \infty$$
$$\mathrm{DF}'(\delta) = 0$$

By using the delta function, the artificially smoothed function is shown in Fig.4.

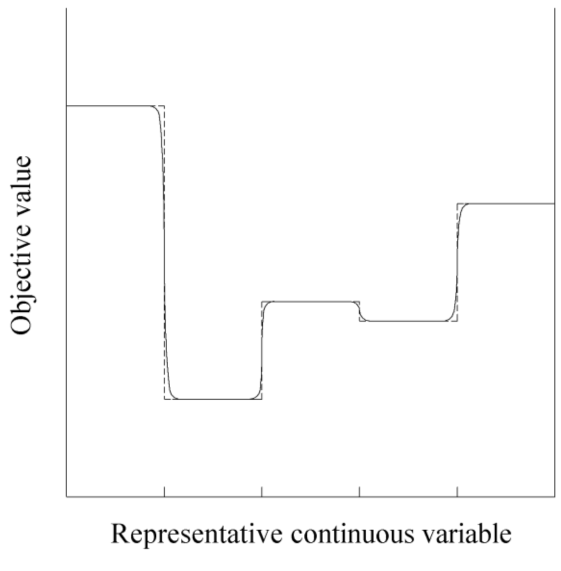

Figure 4. The artificially smoothed function

Therefore, the objective function still becomes almost as the stepwise objective function.

SECTIONING METHOD

For the dense individuals (children) along the axis of complex objective function, the range of this axis is divided into several sections. The individual generated by the genetic algorithm to test the fitness are put in each section in turn and repeatedly. In order to explain sectioning method, if the complex axis of 2-D problem divides into five sections, it could be shown in Fig.5.

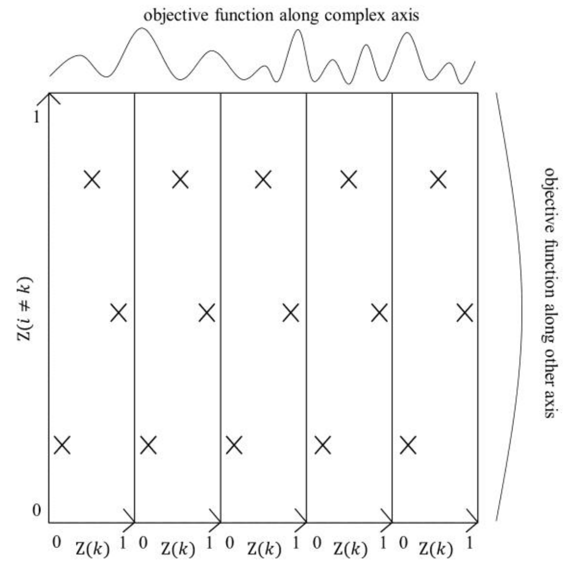

Figure 5. Five sections and individuals (children) fall in each section in turn and repeatedly

After total 15 searches, the variations in k axis are 15 and 3 for other axis.
The relations between global variable $\mathrm{y}(i)$ and dimensionless variable $\mathrm{Z}(i)$ are as follows;

In usual GA:
$\mathrm{y}(i) = y_1(i) + \{y_2(i) - y_1(i)\}\mathrm{Z}(i)$
(i=1~N, $0 \leq \mathrm{Z}(i) \leq 1$)
(N : number of variables)

In sectioning GA with five sections for example:
For the complex axis $i = k$ (k : the axis number of complex objective function)

$$y_{\mathrm{I}}(i) = y_{\mathrm{I}1}(i) + \{y_{\mathrm{I}2}(i) - y_{\mathrm{I}1}(i)\}Z(i)$$
$$y_{\mathrm{II}}(i) = y_{\mathrm{II}1}(i) + \{y_{\mathrm{II}2}(i) - y_{\mathrm{II}1}(i)\}\mathrm{Z}(i)$$
$$\vdots$$
$$y_{\mathrm{V}}(i) = y_{\mathrm{V}1}(i) + \{y_{\mathrm{V}2}(i) - y_{\mathrm{V}1}(i)\}Z(i)$$

For other axis $i \neq k$

$$\mathrm{y}(i) = y_1(i) + \{y_2(i) - y_1(i)\}\mathrm{Z}(i)$$

The sectioning method proposes a way that decreases the number of calculation times. For 4-D problem, there are a large number of searches. For example, if 2-D needs 30 searches in function complex and 10 searches in function simple, 10 searches in function simple becomes 30 searches, because there is no way of different search density, so the total number of searches is $(30)^2$, i.e., 900searches. Moreover, the total number of searches for 4-D are $(30)^4$, i.e., 810,000 searches. To analyze the 4-D problem, prolonged time is required. However, by using the sectioning method of three sections, the number of cases is reduced efficiently as follows.

Usual method

$$N_{usual} = \left(N_{rep} = N_{com} \times n_{sect}\right)^{n_{dim}} \quad (2)$$

Sectioning method

$$N_{sect} = {N_{com}}^{n_{dim}} \times n_{sect} \quad (3)$$

Reduced rate

$$R_{red} = \frac{n_{sect}}{{n_{sect}}^{n_{dim}}} = \frac{1}{{n_{sect}}^{n_{dim-1}}} \quad (4)$$

where

N_{usual} : search no. for usual method
N_{sect} : search no. for sectioning method
N_{rep} : search no. along representative variable
N_{com} : search no. along complex variable
n_{sect} : sectioning no.
n_{dim} : dimension no.
R_{red} : reduced rate

If we divide function complex, the total number of cases for 4-D problem reduces considerably as shown in Table 2.

Table 2. Search numbers for 2-D and 4-D problem

Dimension of problem	Not using sectioning method	Using sectioning method	R_{red}
2-D	$(30)^2$ searches = 900 searches	$3 \times (10)^2$ searches = 300 searches	1/3
4-D	$(30)^4$ searches = 810,000 searches	$3 \times (10)^4$ searches = 30,000 searches	1/27

Search number of 2-D problem reduces 1/3 by using sectioning method. In addition, search number of 4-D reduces 1/27. The sectioning method would be much more effective on high dimensional problem.

VERIFICATION USING FORTRAN

The objective function was classified as a 1-D, 2-D, and 4-D problem depending on the number of variables of the objective function. Then, the results of optimization accuracy that used MGA [7] or SZGA (successive zooming genetic algorithm [8-9]) for stepwise or converted objective function were compared with each other. This analysis was carried out using FORTRAN. The population sizes range from 2 to 40,000 in terms of variables. The generation number for all cases is set as 600. If the optimized value, which was set previously on objective function, was not found, the population number was increased.

1-D problem

The 1-D problem is relatively simple. In this case, MGA was used for the stepwise, AC, and AS objective functions. Eq. (5) shows a function of 1-D problem. The optimum value is set as 1.00000, and the verification result is shown in Table 3.

$$f = f(x) = \begin{cases} 10 & (0 < x < 0.25) \\ 4 & (0.25 < x < 0.5) \\ 1 & (0.5 < x < 0.75) \\ 6 & (0.75 < x < 1) \end{cases} \quad (5)$$

Table 3. The results of 1-D problem

Objective function	Population size	Optimum value
Stepwise	2	1.00000
Artificially Continuous(AC)	2	1.00588
	3	1.00313
	4	1.00210
	10	1.00054
	15	1.00279
	20	1.00123
	100	1.00007
	200	1.00026
Artificially Smoothed(AS)	2	1.00000

The MGA finds optimized value 1.00000 directly by using population size 2 on stepwise and AS objective functions with 600 generations. However, MGA cannot find optimized value 1.00000 on AC objective function, even though the population size is increased to 200.

2-D problem

The analysis for the 2-D problem is verified similarly. The 2-D problem has the same number of generations of the 1-D problem. Eq. (6) shows a function of 2-D problem. The optimum value is also 1.00000, and the result of verification is shown in Table 4.

$$f = f(x, y) = g(x) + \{y - y_0\}^2 \quad (6)$$

where

$$g(x) = \begin{cases} 10 & (0 < x < 0.25) \\ 4 & (0.25 < x < 0.5) \\ 1 & (0.5 < x < 0.75) \\ 6 & (0.75 < x < 1) \end{cases}$$

$$y_0 = \begin{cases} 0.2 & (0 < x < 0.25) \\ 0.4 & (0.25 < x < 0.5) \\ 0.6 & (0.5 < x < 0.75) \\ 0.8 & (0.75 < x < 1) \end{cases}$$

Table 4. The results of 2-D problem

Objective function	Population size	Optimum value
Stepwise	4	1.00000
Artificially Continuous(AC)	4	1.00981
	6	1.01469
	8	1.00887
	20	1.00328
	30	1.00571
	40	1.00444
	100	1.00105
Artificially Smoothed(AS)	4	1.00000

This result has the same tendency as that in Table 3. We can ensure that MGA is more effective for stepwise and AS objective functions than for the AC objective function.

4-D problem

The 4-D problem is fairly complicated because it has a large number of cases. Eq. (7) shows a function of 4-D problem. The optimum value is set as 2.00000, and the result of verification is shown in Table 5.

$$f = f(x, y, z, w) = g(x) + \{y - y_0\}^2 + \{z - z_0\}^2 + \{w - w_0\}^2 \quad (7)$$

where

$$g(x) = \begin{cases} 4 & (0 < x < 1/6) \\ 2 & (1/6 < x < 2/6) \\ 5 & (2/6 < x < 3/6) \\ 2.1 & (3/6 < x < 4/6) \\ 5.1 & (4/6 < x < 5/6) \\ 6 & (5/6 < x < 1) \end{cases}$$

$$y_0 = \begin{cases} 0.2 & (0 < x < 1/6) \\ 0.3 & (1/6 < x < 2/6) \\ 0.1 & (2/6 < x < 3/6) \\ 0.4 & (3/6 < x < 4/6) \\ 0.5 & (4/6 < x < 5/6) \\ 0.5 & (5/6 < x < 1) \end{cases}$$

$$z_0 = \begin{cases} 0.4 & (0 < x < 1/6) \\ 0.5 & (1/6 < x < 2/6) \\ 0.3 & (2/6 < x < 3/6) \\ 0.6 & (3/6 < x < 4/6) \\ 0.7 & (4/6 < x < 5/6) \\ 0.5 & (5/6 < x < 1) \end{cases}$$

$$w_0 = \begin{cases} 0.6 & (0 < x < 1/6) \\ 0.7 & (1/6 < x < 2/6) \\ 0.5 & (2/6 < x < 3/6) \\ 0.8 & (3/6 < x < 4/6) \\ 0.9 & (4/6 < x < 5/6) \\ 0.5 & (5/6 < x < 1) \end{cases}$$

Table 5. The results of 4-D problem

Objective function	Population size	Optimum value
Stepwise	8000	2.00004
Artificially Continuous(AC)	8000	2.00466
	12000	2.00261
	40000	2.00124
Artificially Smoothed(AS)	8000	2.00003

We found optimum values near 2.00000 by stepwise and AS objective functions with the same number of generation; however, AC objective function found optimum values far from 2.00000. Even though the population size was increased to 40000 in AC objective function, the results were not better than those of stepwise and AS objective functions. Table 3, 4, and 5 show that the stepwise and AS functions are more effective for MGA than the AC function.

Comparison of the results of MGA and SZGA on AS objective function with sectioning method

In the previous analysis, only MGA was used. The MGA found the exact optimum value 1.00000 on the 1-D and 2-D stepwise and AS objective functions well. However, MGA could not find the exact optimum value 2.00000 on 4-D stepwise and AS objective functions. In this section, SZGA is used to find an optimum value for the AS objective function. Comparison of the results of MGA and SZGA on AS objective function is shown in Table 6.

Table 6. The results of MGA & SZGA applied on AS objective function

Problem	Population size	MGA	SZGA
1-D	2	1.00000	1.00000
2-D	4	1.00000	1.00000
4-D	8000	2.00003	2.00000

Both, SZGA and MGA found the exact optimum value 1.00000 for 1-D and 2-D problem. Moreover, SZGA found the exact optimum value 2.00000 for 4-D problem although MGA could not find it.

In addition, the sectioning method is applied to the final case, which used SZGA on a 4-D problem. The result of optimization was 2.00000 by using a population size of only 300. Even though the sectioning method reduces iteration, the optimum value of 2.00000 was found.

APPLICATION USING PIAnO

PIAnO is a commercial package for executing process-integrated optimal design [10]. It changes design variables for an objective function, executes finite element analyses, conducts a test of the objective function, checks the constraint conditions, and repeats this process as necessary to find an optimal solution.

Using an AS objective function with MGA followed by gradient method (hybrid method) improves the accuracy for discrete optimization problems. For verification, this method was applied to the position and thickness optimization problem of the steel ventilation covering. This was studied by Kwon et al. [11]. The objective of this problem is to determine the most appropriate locations and optimal thickness for ribs of ventilation covering to minimize their total weight. After completing the optimization process, an optimal solution satisfying all of the imposed requirements was acquired. Fig. 6 shows possible locations of ribs assigned numbers between 5 and 14. As summarized in Table 7, the safety factor of the structure is 5 with regard to the yield stress of SM490.

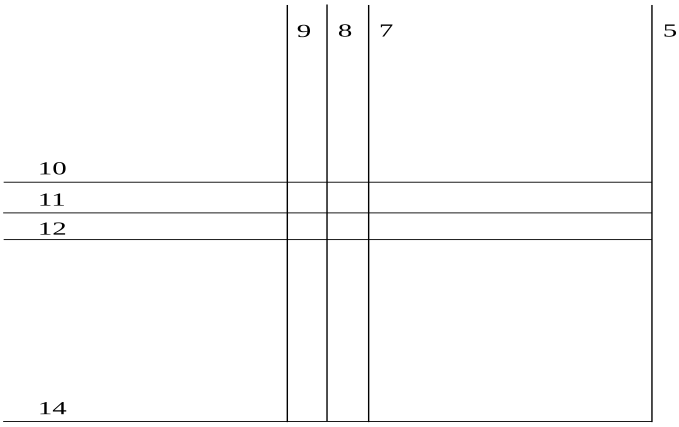

Figure 6. Possible locations of ribs

Table 7. The optimal solution using MGA with AS object function

Evaluation index	Design parameters	Initial state	MGA(AC)	Hybrid(AS)
Design variables	thickness (x_α , mm)	10	7.062	8.058
	thickness (x_β , mm)	10	4.859	7.825
	thickness (x_γ , mm)	10	3.496	3.394
	thickness (x_δ , mm)	10	7.778	3.350
	position set (α, β, γ, δ)	(5,8,11,14)	(5,7,12,14)	(5,7,12,14)
Objective	Weight (W_{rib}, Mg)	0.195	0.113	0.110
Constraints	von Mises stress (σ, MPa) (< 50.000 MPa)	33.43	49.93	49.68
Iteration	-	-	2000	264

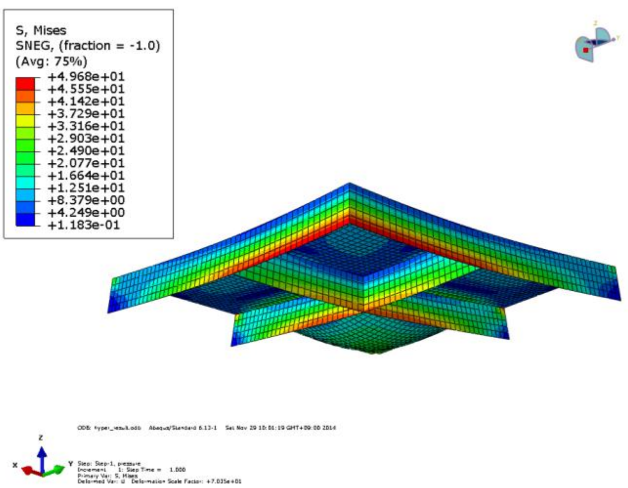

Figure 7. The result of FEM using hybrid method

As shown in Fig. 7, the maximum von Mises stress evaluated for the structure using MGA with gradient method on the AS objective function is 49.68 MPa, which is very close to the upper limit of 50 MPa. As a result, the optimal position set of the ribs was revealed to be (5, 7, 12, 14), with thicknesses of 8.058 mm, 7.825 mm, 3.394 mm, and 3.350 mm, respectively. The result using hybrid method is same as the result of using MGA in position set, however, different in thicknesses. The iteration of the hybrid method was just 264 which is smaller than the iteration of MGA, which was almost 8 times larger.

CONCLUSION

The discrete optimization problem can be converted to a continuous optimization problem by using a representative continuous variable mapping the set of discrete variables. AC (artificially continuous) objective function method has been proposed to adopt the gradient method, which is effective in local search problems, but not as effective as the original stepwise objective function generally is in global search problems. To extend the progress up-to-date, further, in this study,

- AS (artificially smoothed) objective function is proposed for the effectiveness in both the local and global searches. AS objective function, which adopts the SZGA (successive zooming genetic algorithm), exhibits fast and precise convergence.

- Sectioning method that uses denser population for the representative continuous variable mapping discrete variable, reduced much computational load than overall even searches. It would be also effective in other problems of continuous variables.

- A hybrid method adopting the global search followed by a gradient search reduced many FEM computations by not falling in local optima but showing fast convergence at the later stage.

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