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Analysis and comparison of different methods to determine Weibull parameters for wind energy potential prediction for Tetouan, northern Morocco

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Abstract

In this paper, five methods of Weibull parameters estimation have been carried out. The meteorological data of Tetouan city in Morocco measured during 3 years (12 May 2010 – to - 12 May 2012).These measurements are collected using a measuring station located on the roof of the Physics Department of Sciences Faculty in Tetouan (FST) -Morocco. The realized study showed that the maximum likelihood method presents a best adjustment of two Weibull Distribution Probability Function (W. PDF) parameters. The different results has been tested by the determination of R^2 coefficient, chi-square coefficient as well as the Root Mean Square Error (RMSE) in order to determine efficiently the wind energy potential for different Moroccan sites.

Keywords: Weibull. Probability Distribution Function (W. PDF), W. PDF parameters, Wind speed, Wind energy

INTRODUCTION

Nowadays, the current energy landscape is characterized by the continuous increase in global demand, declining reserves of fossil fuels, rising prices, and climate change. Thus Morocco, like many non-producing oil countries, has to look for others sources of energy for sustainable development. Thus, he has launched several development projects of renewable energy. These energies are clean, showing no danger for the environment and are sustainable.

Morocco has important clean and renewable energy resources. Among the most abundant resources and best indicated for a large-scale operation was wind energy. However, identifying and effectively estimate the wind potential at a given site, is a must and an essential step. Among the methods used for this, we use the probability density functions such as the Weibull distribution. In this work, we provide a comparison of five methods for determining the parameters of Weibull. Several references are used to complete our work [1], [2], [3].

WEIBULL PROBABILITY DISTRIBUTION FUNCTION (W. PDF)

The expression of the Weibull distribution has two parameters:

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^{k}\right) \quad (1)$$

Where v indicated the wind speed (m / s), k denotes the shape parameters, while c (m/s) correspond to the value where W. PDF is maximal and it called scale parameter.

The expression of the cumulative function can be given by:

$$F(v) = \int_0^{v_x} f(v)dv = 1 - exp\left(-\left(\frac{v_x}{c}\right)^{k}\right) \quad (2)$$

STATISTICAL STUDY

In order to determine the wind factors that characterized the site, we identify the following parameters: $\bar{v}$ denotes the mean wind speed, v_f the frequency wind speed, and v_d is the energetic wind speed, P_d is the power density and σ^2 is the variance of the distribution of wind speed:

TABLE 1. EXPRESSIONS MEAN FREQUENCY AND ENERGETIC WIND SPEEDS DEPENDING ON THE MODEL.

Distribution	Mean wind speed $\bar{v}$	frequency wind speed v_f	energetic wind speed v_d
Arithmetic	$\sum_{i=1}^{n} v_i f_i$	$v[f(v)_{max}]$	$v[P_d(v)_{max}]$
Weibull	$C\Gamma\left(1+\frac{1}{k}\right)$	$C\left(1+\frac{1}{k}\right)^{1/k}$	$C\left(1+\frac{2}{k}\right)^{1/k}$

Where Γ , the standard gamma function is given by:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{(-t)} dt \quad (3)$$

The determination of the energetic wind speed v_d giving the maximum energy is essential in order to dimension the wind installation correctly. In fact, it allows to define a nominal operating speed which will avoid under sizing or over-dimensioning the rotor. The energetic wind speed v_d can be determined analytically using the expressions derived from the Weibull distribution by deriving the expression of energy with respect to the speed v, and then determines the value v_d which cancels this derivative, Table1.

The energetic wind speed $\boldsymbol{v_d}$ can be calculated by the arithmetic method by computing the power density P_d (Table2.) for the centers of the classes of wind speed ([0-1], [1-2], ..., [21-22]). Thus $\boldsymbol{v_d}$ corresponds to the wind speed of the highest value of P_d.

TABLE 2. EXPRESSIONS OF THE POWERS DENSITY AND VARIANCES DEPENDING ON THE MODEL.

Distribution	Power density $\boldsymbol{P_d}$	Variance $\boldsymbol{\sigma^2}$
Arithmetic	$\frac{1}{2}\frac{16}{27}\rho\sum_{i=1}^{n} {v_i}^3 f_{(v_i)}$	$\frac{1}{n}\sum_{i=1}^{n} f_i\,(v_i-\bar{v})^2$
Weibull	$\frac{1}{2}\frac{16}{27}\rho C^3\Gamma\left(1+\frac{3}{k}\right)$	$C^2\left[\Gamma\left(1+\frac{2}{k}\right)-\Gamma^2\left(1+\frac{1}{k}\right)\right]$

ρ: air density (kg/m^3) at the site; is often written in a simple form [1]:

$$\rho=\rho 0-(1.194\times 10^{-4}\,H_m) \quad (4)$$

Where, H_m is the site elevation in meters and ρ_0 is the air density value at sea level (ρ_0 = 1.225).

ADJUSTMENT METHODS

Several methods have been used to adjust the Weibull distribution and to find out scale and shape parameters.

1. *Least-Square Method (LSM).*

The least square method (LSM) allow to a best adjustment by simple linear regression. It can be served to class measurements hierarchically then to use it in a Weibull Cumulated Distribution Frequencies.

By using equation 2, we can write

$$ln|-ln(1-F(v))|=k\ln(v)-k\ln(c) \quad (5)$$

If we replace y= $ln|-ln(1-F(v))|$ and $x=\ln(v)$ then equation (5) becomes:

$$y=ax+b \quad (6)$$

Where: a=k and b=-k ln(c)

Then

$$c=exp\left(-\frac{b}{a}\right) \quad (7)$$

2. *Maximum Likelihood Method (MLM).*

The maximum likelihood method, applied to the Weibull distribution, can be written as:

$$L_v=\prod_{i=1}^{N} f(v_i)=\prod_{i=1}^{N}\left(\left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^{k}\right)\right) \quad (8)$$

By applying a logarithmic function to equation (8), then we calculate:

$$\frac{\partial(L_v)}{\partial k}=0 \qquad \frac{\partial(L_v)}{\partial C}=0$$

Then :

$$c=\left(\frac{1}{N}\sum_{i=1}^{N} {v_i}^k\right)^{1/k} \quad (9)$$

And

$$k=\left(\frac{\sum_{i=1}^{N} {v_i}^k \ln v_i}{\sum_{i=1}^{N} {v_i}^k}-\frac{\sum_{i=1}^{N}\ln v_i}{N}\right)^{-1} \quad (10)$$

N represents the number of total non-zero observations and v_i is the ith saved mean wind speed.

The system of equations composed by equations (9) and (10) can be resolved by successive iterations using an optimization method (Newton Raphson).

3. *Modified Maximum Likelihood Method (MMLM).*

When the wind speed data are available as a form of frequency distribution, a variation of the method of Maximum likelihood may be applied. Therefore, the Weibull parameters are estimated by using the following equations [2]:

$$k=\left(\frac{\sum_{i=1}^{N} {v_i}^k \ln(v_i)P(v_i)}{\sum_{i=1}^{N} {v_i}^k\,P(v_i)}-\frac{\sum_{i=1}^{N}\ln(v_i)P(v_i)}{P(v\geq 0)}\right)^{-1} \quad (11)$$

And

$$c=\left(\frac{1}{P(v\geq 0)}\sum_{i=1}^{N} {v_i}^k P(v_i)\right)^{1/k} \quad (12)$$

Where vi is the wind speed, $P(vi)$ represent the Weibull frequency, $P(v \geq 0)$ is the probability where the wind speed is positive.

An iterative process resolves equation (11), thus equation (12) will be resolved by the explicit scheme.

4. *Moment Method (MM).*

The moment method applied to two parameters W. PDF consists to equalize the first moments; the first moment ($\mu 1(v)$ with the value of the mean wind speed and the second moment ($\mu 2(v)$) with the Variance of the correspondent theoretical measurements:

$$\bar{v}=\mu 1(v) \quad (13) \qquad \bar{v}=C\Gamma\left(1+\frac{1}{k}\right) \quad (14)$$

$$\sigma^2 = \mu 2(v) \tag{15}$$

$$\sigma = C\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^{1/2} \tag{16}$$

By substituting the parameter C of equations (15) and (16) and by rating respectively these two equations we obtain an equation witch depend only on the form parameter k:

$$\frac{\sigma^2}{\bar{v}^2} = \frac{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} \tag{17}$$

In order to resolve equation (17) it has been used an iterative procedure (Newton-Raphson method) for determine the mentioned parameters k and c by using equation (14).

5. Method 5

The Maximum likelihood Method has been modified in order to replace the iterative calculation of the form parameter by the following [3]:

$$k = \frac{\pi}{\sqrt{6}}\left(\frac{N(N-1)}{N(\sum_{i=1}^{N}\ln^2(v_i)) - (\sum_{i=1}^{N}\ln(v_i))^2}\right)^{0.5} \tag{18}$$

Where N is the number of the observation.

The scale parameter c is determined by equation (9).

MODEL VALIDATION (STATISTICAL ANALYSIS)

In order to test the results by five methods, we have used statistical tests: *RMSE*, R^2, and Chi-square χ^2. These statistical test are defined by [4]:

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{\frac{1}{2}} \tag{19}$$

$$\chi^2 = \frac{\sum_{i=1}^{N}(y_i - x_i)^2}{N-n} \tag{20}$$

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - z_i)^2 - \sum_{1=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(y_i - z_i)^2} \tag{21}$$

Where y_i is the ith value of the real data probability, z_i is the mean value of real data, x_i is the ith estimated value by the W. PDF, N represents the number of observations and n is the number of the used constants (two for our case: k and c).

The best results are characterized by the high value of R^2, and low values of $\chi^2$and RMSE.

WIND DATA SAVING

Generally wind speed data is represented as chronological series types and saved for a small period of time, where each value of this period correspond to the mean value released by instantaneous values.

Table 3 represents the data of frequency distribution of the wind speeds and average speeds depending on the speed classes for the 2010, 2011 and 2012, which are used to evaluate the W. PDF parameters.

TABLE 3. FREQUENCY DISTRIBUTION OF WIND SPEEDS

Classes of wind speeds	**Frequency (%)**			
	2010-2012	***2010***	***2011***	***2012***
0-1	7,34	7,59	7,39	6,82
1-2	20,28	18,69	20,79	21,36
2-3	26,82	26,64	26,69	27,47
3-4	23,83	22,70	24,74	23,12
4-5	13,11	14,39	12,58	12,56
5-6	5,44	6,69	4,90	4,97
6-7	2,06	2,06	1,86	2,59
7-8	0,74	0,80	0,69	0,80
8-9	0,22	0,36	0,14	0,22
9-10	0,08	0,08	0,08	0,09
10-11	0,01	0,00	0,02	0,00
11-12	0,01	0,00	0,02	0,00
12-13	0,01	0,00	0,01	0,00
13-14	0,01	0,00	0,02	0,00
14-15	0,00	0,00	0,00	0,00
15-16	0,00	0,00	0,00	0,00
16-17	0,01	0,00	0,02	0,00
17-18	0,01	0,00	0,01	0,00
18-19	0,01	0,00	0,01	0,00
19-20	0,01	0,00	0,02	0,00
21-22	0,00	0,00	0,00	0,00
V_{moy} (m/s)	**3,0**	**3,0**	**2,9**	**2,9**

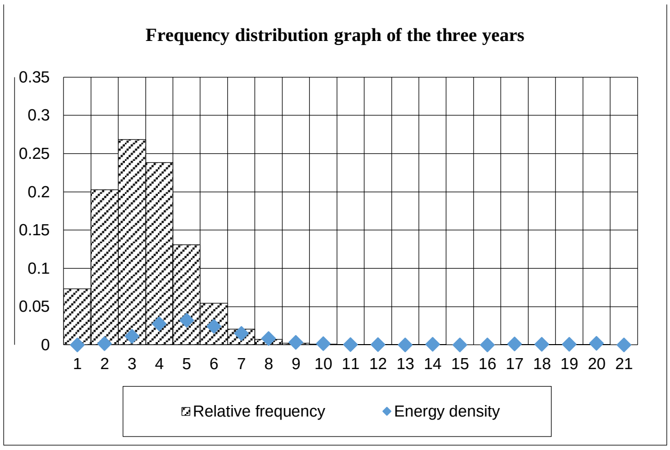

Figure 1. Frequency distribution and density energy of wind speeds of 2010-2012.

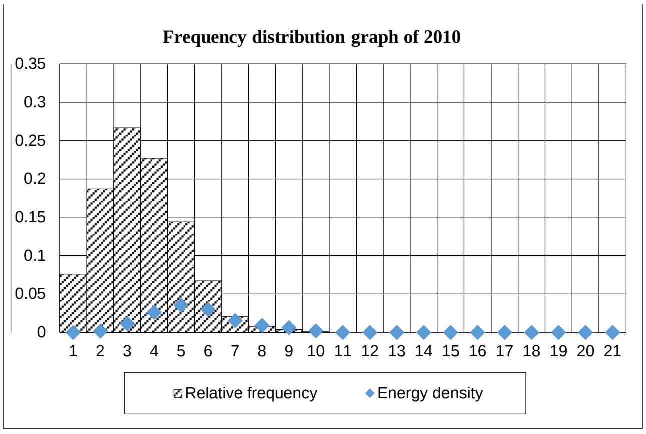

Figure 2. Frequency distribution and density energy of wind speeds of 2010.

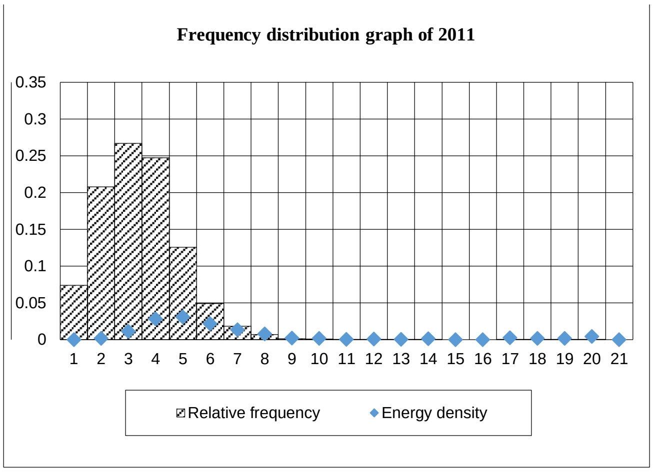

Figure 3. Frequency distribution and density energy of wind speeds of 2011.

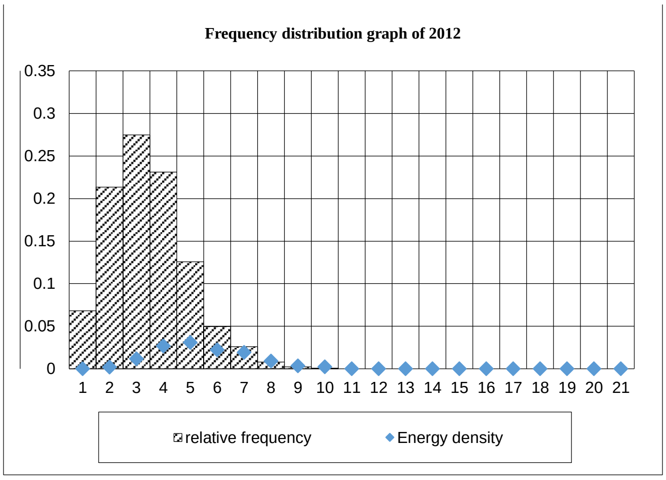

Figure 4. Frequency distribution and density energy of wind speeds of 2012.

RESULTS AND DISCUSSION

The table1. Presents the mean, frequency and energetic wind speeds formulas derived from the arithmetic method and the Weibull distribution.

The Table 2. presents the power density and variance formulas derived from the arithmetic method and the Weibull distribution.

The figures 1, 2, 3 and 4. show that the wind speed of the site studied is low at the height of 10 m (height of the measuring station). Indeed, and from these figures 1, 2, 3 and 4, it is noted that the wind speed distribution for the site studied is unimodal for the three years, with maximum frequencies belonging to the class [2 m/s-3 m/s], with a frequency of 26.82% for the three years 2010-2012, 26.64% for 2010, 26.69% for 2011 and 27.47% for 2012. Thus, the value of the frequency speed of the three years belong to this class.

The figures 5, 6, 7 and 8 represent the W. PDF and the frequency histograms vs. wind speed that corresponds to the years 2010, 2011 and 2012 for the site of Science Faculty in Tetouan – Morocco, by using the five mentioned methods in this paper.

Furthermore, we observe that MLM and MMLM and the Method 5 represent a best adjustment for the W. PDF, thus these methods still the best adapted for the saved data in the site.

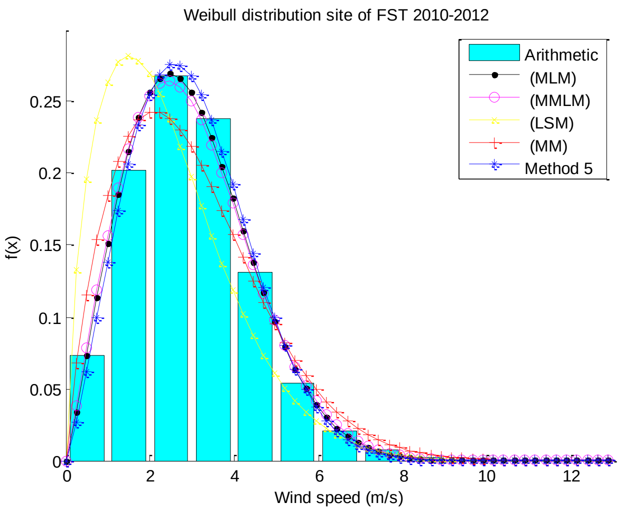

Figure 5. Weibull distribution of 2010, 2011 and 2012

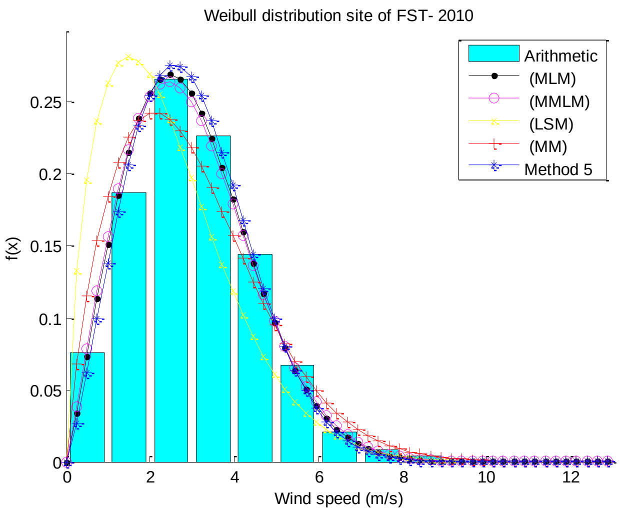

Figure 6. Weibull distribution of 2010

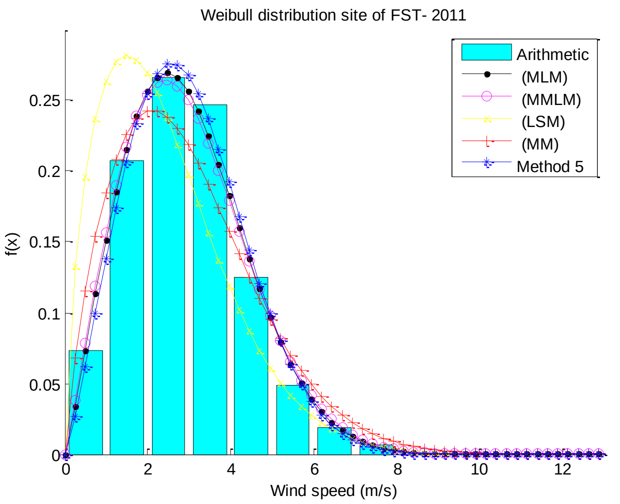

Figure 7. Weibull distribution for 2011

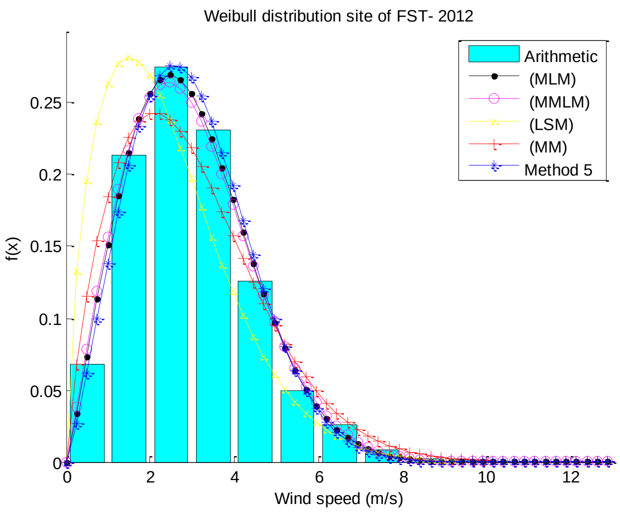

Figure 8. Weibull distribution of 2012

The calculated values of k and c parameters for each proposed method are presented in the table 4.

In one hand, we observe that MLM, MMLM, MM, and Method 5 provide the same value for the C parameter. On the other hand, for the scale parameter k, the methods which give the same values are MLM, MMLM, and Method 5 especially for the two years 2010 and 2012. However, the value of k in 2011 for the method5 has a significant gap with MLM and MMLM.

TABLE 4. PARAMETER OF W. PDF OF 2010, 2011 AND 2012

Methods	**2010**		**2011**		**2012**	
	C(m/s)	***k***	***C(m/s)***	***k***	***C(m/s)***	***k***
MLM	3,42	2,15	3,38	1,63	3,32	2,13
MMLM	3,41	2,08	3,37	1,68	3,32	2,08
LSM	2,76	1,58	2,73	0,95	2,71	1,62
MM	3,40	1,80	3,36	1,68	3,31	1,80
Method 5	3,44	2,21	3,79	2,13	3,36	2,24

Tables 5, 6 and 7 summurize the statistical analysis results for different used methods for W. PDF parameters determination.

TABLE 5. PARAMETERS STATISTIC OF 2010

2010			
Methods	***R^2***	***χ^2***	***RMSE***
MLM	0,999994138	8,9956E-08	0,00029987
MMLM	0,999996688	1,4812E-07	0,00038479
LSM	0,998239402	2,7855E-05	0,00527671
MM	0,99997583	1,0807E-06	0,00103935
Method 5	0,999994226	8,8605E-08	0,00029761

TABLE 6. PARAMETERS STATISTIC OF 2011

2011			
Methods	***R^2***	***χ2***	***RMSE***
MLM	0,999948649	1,566E-06	0,00125127
MMLM	0,9999477	1,3233E-06	0,00115021
LSM	0,958337135	0,00051235	0,02263262
MM	0,999947765	1,3216E-06	0,00114949
Method 5	0,999971644	8,6497E-07	0,00092993

TABLE 7. PARAMETERS STATISTIC OF 2012

2012			
Methods	***R^2***	***χ2***	***RMSE***
MLM	0,999994707	1,178E-07	0,00034311
MMLM	0,999996726	2,1267E-07	0,00046101
LSM	0,999191237	1,8587E-05	0,00430999
MM	0,999971114	0,00512815	0,00136927
Method 5	0,999993865	1,3657E-07	0,00036943

Statistical analysis prove the graphical results since the methods *MLM*, *MMLM*, and *Method 5* represent the highest values of the determination coefficient R^2 (≈1) and the low values for two other coefficients Chi-square χ^2 and for Root Mean Square Error RMSE followed by the *ML* method then *LSM*.

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Tables 8, 10 and 12 present the arithmetic mean values and the standard deviation of the W. PDF of three years for five methods.

TABLE 8. MEAN WIND SPEED AND STANDARD DEVIATION OF THE WEIBULL DISTRIBUTION AND ARITHMETIC OF 2010

2010				
Methods	***s_mean***	***wei_ mean***	***s_ std***	***wei_std***
MLM	3,03	3,03	1,49	1,49
MMLM	3,03	3,02	1,49	1,53
LSM	3,03	2,47	1,49	1,60
MM	3,03	3,03	1,49	1,74
Method 5	3,03	3,05	1,49	1,46

TABLE 9. FREQUENCY AND ENERGETIC WIND SPEED OF 2010

2010		
Methods	***vi_frequency (m/s)***	***vi_ energetic (m/s)***
MLM	2,34	3,08
MMLM	2,44	3,23
LSM	2,85	3,95
MM	2,94	3,99
Method 5	2,27	2,97
Arithmetic	2,5	4,5

TABLE 10. MEAN WIND SPEED AND STANDARD DEVIATION OF THE WEIBULL DISTRIBUTION AND ARITHMETIC OF 2011

2011				
Methods	***s_mean***	***wei_ mean***	***s_ std***	***wei_std***
MLM	3,02	3,03	2,09	1,90
MMLM	3,02	3,01	2,09	1,84
LSM	3,02	2,79	2,09	2,92
MM	3,02	3,00	2,09	1,84
Method 5	3,02	3,36	2,09	1,66

TABLE 11. FREQUENCY AND ENERGETIC WIND SPEED OF 2011

2011		
Methods	***vi_frequency (m/s)***	***vi_ energetic (m/s)***
MLM	3,34	4,60
MMLM	3,20	4,40
LSM	5,86	8,86
MM	3,20	4,39
Method 5	2,62	3,45
Arithmetic	2.5	4,5

Table 9, 11, and 13 show the values of wind speed frequencies and energy potential calculated from the W. PDF for three years by the five methods and arithmetic method.

TABLE 12. MEAN WIND SPEED AND STANDARD DEVIATION OF THE WEIBULL DISTRIBUTION AND ARITHMETIC OF 2012

2012				
Methods	***s_mean***	***wei_ mean***	***s_ std***	***wei_std***
MLM	2,93	2,94	1,45	1,45
MMLM	2,93	2,94	1,45	1,48
LSM	2,93	2,42	1,45	1,53
MM	2,93	2,94	1,45	1,69
Method 5	2,93	2,97	1,45	1,40

TABLE 13. FREQUENCY AND ENERGETIC WIND SPEED OF 2012

2012		
Methods	***vi_frequency (m/s)***	***vi_ energetic (m/s)***
MLM	2,29	3,02
MMLM	2,37	3,14
LSM	2,71	3,74
MM	2,86	3,88
Method 5	2,17	2,84
Arithmetic	2,5	4,5

The analysis of tables 9, 11 and 13 shows that the difference between the value of the frequency speed rate estimated by the arihtmetic method and the methods MLM, MMLM and methode5 is low especially for 2010 and 2012. In 2011, the value of the energy speed estimated by the method 5 is roughly equal to that estimated by the arithmetic method.

Therefore, this analysis proves that the methods which estimate energy speed close to those estimated by the arithmetic method are LSM and MM for the year 2010; MMLM, MLM and MM for the year 2011; MM and LSM for the year 2012. We note that the values estimated by the MM method for the three years are roughly equal to that estimated by the arithmetic method for the three years.

CONCLUSION

In this paper, we analyzed and compared the results of five numerical methods for determining Weibull parameters. The data used are the measurements of the wind speed of the FST site for the three years 2010 - 2012. We tested these different results using the coefficient of determination R^2, the Chi-square test $\chi 2$ and the root mean square error RMSE.

The analysis of the various results obtained shows that:

The Maximum Likelihood Method MLM and its derivative MMLM as well as Method 5 still the best efficient methods for determining the shape and the extent of the W. PDF and the best adjustment of wind speed measurements.

Although the Maximum Likelihood ML presents the best adjustments, we observe that all the other statistical parameters for different methods still similar.

The Least Square Method (graphic adjustment) is a less effective method for determining the parameters k and c in Weibull distribution, especially for the low speeds.

The numerical methods which use the iterative operations for determining the shape and the scale parameters adjust better the W. PDF, so it can be used if a high precision degree is required.

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