Ben-Dor's works [11] with a previous edition of the digest [7] demonstrates the evolution of views on the hysteresis problem. By using numerical methods, Ben-Dor, Elperin, and Vasilev have studied the hysteresis during interactions of conical shocks [13]. It has been shown in these works that a hysteresis exists during the change in the wedge angle and Mach number. However, the transition from RR to IR in numerical and physical experiments does differ significantly from theoretical values forecasted by the detachment and SMC criteria.

In the doctoral thesis of Vasilev [14], the hysteresis was studied by numerical methods with allocation of discontinuities. It was shown that an increasing-grid-density approach numerically results in the analytical results obtained according to the detachment and SMC criteria. However, it has not been determined, if the solution matches with one forecasted by theory or with some other. In addition, a series of calculations based on conservative differential schemes has led to a paradoxical result–the transition from RR to IR occurs below the M_R curve. However, inside this region the conditions of dynamic compatibility, derived from preservation laws, forbid the existence of the RR and the conservatism condition of differential schemes assumes a strict realization of the preservation laws. It became obvious that the result of numerical solutions is significantly affected by the numerical viscosity and the approximation order used in the differential scheme.

In the thesis by Hotyanovsky [15], the hysteresis phenomenon has been studied by means of essentially non oscillatory (ENO) and weighted essentially non-oscillatory (WENO) differential schemes with increasing accuracy classes. The data on early experimental and numerical research has been presented. The obtained results indicate a narrower hysteresis region than forecasted by theory. However, if the transition from IR to RR is located near the curve that corresponds to the SMC criterion, the deviation during the transition from RR to IR according to the detachment criterion is even bigger.

The research of Hotyanovsky has found its development in the doctoral thesis of Kudryatsev [16] wherein the numerical methods, the mathematical model, and the description of the phenomena are presented in most detail. The symmetric and asymmetric interactions of counterclaims shocks have been studied. The influence of interaction parameters on the width of the hysteresis zone has been studied. The existence of a hysteresis in the two-value solution region has been proven experimentally. It has been shown that a bending of the compression shock upon a rapid change of the wedge angle has a significant influence on the width of the hysteresis region. The influence of the rate of the Mach number change for a fixed wedge angle has not been studied.

Finally, in the thesis by Shoev [17], the influence of shock blurring on the solution of the problem about shocks interaction has been studied. A direct simulation method for the solution of the Boltzmann equation was employed. Shock blurring influenced by numerical or physical viscosities leads to blurring of shock polars [17]. In addition, main and secondary polars are replaced with some envelope line. Consequently, the solution corresponding to the intersection of polars is shifted to the point at which the envelope overbends.

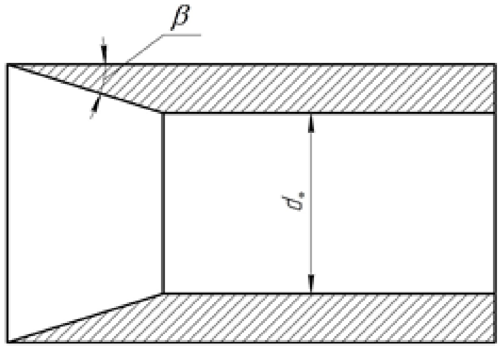
Thus, the aforementioned studies have proven the existence of a hysteresis and the similarity of the results acquired by numerical methods and experiments of the RR ⇆ IR to values forecasted by theory. The influence of the numerical and physical viscosities on the width of the hysteresis region has been determined. Simultaneously, a list of important questions is left unanswered:

1. What is the influence of shock blurring at difference grid on the calculation precision of the RR ⇆ IR transition moment?
2. Does the numerical solution match with the one forecasted by theory for an infinite grid reduction?
3. Why can the hysteresis of the Mach number be seen at wedge angles higher than the critical one even though this it should not be the case (Fig. 2)?
4. Does the rate of the Mach number change affect the hysteresis at a fixed wedge angle?
5. Upon transition from IR to RR in a region of the M_0 curve, is the initial moment of transition to RR defined by some physical reasons or is the Mach stem so small that it cannot be seen on an insufficiently small grid?
6. What criterion or parameter can be used to identify the moment of transition during a smooth transition from IR to RR in a region of the M_0 curve if the Mach stem is small.

MATERIALS AND METHODS

The numerical method:

The modeling was conducted inside a two-dimensional computational region comprising an empty plane in-between two wedges (Fig. 3). An equal spread of Mach numbers is set on the left boundary of the plane; consequently, a supersonic flow with a set value M is propagated onto the wedges. The plane in question comprises a contracting section and a section with a constant cross-section of a characteristic size d_*. The value of d_* influences the volumetric consumption of the medium that passes between the wedges and the possibility of a shock-wave structure existence inside the channel. At low Mach numbers, at the channel entrance, an outcoming shock wave appears, and flow within the channel is subsonic.



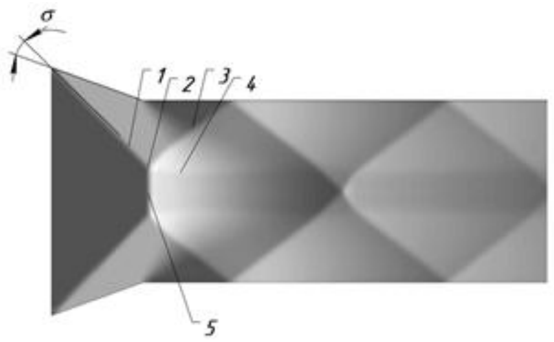


Figure 3. Schematic of the computation region. d_* − distance between wedges, β – angle of flow deflection of the shock wave (wedge angle), σ – compression shock angle. 1 – counterclaims compression shocks, 2 – triple point, 3 – reflected compression shock, 4 – tangential discontinuity behind the triple point, 5 – Mach stem.

The wedge angle β that corresponds to a flow deflection angle for an oblique compression shock is a characteristic geometric parameter that influences the shock-wave picture of the flow. Values of β and d_* were chosen in a way that allows to study RR, IR, and the hysteresis.

When solving the problem of jet streams with shock waves, the choice of the turbulence model can significantly affect the geometry of the SWS. Thus, the numerical modeling was conducted using an ideal gas model. The use of an ideal gas model can be supported by the fact that the RR ⇆ IR transition can occur with an abrupt reformation of the SWS, i.e., in a non-stationary process. The turbulence models use the average turbulent flow with time; thus, their application to the modeling of the rapid process is not theoretically justified [18].

During the calculation, two variants of the problem statement were used. The first one — the quasi-stationary statement in which the time-based parameter (in this case, the value of M at the channel entrance) changes discretely. In addition, the solution acquired at a previous step is used as initial data. Such technique imitates an infinitely slow and gradual change in the Mach numbers.

The second variant is the fully non-stationary problem statement in which a time variable is introduced and parameter changes occur monotonously. This technique imitates a relatively rapid change of the Mach number. This model variant allows determining the hysteresis value for the non-stationary case and comparing it with a variant of the infinitely slow change of the Mach number.

The calculations were performed on four different structured difference grids. The grossest grid had 60 cells across the flow. Others had 240, 480, and 960 cells. The grid was aligned with a boundary of the computational region. The Mach number was set on the left boundary, the impermeability condition was set on rigid boundaries, and the static pressure was set on the right boundary.

The transition moment to IR was monitored visually by the presence of two tangential discontinuities behinds the Mach stem and additionally by a curvature change of the reflected compression shock. From the solution of the first order problem of the GDD interference [19, 20], during a regular intersection of shocks, the reflected shock has a positive curvature; however, upon the appearance of the triple points it changes to negative [21]. Such a combined approach allows to identify the IR even when the Mach stem and tangential discontinuities on it are too blurry to be seen.

The analytical method:

Shock blurring on a difference grid can be interpreted as some indifference from the setting of the compression shock angles. For the case shown in Fig. 4, it is 0.9°.

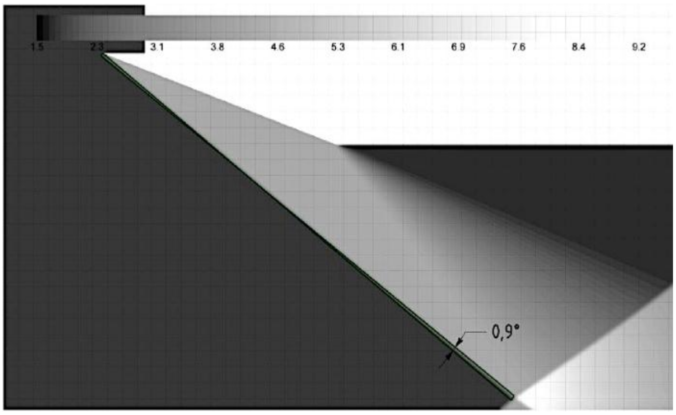


Figure 4. Imitation of shock blurring by introducing indifference into the setting of its angle.

Let us assume that the compression shock angle has an uncertainty $\sigma \pm \Delta\sigma$, then, the point corresponding to an oblique compression shock on the main polar would turn into a multitude of points laying on the polar. For instance, for a heat ratio of $\gamma = 1.4$, a Mach number of $M = 3$ and a fall angle of an oblique shock of $\sigma = 40° \pm 1°$, the shock intensity J lies in the range from 4 to 4.35. Then, the secondary polar would have a finite thickness and could no longer be considered to be infinitely thin (Fig. 5). Its intersection with the ordinate and the main polar would constitute not a single point but a multitude.

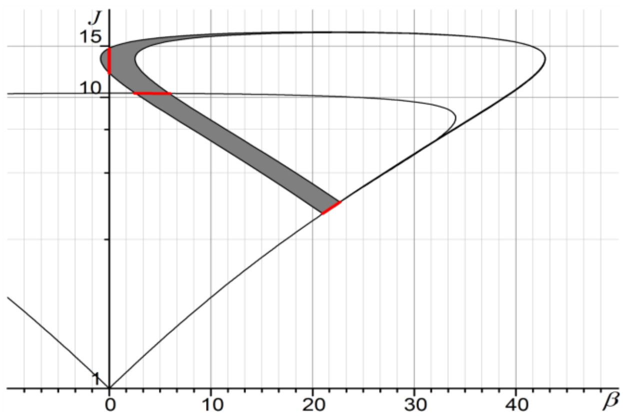
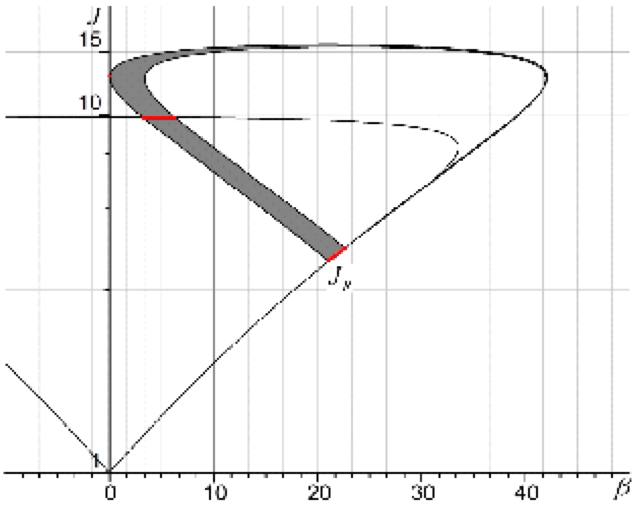
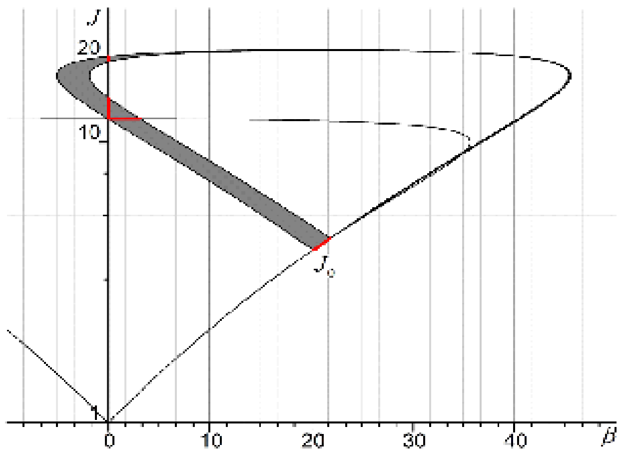


Figure 5. Intersection of the “thick” polar corresponding to an oblique compression shock with the main polar and ordinate.

Then, the regular reflection of such blurry shock would correspond to a multitude of points on the ordinate that correspond to intersections of the ordinate with a "thick" polar. The irregular reflection corresponds to a multitude of points located on a subsonic branch of the main polar that corresponds to its intersection with the "thick" polar. These multitudes are colored red in Fig. 5. A "thick" polar touching an ordinate corresponds to the detachment criterion (Fig. 6a). The SMC criterion corresponds to an intersection of the lower boundary of the "thick" polar with an apex of the main polar that correspond to a Mach number before an oblique compression shock.



(a)



(b)

Figure 6. Determination of characteristic intensity JR of a blurry oblique shock that corresponds to the detachment criterion (a) and J0 corresponding to the SMC criterion (b).

Thus, by using difference cells, the degree of the shock blurring can be determined and the indifference can be introduced into the setting of the shock angle for each rank of grid. This allows to build the dependencies of M_0 and M_R on the wedge angle β with account of the indifference caused by the shock blurring on difference cells (Fig. 7) and to determine the hysteresis region for a fixed angle β using these graphs. Each rank of the difference grid has a different width of the hysteresis region.

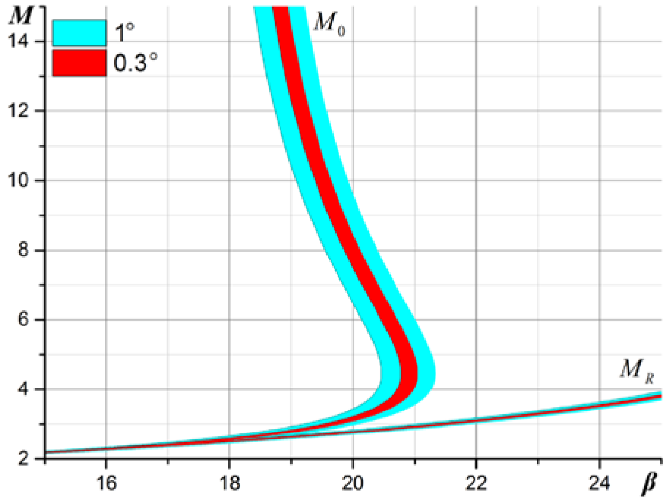


Figure 7. Existence regions of RR and IR with account for the "blurring" of compression shocks.

In comparison to Fig. 2, it can be seen that the appearance of indifference regions thins out the two-value region wherein the existence of RR, IR, and the hysteresis is possible. The thinning increases with the shock blurring. Moreover, it can be seen that the curve M_0 is more blurry than M_R.
Because of the properties of the numerical method, the degree of shock blurring of the difference cells is usually known, which allows to easily correct the calculation results for the RR⇄IR transitions using the method described above.

RESULTS AND DISCUSSION

The calculation results for the interference of counterclaims shocks on four different difference grids are listed below.

The hysteresis range:

For practical use, it is useful to build area of ambiguity in M-β coordinates. Actually, during acceleration of the aircraft in flight Mach number changes, and the angle of the air intake wedge remains constant. At regulation of the air intake, on the contrary, the wedge angle is changed, while the flight Mach number usually remains constant. If you consistently change the Mach number, it is possible to calculate J_0, J_R and their corresponding flow turning angles β for each M. Thus, on the plane M-β we can build parametric curves $M_0(\beta)$, corresponding to SMC criterion, and $M_R(\beta)$ corresponding to criterion of disconnection. Curve M_0 meets the SMC criterion. Secondary shock polar intersects with both the main shock polar and the y-axis between curves M_R and M_0, thus the dynamic compatibility conditions allow both RI and MI. This is a region of solution ambiguity. Figure 2, 7 show that there is a minimum angle of the wedge βmin, in which lines M_0 and M_R merge. This point has its own corresponding Mach number M_{0R}. Respectively, when β<βmin hysteresis cannot exist, below M_{0R} line is the area of MR, above M_{0R} line - RR. The transition of MR to RR with an increase in Mach number occurs on the M_{0R} line. Fig. 7 also clearly shows that there is the second limit angle βmax at the line M_0. There is also a corner β_∞, to which M_0 line tends at $M \to \infty$. Thus, the whole plane is divided into three ranges of angle β: 1 – $\beta<\beta_{min}$, 2 – $\beta_{min}<\beta<\beta_{max}$, 3 – $\beta>\beta_{max}$. In the range 1 below M0R line MI is realized, above M_{0R} – RI, in the range 2 below M_{0R} – MI, above the M_0 – RI, between these curves is the hysteresis area. With the increase of adiabatic index areas of ambiguity significantly narrow.

Modeling was performed in a two-dimensional computational domain, located between the two wedges. Mach number of the incident flow was varied. Since in the problems of calculation of jet flows with shock waves choice of turbulence model can have a significant impact on the geometry of the SWS, numerical simulation was performed within the model of the ideal gas.

Hysteresis ranges were determined using images of Mach number isolines. By using the method described above, the amount of hysteresis varies with the amount of shock blurring on the difference cells corresponding to difference grids. The results for a wedge angle of 20° are compiled in Table 1. In process of Mach number increase the transition to the regular reflection is in accordance with the SMC criterion, in process of Mach number decrease - in accordance with the von Neumann criterion. Physical and scheme viscosity leads to narrowing of the solution ambiguity region and reduce of hysteresis. The reason is shock blurring. As the number of the difference grid cells grow, numerical solution for the transition points from regular interference to Mach interference and vice versa exactly converges to the theoretical von Neumann criteria and stationary Mach configuration.

Table 1. Hysteresis range

Numerical method			Analytical method	
Number of cells	PO → HO	PO ← HO	PO → HO	PO ← HO
60	2.6	2.7	2.612 – 2.975	2.745
240	2.7	2.8	2.715 – 2.835	2.978 – 3.436
480	2.75	3.0	2.753 – 2.789	3.092 – 3.22
960	2.8	3.15	2.762 – 2.78	3.12 – 3.185
Theoretical value	2.77	3.149	2.77	3.149

Convergence by the grid difference:
It can be seen that numerical results precisely match with theoretical values. An analytical solution with account for the shock blurring allows determining the transition moment from one type of interference to the other with higher precision than the visual assessment using a distribution image of isolines. The determination of the transition moment from RR←IR using the gross grid has a larger margin of error than the RR→IR transition. Fig. 8 shows results acquired by using the finest grid.

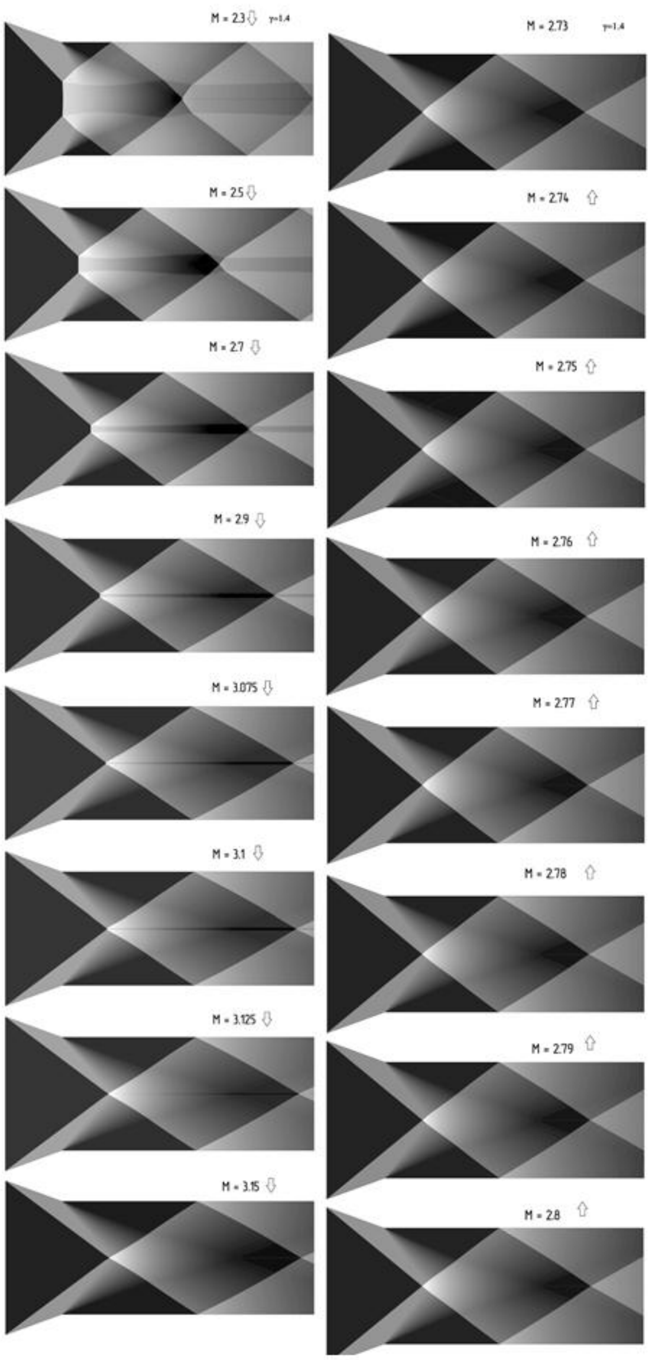


Figure 8. Hysteresis acquired with a difference grid with 960 cells across the flow. $\beta = 20°$.

It can be seen that an increase in the Mach number from $M = 2.3$ to $M = 3.2$ leads to a gradual decrease of the Mach stem. The transition to a regular reflection occurs in a range of Mach numbers between $M = 3.125$ and $M = 3.15$, which corresponds to the SMC criterion. When the Mach number decreases starting from $M = 3.2$, the transition to an irregular reflection occurs in the range of Mach numbers between $M = 2.78$ and $M = 2.76$, which corresponds to the detachment criteria. This transition is identified by a bending of the reflected shock. Thus, the convergence of the numerical solution to the analytical one upon a decrease of the grid size can be considered to be proven.

Shock blurring influence:

Fig, 9 shows results acquired by using the grossest grid. It can be seen that the range of the hysteresis is significantly narrower than that for the fine grid. The transitions occur for Mach numbers between $M = 2.5$ and $M = 2.6$. The narrowing for the grossest grid is mostly attributed to a larger margin of error for the determination of the RR←IR transition according to the SMC criterion. Upon comparison of Figs. 8 and 9 with results of analytical calculations, listed in Table 1, the following conclusion can be made: The detachment criterion during the RR→IR transition corresponds to a line contact between the "thick" polar's left boundary and the ordinate (Fig. 6a), i.e., a decrease in one of the Mach numbers listed in the corresponding column. The SMC criterion during the RR ← IR transition corresponds to the intersection of the main polar's apex with the middle of a "thick" polar. The results of numerical calculations, however, are closer to the left boundary of the "thick" polar (Fig. 6b), i.e., closer to the lower Mach numbers present in the indifference region caused by shock blurring.

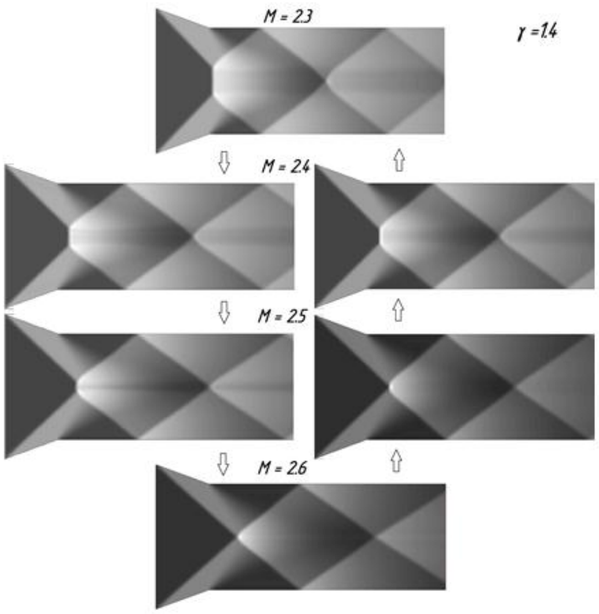


Figure 9. Hysteresis on the gross grid with 60 cells across the flow. $\beta = 20°$.

Height of Mach stem and hysteresis:

A hysteresis is constituted not only by the mismatch of transition moments from RR to IR and vice versa but also by the influence of the direction of the M change on the Mach stem during IR. The reason lies in the dependence of the Mach stem height on the flow conditions inside the "virtual nozzle" constituted by tangential discontinuities behind triple points (Fig. 10) and is defined by the equilibrium of the gas consumption trough the Mach stem and the critical cross-section of "virtual nozzle" [22].

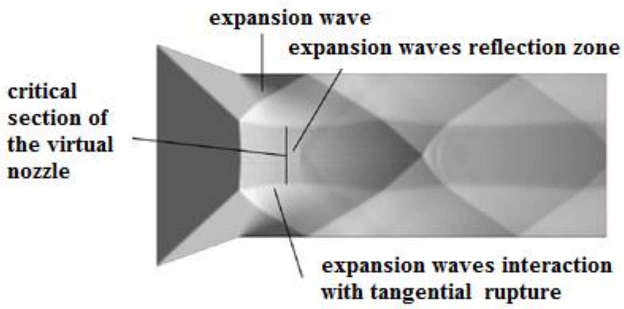


Figure 10. Interaction of rarefaction waves with the flow region behind the Mach stem.

The discharge equilibrium can be established for two different height values of the Mach stem. Thus, the height of the Mach stem depends on the initial conditions. If, during IR, the Mach number increases and the height of Mach stem decreases, then, at a set value of M, the height of Mach stem would be higher than that during the movement from a side of the regular reflection with a decrease in M.

Hysteresis at $\beta > \beta_0$

If a wedge angle $\beta > \beta_0$ is chosen, then the whole region of Mach number changes would be split into two by the M_R value: a region with $M < M_R$, wherein only IR is possible and a region with $M > M_R$, wherein both RR and IR are possible. The calculations show that IR is realized. The Mach stem is always present, but its size decreases as other parameters and angles change. The secondary polar intersects the main polar near its apex but the SMC cannot be achieved for any value of M. The dependence of the Mach stem on the initial parameters is preserved, i.e., in this sense the hysteresis exists. Thus, for $\beta > \beta_0$, only IR is realized and the transition from IR to RR is absent throughout the whole range of M (Fig. 11).

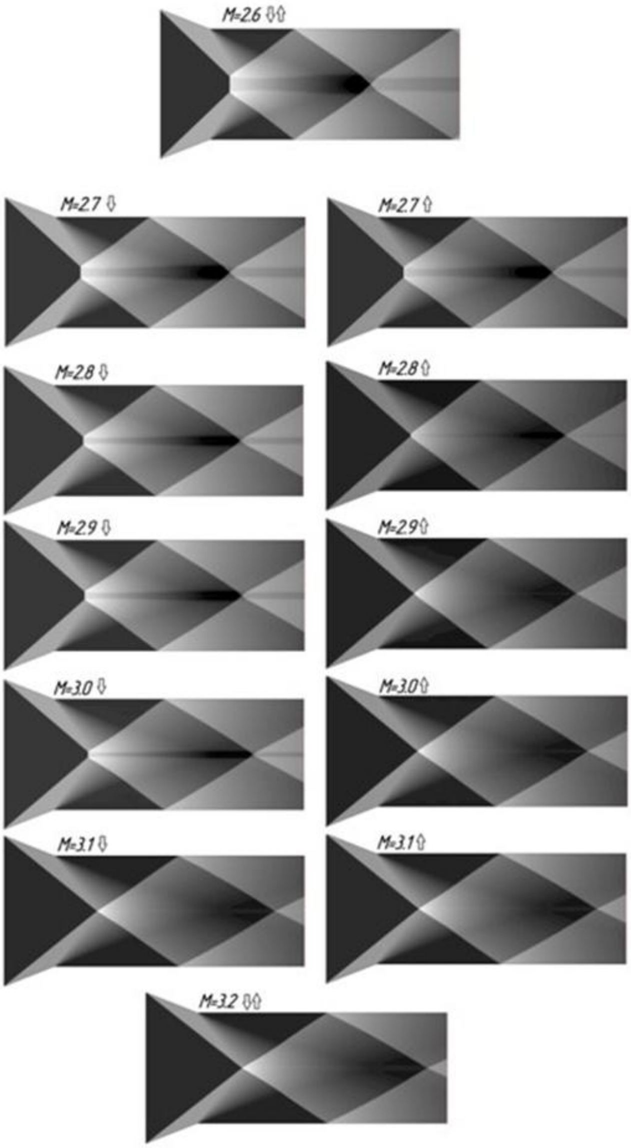


Figure 11. Interference of counterclaims shocks for $\beta > \beta_0$ *(21.5°).*

Influence of the Mach number change rate:

It is known that during a rapid change of the wedge angle, a bending of an oblique compression shock occurs, which influences the transition moment from IR to RR and vice versa. Research to evaluate the influence of the Mach number change rate has not been conducted before. The calculations were conducted on the finest grid with 960 cells across the flow. The spread of M across the flow is uniform, i.e., a bending of the oblique shock induced by a non-stationary occurrence is excluded. Thus, conditions for the determination of only non-stationary flows were created.

The calculations conducted on various difference grids showed identical results for each grid, i.e., a non-stationary flow does not affect the transition moment between the regular and irregular interaction of shocks.

CONCLUSIONS

As shown by analytical and numerical calculations, shock blurring and a transition from a fine difference grid to a more gross one leads to the narrowing of the hysteresis region, i.e., the transition moments from regular to irregular reflections and vice versa deviate from those forecasted by theory. Reduction of the grid size leads to a matching of the results with those forecasted by the theory of interference of stationary gas-dynamic discontinuities. A hysteresis is constituted by a change of the transition moments from regular to irregular interactions and in dependence of the Mach stem height on the initial conditions and the direction in which the M value changes. Non-stationarity by Mach number does not affect the hysteresis. If the wedge angle is larger than some critical value, then only an irregular reflection is realized throughout the whole range of M.

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