Performance Evaluation of Passive and Active Suspension for Armoured Fighting Vehicles

Abstract-
The main aim and fundamental requirements which must be met by all suspensions are to provide good ride comfort by isolating the passengers from road disturbances, it must support the entire mass of vehicle and also to maintain continuous contact between wheels and ground surface. High-mobility Armoured Fighting Vehicles such as main battle tanks and armoured personal carriers are designed for mobility over rough road terrain surfaces. The mobility performance of these vehicles is often limited by the crew’s endurance to withstand the transmitted shocks and vibrations and also his ability to maintain control. These high mobility vehicles are generally fitted with passive suspension systems utilizing torsion bars and shock absorbers to attenuate the terrain-induced shocks and heavy vibrations. But, these suspensions have been found incapable of isolating the vehicle from road vibrations and shocks, while moving through a rough terrain at high speed. Hence, the hydro-pneumatic suspension system has proved to be an effective suspension system for the latest armoured fighting vehicles. A quarter-car dynamic models of hydro-pneumatic suspension and active suspension system have been developed and derived the equations of motion. The relative displacement of sprung mass and unsprung mass of hydro-gas suspension system with and without controller for unit force and step input of road disturbance have been compared with MATLAB 7.0.1 software.

Keywords: Active Suspension System, Hydro – pneumatic Suspension Unit, Quarter-car Dynamic Model, Passive Suspension System, PID Controller, Armoured Fighting Vehicles

Introduction
The primary aim of suspension system in an Armoured Fighting Vehicle (AFV) is to provide support to the entire static mass of the vehicle and also to provide stable platform to various sophisticated systems by isolating the vibrations induced by the road irregularities, thereby reducing the crew’s fatigue and maximize ride comfort. Generally AFV’s are designed for high mobility over rough off road terrain surfaces like sandy, rocky and river bed, etc.. The mobility performance of these tracked vehicles is often limited by the operator’s endurance to withstand the transmitted shocks and vibrations and his ability to maintain control. The maximum allowable vehicle speed varies with the roughness of a particular terrain and is primarily influenced by the suspension system design. The objective of this paper is to study the performance evaluation of passive suspension system of Hydro-pneumatic Suspension [1,2] with different road inputs. The performance evaluation results of active and passive suspension system with and without PID controller are compared and verified with MATLAB and Simulink software.

Hydro-Pneumatic Suspension Unit of AFV
High-mobility Armoured Fighting Vehicles are generally fitted with passive suspension systems like torsion bars and shock absorbers to absorb the shocks and vibrations. But, they have been found incapable of isolating the vehicle from low-frequency vibrations while moving through a rough terrain at high speed. Fig. 1 shows a Hydro pneumatic Suspension [1] Unit (HSU) consists of a stationary housing, crank, axle arm, connecting rod and two pistons (namely actuator piston and accumulator piston) inserted into the inside of two cylinders namely accumulator cylinder and actuator cylinder. The crank pin splined into the crank, connecting rod and actuator piston form a four bar link slider - crank mechanism to convert the rotary movement of the axle arm due to road wheel lift into linear displacement actuator piston, thereby compressing the gas medium in the accumulator cylinder. A damper is mounted in between the actuator cylinder and accumulator cylinder. When the vehicle negotiates an obstacle, the road wheel gets lifted up and causes the axle arm to swing about the splined crank pin axis. The crank, connected with the axle arm through crank pin rotates the connecting rod with actuator piston assembly connected to it moves forward thus displacing the hydraulic oil. The dislodged oil passes through orifices housed in the damper and pushes the accumulator piston. The gas filled on the other side of the accumulator cylinder gets compressed resulting in increased higher pressure. When the obstacle is crossed over by the road wheel, the high pressure compressed gas expands and pushes the hydraulic oil through damper and then the road wheel comes back to the static position.
road wheel tyre spring $k_2$ (linear) and tyre damping $C_t$ (linear) considered as zero. A sinusoidal road wheel displacement $x_r$ is considered as an input excitation for the model with two degree of freedom $x_1$ and $x_2$.

$$\text{Sprung mass (m_s)}$$

$$\text{Un-sprung mass (m_u)}$$

$$\text{Road Profile}$$

$$\text{Tyre}$$

$$\text{x}_1$$

$$\text{x}_2$$

Fig.3. Quarter-car Model of a HSU

For sprung mass (m_s)

$$m_s x_1 + k_1 (x_1 - x_2) + c_s (x_1 - x_2) = 0$$

$$m_s x_1 = -k_1 (x_1 - x_2) - c_s (x_1 - x_2)$$

$$x_1 = -k_1 m_s^{-1} x_1 + k_1 m_s^{-1} x_2 - c_s m_s^{-1} x_1 + c_s m_s^{-1} x_2$$

---(1)

For Unsprung mass (m_u)

$$m_u x_2 - k_2 (x_2 - x_r) - c_t (x_2 - x_r) = 0$$

$$m_u x_2 = k_2 (x_2 - x_r) + c_t (x_2 - x_r) - k_2 (x_2 - x_r)$$

$$x_2 = k_2 m_u^{-1} x_2 - k_2 m_u^{-1} x_2 + c_t m_u^{-1} x_2 - k_2 m_u^{-1} x_2 + k_2 m_u^{-1} x_2$$

Assume the state variables as given below

$$x_1 = x_1$$

$$x_1 = z_1$$

$$x_1 = z_1$$

$$x_1 = z_1$$

$$x_2 = z_2$$

$$x_2 = z_2$$

$$x_2 = z_2$$

$$x_2 = z_2$$

$$x_2 = z_2$$

Substitute the above assumed state variables into the equation (1) and (2).

$$z_2 = -k_1 m_s^{-1} z_1 + k_1 m_s^{-1} z_3 - c_s m_s^{-1} z_2 + c_s m_s^{-1} z_4$$

----- (3)

$$z_4 = k_2 m_u^{-1} z_3 + c_t m_u^{-1} z_4 - c_t m_u^{-1} z_4 - k_2 m_u^{-1} z_3 + k_2 m_u^{-1} z_4$$

----- (4)

State-Space Representation

To solve the above problem the state-space [3] method is used. The state-space representation of a passive suspension is given by the equations

$$x = Ax + Bu$$

----- (5)

$$y = Cx + Du$$

Where, ‘x’ is the vector representing the state (i.e. position & velocity variable) and ‘u’ is the scalar quantity representing
the input (i.e. force, road displacement) and matrices A, B are given below

\[
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix} =
A
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix} +
B
\begin{bmatrix}
    u
\end{bmatrix}
\]

Substitute the values for the assumed state vector \(z\) and matrices A, B and the scalar input (u) quantity in the state equation (5)

\[
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix} =
A
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix} +
B
\begin{bmatrix}
    u
\end{bmatrix} \quad \text{------ (6)}
\]

Similarly, substitute the assumed values in the output equation (5) as given below,

\[
y_1 = x_1 = z_1 \\
y_2 = x_2 = z_3 \\
z_1 = x_1 = z_2 \\
z_3 = x_2 = z_4
\]

\[
y = Cz + Du
\]

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}
\]

Then, the output equation becomes, \(y = Cz\) \quad \text{------ (7)}

After substitution of these state variable values of \(k_1, k_2, C_x, m_1\) and \(m_2\) in state space equations (6) by using Matlab, to find the sprung mass displacement \((x_1/x_2)\), unsprung mass displacement \((x_1/x_2)\) and relative displacement \((x_1-x_2)/x_2\) of a passive suspension. Fig 5 shows the displacement of sprung, unsprung mass and relative displacement of passive suspension against 0.1m step input. It is observed that the sprung mass displacement takes more settling time (above 10 seconds) with high overshoot (0.15m), similarly the relative displacements of sprung and unsprung mass also takes more settling time with high overshoot.

**Active Suspension System**

The section below presents the mathematical modeling [4] of active suspension system for AFV and their performance analysis of sprung mass, unsprung mass displacement and relative displacement is predicted with and without PID controller strategies [3,4]. The principle element of an active suspension is an actuator that is powered by hydraulic, pneumatic or electro-dynamic forces and controlled by a dedicated microcontroller. An active suspension system has the capability to adjust itself continuously to respond to different terrain conditions. By changing its characteristics, an active suspension offers superior handling, road feel, responsiveness and safety. The advantage of controlled suspension is that a better set of design trade-offs are possible compared with passive suspension.

**Mathematical Modeling of Active Suspension of AFV**
For Sprung Mass \((m_u)\)
\[
m_u z_r s + c_r (z_r - z_w) + c_r (z_r - z_m) - u = 0 \quad \text{------- (8)}
\]
For Unsprung Mass \((m_u)\)
\[
m_u z_w s + k_r (z_r - z_m) - c_r (z_r - z_w) + u = 0 \quad \text{------- (9)}
\]
After taking Laplace Transformation, the equations (8) & (9) can be written as
\[
(m_u s^2 + c_r s + k_r) z_r(s) - (c_r s + k_r) z_w(s) = U(s) \quad \text{------- (10)}
\]
\[
- (c_r s + k_r) z_r(s) + (m_u s^2 + c_r s + (k_r + k_w)) z_w(s) = k_r z_d(s) - U(s) \quad \text{------- (11)}
\]
The equations 10 and 11 can be written in the matrix form
\[
\begin{bmatrix}
z_r(s) \\
z_w(s)
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
(m_u s^2 + c_r s + k_r) & (c_r s + k_r) \\
(c_r s + k_r) & (m_u s^2 + c_r s + (k_r + k_w))
\end{bmatrix} \begin{bmatrix}
U(s) \\
k_r z_r(s) - U(s)
\end{bmatrix}
\quad \text{------- (12)}
\]
After multiplication the \(z_r(s)\) and \(z_w(s)\) will be given in equation (13)
\[
\begin{bmatrix}
z_r(s) \\
z_w(s)
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
(m_u s^2 + c_r s + k_r) & (c_r s + k_r) \\
(c_r s + k_r) & (m_u s^2 + c_r s + (k_r + k_w))
\end{bmatrix} \begin{bmatrix}
u(s) \\
z_r(s)
\end{bmatrix}
\quad \text{------- (13)}
\]
The suspension travel or the relative displacement of sprung and unsprung mass due to actuator force \(U(s)\) can be determined by considering the force \(U(s)\) as input and set road disturbance \(z_d(s) = 0\). Then, we get one set of transfer function, \(G_1(s)\)
\[
G_1(s) = \frac{z_r(s) - z_w(s)}{u(s)} = \frac{(m_u s^2 + k_w)}{\Delta} \quad \text{------- (14)}
\]
The suspension travel or the relative displacement of sprung and unsprung mass due to road disturbance \(z_r(s)\) can be determined by considering the step input and set the force \(U(s) = 0\). Then, we get one set of transfer function, \(G_2(s)\)
\[
G_2(s) = \frac{z_r(s) - z_w(s)}{z_r(s)} = \frac{(m_u s^2 + k_w)}{\Delta} \quad \text{------- (15)}
\]
The suspension mass displacement due to road disturbance, then we consider \(z_r(s)\) (road disturbance) as input, we set \(U(s) = 0\), thus we get one set of transfer function, \(G_3(s)\)
\[
G_3(s) = \frac{z_r(s)}{z_r(s)} = \frac{(C_s s + k_r)}{\Delta} \quad \text{------- (16)}
\]
The unsprung mass displacement due to road disturbance, then we consider \(z_r(s)\) (road disturbance) as input, we set \(U(s) = 0\), thus we get one set of transfer function, \(G_4(s)\)
\[
G_4(s) = \frac{z_r(s)}{z_r(s)} = \frac{(m_u s^2 + (C_s s + k_r) s + k_r)}{\Delta} \quad \text{------- (17)}
\]
### Proportional Integral Derivative (PID) Controller Design

The PID controller [5,6] technique design is the most popular feedback controller used. It is a robust easily understood that can provide excel lent control performance despite the varied dynamics characteristics of processes. As the name implies the PID controller produces an output signal consisting of three terms: first one is proportional to error signal second is proportional to the integral of the error signal and the third one is proportional to integral and derivative of the error signal.

![Fig.6. Block Diagram of Proportional-Integral-Derivative Controller](image)

- In the S-domain:
  \[
  U(s) = \left( K_p \frac{1}{s} + K_i s + K_d s \right) E(s)
  \]
  or
  \[
  U(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) E(s)
  \]
  \[
  G_{PID}(s) = \frac{U(s)}{E(s)} = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right)
  \]

- In the time domain:
  \[
  u(t) = K_p e(t) + \int e(t) dt + K_d \frac{de(t)}{dt}
  \]
  \[
  = K_p e(t) + \left( \frac{1}{T_i} \int e(t) dt + T_d \frac{de(t)}{dt} \right)
  \quad \text{------- (19)}
  \]

![Fig.7. Relative Displacement of Sprung and Unsprung mass with PI Controller](image)
The relative displacement of sprung and unsprung mass of passive suspension system, due to the step input is minimized to an optimum level by adding different controller strategies like PI, PD and PID controllers. The relative displacement [7] of sprung and unsprung mass with PI, PD and PID are shown in the above figures from (7) to (10).

**Conclusion**

A comparison of performance study on passive and active suspension of an AFV has been carried out with a mathematical model and MATLAB. Relative displacements of sprung and unsprung masses of both passive and active suspensions have been compared by tuning the PID controller strategies. The following conclusions are drawn.

The relative displacement due to step input (0.1m) of the sprung mass and unsprung mass of active suspension system with proportional integral (PI) controller has the effect of eliminating steady-state error, but takes more settling time (50 seconds) with increase in amplitude (0.08m). The proportional derivative (PD) controller will have the effect of reducing both the overshoot with amplitude of 0.06m and the settling time below 25 seconds. In fact, changing one of these variables can change the effect of the other two. The PID controller has the advantages over the three individual (P, PI and PD) control actions. It has the minimized overshoot of 65% and the amplitude raise is reduced to less than 0.04m and also the settling time is well below (6 seconds) that of other three individual control actions. By fine tuning of PID controller with different ‘k’ values, the settling time can be reduced from seconds to milli seconds.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m₁</td>
<td>Quarter car sprung mass, (4500 kg)</td>
</tr>
<tr>
<td>m₂</td>
<td>Unsprung mass of single station, (320 kg)</td>
</tr>
<tr>
<td>Cₛ</td>
<td>Damping coefficient, (1000 N s/m)</td>
</tr>
<tr>
<td>Fₐ</td>
<td>Actuator force, KN</td>
</tr>
<tr>
<td>k₁</td>
<td>Spring stiffness, (80000 KN/m)</td>
</tr>
<tr>
<td>k₂</td>
<td>Road wheel stiffness, (900000 KN/m)</td>
</tr>
<tr>
<td>kₚ</td>
<td>Proportional gain</td>
</tr>
<tr>
<td>kᵢ</td>
<td>Integral gain</td>
</tr>
<tr>
<td>k₅</td>
<td>Derivative gain</td>
</tr>
<tr>
<td>x₁</td>
<td>Sprung mass vertical displacement, m</td>
</tr>
<tr>
<td>x₂</td>
<td>Unsprung mass vertical displacement, m</td>
</tr>
<tr>
<td>t</td>
<td>Time, (5 sec),</td>
</tr>
</tbody>
</table>

**References**

