

Wavelet Based Fluctuation Analysis on ECG Time Series

Mayukha Pal^{a,b,c}, P. Madhusudana Rao^b and P. Manimaran^{a,*}

^a*C R Rao Advanced Institute of Mathematics, Statistics and Computer Science, Hyderabad-500046, India.*

^b*College of Engineering, Jawaharlal Nehru Technological University, Hyderabad-500085, India.*

^c*India Innovation Center, General Electric Company, Secunderabad-500003, Andhra Pradesh, India.*

Abstract

In this paper the correlation behavior and multifractal properties of electrocardiogram (ECG) signals were investigated through the wavelet based fluctuation analysis method to classify patients and healthy subjects. For this purpose the ECG time series data of five patients suffering from congestive heart failure (CHF) and five healthy human-being were obtained from Physionet online database. From the results, we observe that the presence of persistent behavior and strong multifractal nature in all the time series. Also we found that the calculated Hurst scaling exponents distinguishes the healthy signals from patients with CHF. We suggest that this approach may be useful for diagnosis and prognosis of heart disease.

Keyword: ECG signals, wavelet analysis, Hurst exponent, classification.

INTRODUCTION

Fractals analysis on natural time series has made tremendous applications in areas ranging from financial markets to physiological systems [1-4]. It characterizes the fluctuations dynamics through scaling laws. In recent years, the study of scaling and correlation behavior in time series analysis is an active area of research. Various methods have been developed to characterize the time series featuring irregular dynamics with sudden and intense bursts of high frequency fluctuations. Among those, rescaled range analysis [5], structure function [14], wavelet transform modulus maxima [8], average wavelet coefficient [9], detrended fluctuation analysis and its variants [6,7], detrended moving average methods and its variants [10,11], wavelet based fluctuation analysis methods [12,13] etc. were in use to study the correlation behavior and fractal characteristics in time series analysis [5-13]. These methods were applied in various fields ranging from finance, physiology, engineering and natural sciences [14-27].

A large number of studies have been carried out to analyze electrocardiogram (ECG) signals that provide information about cardiac activity and also in identifying the various heart diseases and heart abnormalities. Among those, the congestive heart failure (CHF) is a high risk disease which is triggered due to damaged heart valves, blocked blood vessels, excess toxic exposure etc. Treating patients with such disease is a challenging task for physicians to categorize the disease severity that in turn motivates the researchers in development of different methodologies to characterize the ECG signals. Until now, various studies have been carried out in analyzing the ECG signals using the methods such as Fourier power spectral analysis [28], detrended fluctuation analysis and its

variants [29-34], cumulative variation amplitude analysis [35], Hilbert and wavelet transform based analysis [36,37], complex networks [38], permutation entropy [39-41], ordinal pattern statistics and symbolic dynamics [42] etc.

In this paper, we investigate the correlation properties and multifractal behavior in the ECG signals using the recently developed wavelet based fluctuation analysis method for classification of patients and healthy subjects. For the purpose, we have obtained the ECG data of congestive heart failure (CHF) and healthy human-being from Physionet online database. In section 2 the information about the data and the wavelet based fluctuation analysis method are described and in section 3 we provide the results and discussion. The conclusion to our study is in section 4.

DATA AND METHODOLOGY

ECG data collection

We have collected the ECG data of five patients suffering from congestive heart failure (CHF) and five healthy human-being from Physionet online database [43]. The obtained data was the filtered version of the original time series where the outliers (i.e. abnormal beats) were removed [44]. The measured RR interval (heart beat) is in seconds for each data set and its lengths ranges between 70,000 to 110,000 data points. The healthy human ECG was recorded from 2 male of age 32 and 45 and 3 female of age 20, 32, 20 where as the ECG of CHF patients were recorded from 5 male of age 71, 63, 48, 51, 61.

Wavelet based fluctuation analysis method

The procedure of wavelet based fluctuation analysis method is as follows:

Let us assume the time series $x(i)$, where $i = 1, 2, \dots, N$ and N is the length of the time series.

Step 1: (Initially $L=1$) Decompose the signal using the desired discrete wavelet filters belonging to Daubechies (Db) family which effectively capture the polynomial trend in a variable window size from the time series. The high-pass and low-pass coefficients represents about the fluctuations and the trend of signal respectively.

Step 2: Discarding the high-pass coefficients perform the reconstruction only with the low-pass coefficients that provides the local trend of the signal. Now, the fluctuations can be extracted at each level by subtracting the trend from the original data.

Step 3: Due to the asymmetric nature and wrap around problem of Daubechies series of wavelets they affect the precision in the extraction of fluctuations. But this can be rectified from the extracted fluctuations by performing the above analysis by reversing the time series. Now, the newly extracted fluctuations are then reversed and averaged with the earlier obtained fluctuations.

Step 4: The extracted fluctuations from the time series data is subdivided into non-overlapping segments $M = \text{int}(N/s)$ where $s=2^{(L-1)}$ W is the wavelet window size at a particular level 'L' of decomposition and W is length of the filter coefficients of the chosen wavelet. During the analysis some of the data point will be discarded, in case of N/s is not an integer. This causes statistical errors in calculating the local variance. In such cases, we have to repeat the step 4 starting from end and going to the beginning to calculate the local variance.

Step 5: By squaring and averaging fluctuations over all segments the q^{th} order fluctuation function $F_q(s)$ can be obtained:

$$F_q(s) = \left[\frac{1}{2M} \sum_{b=1}^{2M} [F^2(b,s)]^{q/2} \right]^{1/q} \quad (1)$$

Here 'q' is the order of moments which takes any real value and $F(b,s)$ is the local variance calculated at each segment 'b'.

Step 6: The above steps 4 and 5 is repeated for variable wavelet window sizes 's' with different values of q.

If the time series has fractal nature, then $F_q(s)$ reveals the power law scaling:

$$F_q(s) \sim s^{h(q)} \quad (2)$$

If the order $q = 0$ then equation (1) leads to divergence of the scaling exponent, in such case the logarithmic averaging has to be performed to find the fluctuation function:

$$F_0(s) = \exp \left\{ \frac{1}{2M} \sum_{b=1}^{2M} \ln [F^2(b,s)]^{q/2} \right\}^{1/q} \quad (3)$$

As is well known, the $h(q)$ values are independent of q for monofractal time series. For multifractal time series, the $h(q)$ values depend on q . The calculated Hurst exponent ($H=h(q=2)$) characterizes the correlation behavior of the time series which varies between 0 and 1 (i.e. $0 < H < 1$). For persistent time series the H value is greater than 0.5 and anti-persistent time series the H value is less than 0.5. For uncorrelated time series $H=0.5$. The strength of the multifractality is evaluated by calculating the singularity spectrum $f(\alpha)$:

$$f(\alpha) \equiv q\alpha - \tau(q) \quad (4)$$

The values of $f(\alpha)$ can be obtained by taking Legendre transform of $\tau(q)$, where $\tau(q)=qh(q)$ and $\alpha \equiv \frac{d\tau(q)}{dq}$. For monofractal time series $\alpha = \text{const}$. For multifractal time series there occurs a distribution of α values. The broader $f(\alpha)$ spectrum reveals the strong multifractality and the narrower $f(\alpha)$ spectrum reveals the weak multifractlity nature present in the analyzed time series.

RESULTS AND DISCUSSION

We have analyzed the time series of five healthy and five CHF patients' time series using wavelet based fluctuation analysis method. Before applying the above method as an

initial step, we have performed wavelet based denoising technique to suppress the noise in the ECG signals [45-46]. For this purpose we have used MATLAB function 'wden' with 5 level decomposition and applied soft threshold to obtain the denoised ECG signals for further analysis [47]. Also we have discarded the first and last hour of the denoised signal for our study and this ensures the extracted signal is free from the artifacts which may occurs during mounting and removal of electrodes [48]. Now, the analysis using wavelet based fluctuation analysis method was carried out on ECG signals of five healthy and five CHF patients.

From the analysis, we observe that the fluctuation function, $F_q(s)$ increases linearly as the size of the scale s increases hence there exists power law behavior which is present in all the time series. The fluctuation function $F_q(s)$ were calculated for various q values which ranges between -10 and +10 with 0.2 step size. Sample plots of fluctuation function for a healthy and a CHF subject is given as **Figure 1**. The scaling exponents $h(q)$ values are obtained by least square fit on the linear regime of the fluctuation function. We found all the time series posses multifractal nature and this is evident from the non-linear behavior of calculated $h(q)$ values. Also the Hurst exponent H ($q=2$) of all time series is greater than 0.5 which shows persistent behavior exist in both healthy and CHF patient data. This is clearly shown in **Figure 2**. We have carried out further analysis to study the singularity spectrum of healthy and CHF time series to identify the strength of multifractal nature (i.e. weak or strong). From the results, it is observed broader spectrum for both healthy and CHF patient time series. The spectrum width for all time series has values greater than 0.9. This is clearly shown in **Figure 3**.

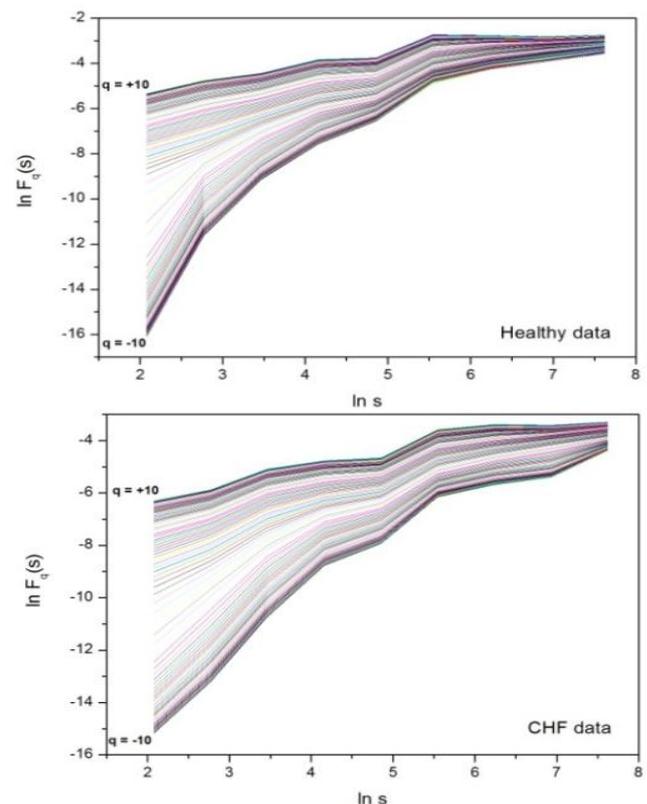


Figure 1: Fluctuation function $F_q(s)$ of a Healthy and a CHF subject.

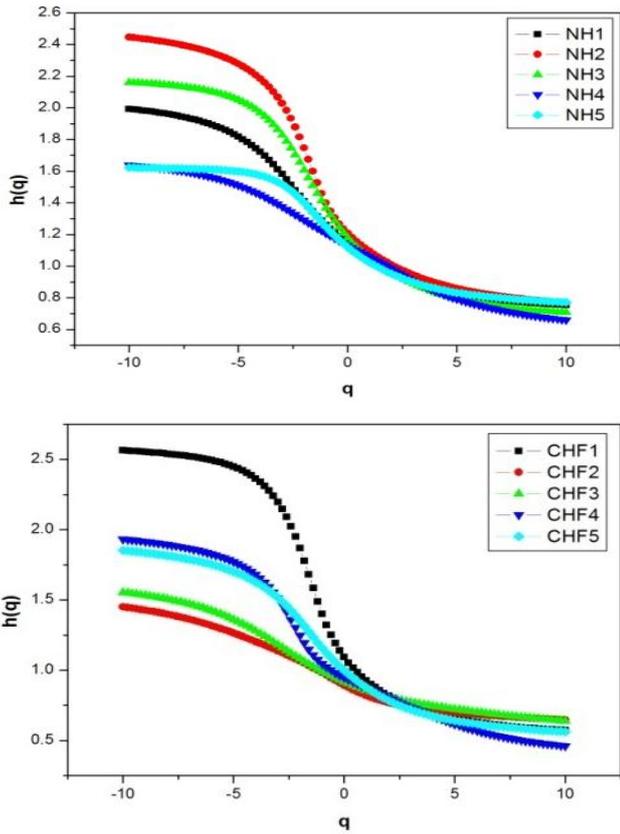


Figure 2: The non-linear behavior of $h(q)$ values for various q values of normal healthy (NH) and congestive heart failure patients (CHF).

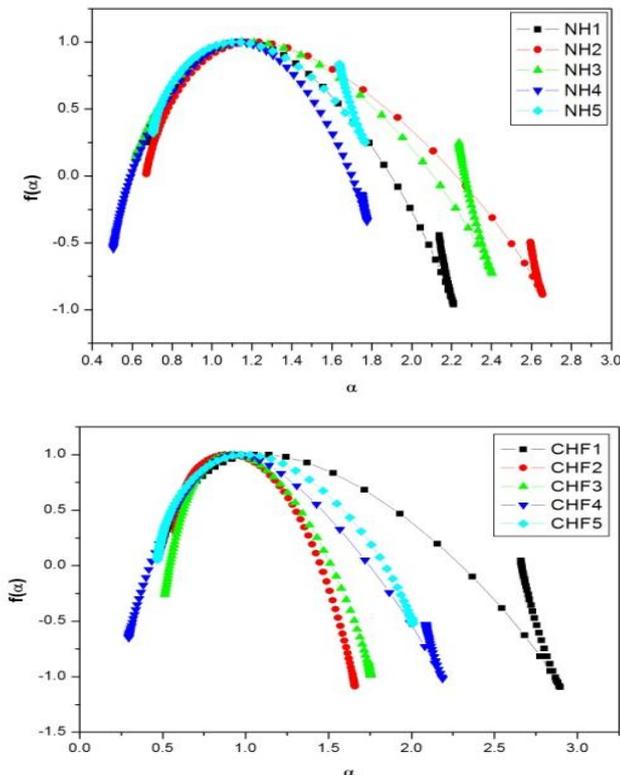


Figure 3: Singularity Spectrum ECG signals of a normal healthy (NH) and congestive heart failure patients (CHF).

From the empirical results obtained through wavelet based fluctuation analysis method, we found that the calculated Hurst scaling exponent shows significant difference between the time series of healthy and CHF patients. The Hurst scaling exponent separates into two regions that distinguish the CHF patients from healthy subjects and this is clearly seen in **Figure 4**.

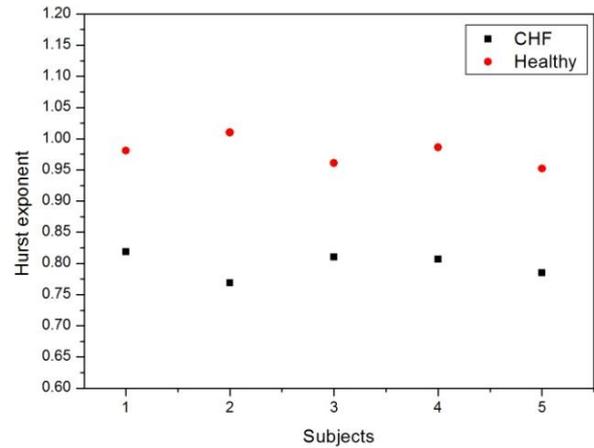


Figure 4: The calculated Hurst exponent (H) of ECG signals of five healthy subjects and five patients with congestive heart failure.

CONCLUSION

From our analysis, we found that all ECG time series signals possess persistent behavior and exhibits strong multifractal characteristics. It is also evident that the Hurst scaling exponents distinguishes the patients and healthy subject and this parameter can be used for prognosis and diagnosis of heart disease. The approach we have used in this study is quite fast and efficient in calculating scaling exponents. It is worth emphasizing that there is no limitation of data length required to use this method. This approach may find applications in financial time series analysis, geophysical time series analysis, genomic sequence analysis, biomedical signal processing, next generation sequence analysis etc.

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