

Solitary Waves In Weakly Inhomogeneous Plasma With Nonthermal Electrons

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Abstract

Propagation of solitary waves in weakly inhomogeneous plasma with non thermally distributed electrons has been studied. For this study we consider the fluid model and the related fluid equations have been treated by the reductive perturbation analysis. In this formulation process we have used a space–time stretched coordinate which is suitable for inhomogeneous plasma. The system of equations has been reduced to a modified Korteweg–de–Vries (mkdV) equation. The Sagdeev pseudo-potential technique is used to carry out the analysis. The soliton solutions are found to be affected by nonthermal electron parameter.

Keywords: Ion–acoustic solitons, inhomogeneous plasma, mkdV equation, nonthermal electron.

Introduction

Theoretical study of non-linear collective processes such as wave propagation is one of the most promising and frontline fields of active research in plasma dynamics. It has received special attention in the past decade mainly due to the realization of their occurrence in both, the laboratory as well as in space environments. One of the pioneering method for mathematical treatment of nonlinear plasma waves was first conceptualized by Washimi and Taniuti^[1] through an equation known as Korteweg – de – Vries (KdV) equation^[2]. A solitary wave is one of such specific types of nonlinear waves which were first observed on the Edinburgh to Glasgow canal by the famous British scientist John Scott Russel^[3] as early as 1834.

Ion-acoustic solitary waves in plasma is one of the basic non-linear wave processes in which phase speed is much smaller than the thermal speed of hot and cold electrons but is much larger than that of ions. In uniform plasma, an ion – acoustic soliton travels without change in amplitude, shape and speed however, in non – uniform plasma the soliton is altered as it propagates. The plasma in all situations, both in laboratory as well as in the space, is to some extent non-uniform (inhomogeneous), Inhomogeneity may be due to density gradient, temperature gradient or due to the magnetic field. The effect of inhomogeneity on the propagation of ion – acoustic soliton have attracted the attention of researchers in various cases such as multi-ion plasma^[4–7], two electron temperature plasma^[8], electron – positron – ion plasma^[9, 10], magnetized plasma^[11, 12], ionized plasma^[12–14] etc.

Based on the relative temperatures of electrons, ions and neutrons, plasmas are divided into thermal and non-thermal plasmas. In general, non-thermal plasmas possess very low temperature compared to thermal plasma. This occurs because the majority of the electrical energy supplied to create plasma from starting gases, primarily excited the electrons within the starting medium, instead of heating the surrounding gas. Thus the electron temperatures are not in thermal equilibrium with the surrounding gas itself. Hence, non-thermal plasma can be defined as plasma that contains energetic electrons^[15–17] or ions^[18, 19] which have been observed in variety of astrophysical plasma environments. The distribution function of energetic electrons and ions revealed to be highly non-thermal and are expected to play an important role in coherent non linear waves and structures like solitons, shocks, double layers, vortices etc. Non-thermal distributions of electrons have been observed in space plasmas by the Viking spacecraft^[20] and Freja satellite^[21].

Later Cairns et al^[22] showed that the presence of non-thermal distribution of electrons may change the ion-acoustic solitary structures, very like those observed by Freja and Viking satellites. Theoretical investigations in such plasmas containing non-thermal electrons have been reported in a number of cases^[15–17]. Almost all of these studies considered for dusty homogeneous plasmas using Sagdeev's pseudopotential method. To the best of our knowledge, studies for inhomogeneous plasmas with non-thermal electrons have never been undertaken. Further the uses of Sagdeev's pseudopotential method in inhomogeneous plasma have never been addressed for the ion-acoustic wave literature. Therefore it seems worthwhile to present a theoretical work to investigate non-thermal ion – acoustic waves in weakly inhomogeneous plasma to identify the conditions that favour their existence.

Methods

A. Basic Equation

Let us consider collisionless, unmagnetized weakly inhomogeneous plasma composed of non-thermal electrons. As the electrons follow the non-thermal distribution, their mass is neglected. We consider the ion temperature to be very small compared to that of electron, hence ion temperature is also neglected. For that logic,

we consider the model of plasma as cold plasma containing inertial cold ions in which the electron temperature may be several thousand degrees, but the ion temperature is near that of ambient. We employ the reductive perturbation technique (RPT) to investigate the propagation of ion-acoustic soliton in our plasma model. We have considered the continuity, momentum and Poisson equations for ions, non-thermal electrons. The non-linear one-dimensional ion-acoustic waves can be governed by following equations in terms of normalized variables as

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} n_i u_i = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i \tag{3}$$

Where non-thermal electron density n_e is given by

$$n_e = 1 - \beta\phi + \beta\phi^2 e^\phi \tag{4}$$

with $\beta = \frac{4\alpha}{1+3\alpha}$. Here α is a parameter determining the number of non-thermal electrons present in our plasma model. We have normalized the ion number density n_i by equilibrium ion density n_i^0 , the ion fluid velocity u_i by ion – acoustic speed $C_s = \left(\frac{KT_e}{m_i}\right)^{1/2}$, time t by the inverse of the characteristic ion plasma frequency $\omega_{pi}^{-1} = \left(\frac{m_i}{4\pi n_i^0 e^2}\right)^{1/2}$, the space variable x by the Debye length $\lambda_{De} = \left(\frac{KT_e}{4\pi n_i^{(0)} e^2}\right)^{1/2}$, the electrostatic potentials ϕ by T_e/e .

B. Derivation of Mkdv Equation:

To derive the mKdV equation we use the reductive perturbation technique [1] and introduce a widely used stretched variable [23] which is appropriate for spatially inhomogeneous plasma, along with the zeroth order fluid velocities. The stretched variable is

$$\xi = \varepsilon^{1/2} \left(\frac{x}{\lambda_0} - t \right), \quad \tau = \varepsilon^{3/2} x \tag{5}$$

Where $\varepsilon \ll 1$ is a small expansion parameter measuring the order of inhomogeneity and λ_0 is the phase velocity.

Using (5), equations (1) – (3) becomes

$$-\frac{\partial n_i}{\partial \xi} + \frac{1}{\lambda_0} \frac{\partial}{\partial \xi} n_i u_i + \varepsilon \frac{\partial}{\partial \tau} n_i u_i = 0 \quad (6)$$

$$-\frac{\partial u_i}{\partial \xi} + \frac{u_i}{\lambda_0} \frac{\partial u_i}{\partial \xi} + \varepsilon u_i \frac{\partial u_i}{\partial \tau} + \frac{1}{\lambda_0} \frac{\partial \phi}{\partial \xi} + \varepsilon \frac{\partial \phi}{\partial \tau} = 0 \quad (7)$$

$$\frac{\varepsilon}{\lambda_0^2} \frac{\partial^2 \phi}{\partial \xi^2} + \frac{2\varepsilon^2}{\lambda_0} \frac{\partial^2 \phi}{\partial \xi \partial \tau} + \varepsilon^3 \frac{\partial^2 \phi}{\partial \tau^2} - 1 - \beta \phi + \beta \phi^2 e^\phi + n_i = 0 \quad (8)$$

The dependent physical parameters n_i , u_i and ϕ are expanded as power series expansion as

$$f = f^{(0)} + \varepsilon f^{(1)} + \varepsilon^2 f^{(2)} + \varepsilon^3 f^{(3)} + \dots \quad (9)$$

Here $f = n_i, u_i, \phi$

Where $n_i^{(0)}$, $u_i^{(0)}$ and $\phi^{(0)}$ are the zeroth order ion density, ion velocity and phase velocity in the presence of weak inhomogeneity. In our present case we have taken all the zeroth order quantities to vary with space and hence, we have retained the terms $\frac{\partial n_i^{(0)}}{\partial \tau}$, $\frac{\partial u_i^{(0)}}{\partial \tau}$ and $\frac{\partial \phi^{(0)}}{\partial \tau}$.

As $n_i^{(0)}$ is a function of x only, we have

$$\frac{\partial n_i^{(0)}}{\partial \xi} = 0 \quad (10)$$

The terms free from of ε gives

$$\frac{\partial u_i^{(0)}}{\partial \xi} = \frac{\partial \phi^{(0)}}{\partial \xi} = 0 \quad (11)$$

This yields following relations for unperturbed quantities

$$\frac{\partial}{\partial \tau} (n_i^{(0)} u_i^{(0)}) = 0 \quad (12)$$

$$u_i^{(0)} \frac{\partial u_i^{(0)}}{\partial \tau} + \frac{\partial \phi^{(0)}}{\partial \tau} = 0 \quad (13)$$

$$\text{And } n_i^{(0)} = 1 + (1 - \beta) \phi^{(0)} + \frac{\phi^{(0)2}}{2} \quad (14)$$

The lowest order of ε gives

$$-\frac{\lambda_0 - u_i^{(0)}}{\lambda_0} \frac{\partial n_i^{(1)}}{\partial \xi} + \frac{n_i^{(0)}}{\lambda_0} \frac{\partial u_i^{(1)}}{\partial \xi} + \frac{\partial}{\partial \tau} (n_i^{(0)} u_i^{(0)}) = 0 \quad (15)$$

$$-\frac{\lambda_0 - u_i^{(0)}}{\lambda_0} \frac{\partial u_i^{(1)}}{\partial \xi} + \frac{1}{\lambda_0} \frac{\partial \phi^{(1)}}{\partial \xi} + u_i^{(0)} \frac{\partial u_i^{(0)}}{\partial \tau} + \frac{\partial \phi^{(0)}}{\partial \tau} = 0 \quad (16)$$

$$n_i^{(1)} = 1 - \beta + \phi^{(0)} \phi^{(1)} \quad (17)$$

Integrating these equations and using boundary conditions $n_i^{(1)}, u_i^{(1)}, \phi^{(1)} \rightarrow 0$ as $|\xi| \rightarrow \infty$ we get

$$u_i^{(1)} = \frac{\lambda_0 - u_i^{(0)}}{n_i^{(0)}} n_i^{(1)}, \quad \phi^{(1)} = \frac{\lambda_0 - u_i^{(0)2}}{n_i^{(0)}} n_i^{(1)} \quad (18)$$

From (17) and (18) we get the following phase velocity relation

$$\lambda_0 = u_i^{(0)} \pm \sqrt{S}, \quad S = \frac{n_i^{(0)}}{1 - \beta + \phi^{(0)}} \quad (19)$$

The above relation shows that λ_0 depends on the quantities $u_i^{(0)}, n_i^{(0)}$ and $\phi^{(0)}$ which are functions of space coordinate. It can realize that λ_0 also becomes a function of space coordinate which yields $\frac{\partial \lambda_0}{\partial \tau} \neq 0$. However, the effect of inhomogeneity entered into λ_0 through the fluid drift and hence λ_0 can be taken to be constant in the stretched variable which is also justified for the weak inhomogeneity.

For second order of ε , we obtain

$$\frac{\lambda_0 - u_i^{(0)}}{\lambda_0} \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{n_i^{(0)}}{\lambda_0} \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{n_i^{(1)}}{\lambda_0} \frac{\partial u_i^{(1)}}{\partial \xi} + \frac{u_i^{(1)}}{\lambda_0} \frac{\partial n_i^{(1)}}{\partial \xi} + \frac{\partial}{\partial \tau} (n_i^{(0)} u_i^{(1)} + n_i^{(1)} u_i^{(0)}) = 0 \quad (20)$$

$$-\frac{\lambda_0 - u_i^{(0)}}{\lambda_0} \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{1}{\lambda_0} \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{u_i^{(1)}}{\lambda_0} \frac{\partial u_i^{(1)}}{\partial \xi} + u_i^{(0)} \frac{\partial u_i^{(1)}}{\partial \tau} + u_i^{(1)} \frac{\partial u_i^{(0)}}{\partial \tau} + \frac{\partial \phi^{(1)}}{\partial \tau} = 0 \quad (21)$$

$$\frac{1}{\lambda_0^2} \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - 1 - \beta + \phi^{(0)} \phi^{(2)} - \frac{1}{2} \phi^{(1)2} + n_i^{(2)} = 0 \quad (22)$$

Differentiating (22) w. r. t. ξ gives

$$\frac{1}{\lambda_0^2} \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} - 1 - \beta + \phi^{(0)} \frac{\partial \phi^{(2)}}{\partial \xi} - \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\partial n^{(2)}}{\partial \xi} = 0 \quad (23)$$

Using equation (19), all second order quantities from equations (20), (21) and (23) are eliminated exactly. After some mathematical treatment, finally we get the following modified KdV equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + C \frac{\partial n_i^{(0)}}{\partial \tau} \phi^{(1)} = 0 \quad (24)$$

Where

$$A = \frac{1}{2\lambda_0^2} \left[\frac{3}{\lambda_0 - u_i^{(0)}} - \frac{S^{3/2}}{n_i^{(0)}} \right], \quad B = \frac{S^{3/2}}{2n_i^{(0)}\lambda_0^4}, \quad C = \frac{1}{n_i^{(0)}\lambda_0} \left[\lambda_0 - u_i^{(0)} - \frac{S}{2u_i^{(0)}} \right]$$

C. Solution of Modified Kdv Equation

To solve the modified KdV equation (24), we introduce the following transformation

$$\phi^{(1)} \xi, \quad \tau = g \tau \psi^{(1)} \quad (25)$$

Where $g \tau = e^{-\int c \frac{\partial n_i^{(0)}}{\partial \tau} d\tau}$, is a new variable.

Then the eqn.(24) takes the form

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A g \tau \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + B \frac{\partial^3 \psi^{(1)}}{\partial \xi^3} = 0 \quad (26)$$

The non-linear co-efficient functionally depends on the space of the plasma. For the sake of simplicity of mathematical treatment, the variations are assumed negligible as compared to the scale length and hence it is assumed that all parameter are locally constant.

To obtain a steady state solution of eqn. (26), transforming the pair of variable ξ, τ to a single variable ζ using

$$\zeta = \xi - U_0 \tau \quad (27)$$

With U_0 as the soliton speed in ζ -space. With this transformation the mKdV eqn. (26) after integrating twice takes the form

$$\begin{aligned} -U_0 \psi^{(1)} + \frac{A}{2} g \zeta \psi^{(1)2} + B \frac{d^2 \psi^{(1)}}{d\zeta^2} &= 0 \\ \Rightarrow B \left(\frac{d\psi^{(1)}}{d\zeta} \right)^2 &= U_0 \psi^{(1)2} - \frac{A}{3} g \zeta \psi^{(1)3} \\ \Rightarrow \left(\frac{d\psi^{(1)}}{d\zeta} \right)^2 &= \frac{U_0}{B} \psi^{(1)2} - \frac{A}{3B} g \zeta \psi^{(1)3} \\ \Rightarrow \frac{1}{2} \left(\frac{d\psi^{(1)}}{d\zeta} \right)^2 &= \frac{U_0}{2B} \psi^{(1)2} - \frac{A}{6B} g \zeta \psi^{(1)3} \\ \Rightarrow \frac{1}{2} \left(\frac{d\psi^{(1)}}{d\zeta} \right)^2 + V \psi^{(1)} &= 0 \end{aligned} \quad (28)$$

Where the Sagdeev Pseudo-potential $V \psi^{(1)}$ is given by

$$V \psi^{(1)} = \frac{A}{6B} g \zeta \psi^{(1)3} - \frac{U_0}{2B} \psi^{(1)2} \quad (29)$$

The equation (28) can be regarded as 'energy integral' of an oscillating particle of unit mass with velocity $\frac{d\psi^{(1)}}{d\zeta}$ and position $\psi^{(1)}$ in a potential well $V \psi^{(1)}$. The same equation (28) can also be considered as a motion of a particle with pseudoposition $\psi^{(1)}$ at a pseudotime ζ with pseudovelocity $\frac{d\psi^{(1)}}{d\zeta}$ in a pseudopotential $V \psi^{(1)}$. Here the pseudoparticle starts at a position $\psi^{(1)} = 0$ with small velocity $\frac{d\psi^{(1)}}{d\zeta}$ and it will be reflected back at some $\psi^{(1)} = \psi_m$ and will come back to $\psi^{(1)} = 0$.

The existence of solitary waves can be determined from the nature of the pseudopotential $V(\psi^{(1)})$. From equation (28) it is obvious that $V(\psi^{(1)})$ must be negative to get real solution. For the two extreme points (roots) 0 and ψ_m of the Sagdeev potential $V(\psi^{(1)})$, the conditions for solitary waves are

1. $V(\psi^{(1)}) = 0$ at $\psi^{(1)} = 0$ and $\psi^{(1)} = \psi_m$.
2. $\left. \frac{dV}{d\psi^{(1)}} \right|_{\psi^{(1)}=0} = 0$ but $\left. \frac{dV}{d\psi^{(1)}} \right|_{\psi^{(1)}=\psi_m} \neq 0$
3. $\left. \frac{d^2V}{d\psi^{(1)2}} \right|_{\psi^{(1)}=0} < 0$

The non-zero ψ_m , the maximum or minimum value of $\psi^{(1)}$ is called the amplitude of the solitary wave.

Here $A, B > 0$, then

$$\begin{aligned} \frac{d\psi^{(1)}}{d\zeta} &= \psi^{(1)} \sqrt{\frac{3U_0 - Ag \zeta \psi^{(1)}}{3B}} \\ \Rightarrow \frac{d\zeta}{\sqrt{3B}} &= \frac{d\psi^{(1)}}{\psi^{(1)} \sqrt{3U_0 - Ag \zeta \psi^{(1)}}} \\ \Rightarrow \frac{\zeta}{\sqrt{3B}} &= \int \frac{d\psi^{(1)}}{\psi^{(1)} \sqrt{3U_0 - Ag \zeta \psi^{(1)}}} = \int \frac{-2zdz}{\psi^{(1)} z} = -\frac{2}{Ag \zeta} \int \frac{dz}{\psi^{(1)}} \end{aligned}$$

Where $3U_0 - Ag \zeta \psi^{(1)} = z^2$

$$\begin{aligned} \Rightarrow \frac{\zeta}{\sqrt{3B}} &= -\frac{2}{Ag \zeta} \int \frac{dz}{\frac{3U_0 - z^2}{Ag \zeta}} \\ &= -2 \int \frac{dz}{3U_0 - z^2} = -\frac{2}{\sqrt{3U_0}} \tanh^{-1} \frac{z}{\sqrt{3U_0}} \\ \Rightarrow \zeta &= -2 \sqrt{\frac{B}{U_0}} \tanh^{-1} \sqrt{\frac{3U_0 - Ag \zeta \psi^{(1)}}{3U_0}} \\ \Rightarrow \sqrt{\frac{U_0}{B}} \frac{\zeta}{2} &= -\tanh^{-1} \sqrt{\frac{3U_0 - Ag \zeta \psi^{(1)}}{3U_0}} \\ \Rightarrow \tanh \left(\frac{\zeta}{2} \sqrt{\frac{U_0}{B}} \right) &= -\left(1 - \frac{Ag \zeta \psi^{(1)}}{3U_0} \right) \\ \Rightarrow 3U_0 \tanh^2 \left(\frac{\zeta}{2} \sqrt{\frac{U_0}{B}} \right) &= 3U_0 - Ag \zeta \psi^{(1)} \end{aligned}$$

$$\begin{aligned}
\Rightarrow Ag \zeta \psi^{(1)} &= 3U_0 \left[1 - \tanh^2 \left(\frac{\zeta}{2} \sqrt{\frac{U_0}{B}} \right) \right] \\
\Rightarrow \psi^{(1)} &= \frac{3U_0}{Ag \zeta} \operatorname{sech}^2 \left(\frac{\zeta}{2} \sqrt{\frac{U_0}{B}} \right) \\
&= \psi_m \operatorname{sech}^2 \zeta \omega_1
\end{aligned} \tag{30}$$

Where ψ_m and ω_1^{-1} gives amplitude and width of the solitons respectively, with

$$\psi_m = \frac{3U_0}{Ag \zeta}, \quad \omega_1 = \frac{1}{2} \sqrt{\frac{U_0}{B}}$$

Results and Discussion

In the present plasma model with nonthermal electron parameter α , we observe the existence of small amplitude compressive solitons. The nonthermal electron parameter α plays an important role in the formation of solitary waves. The existence of solitary waves have determined by plotting equation (29) with $V \psi^{(1)}$ against $\psi^{(1)}$ for different values of nonthermal parameter α (Fig. 1). Here we observed that as α increases, amplitude is also increases in specific ranges of some affecting parameters.

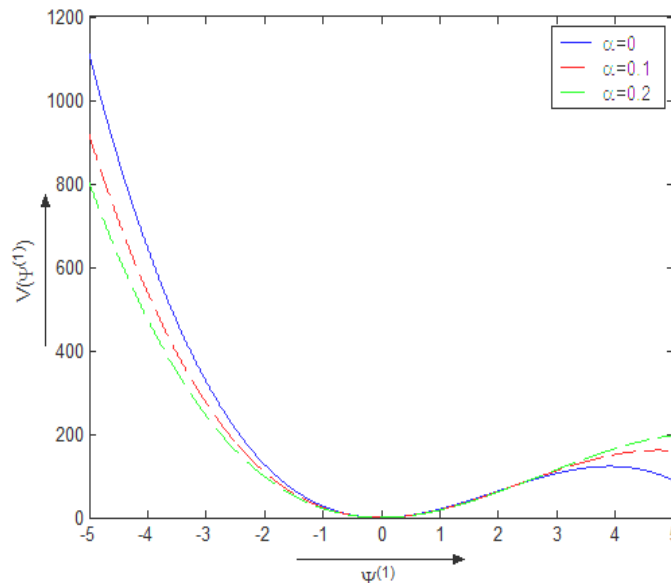


Figure 1: Plot of $V \psi^{(1)}$ versus $\psi^{(1)}$ showing the formation of solitons for $U_0=1$, $n_i^{(0)}=0.5$ and $\phi^{(0)}=0.5$

In Fig. 2 and Fig. 3, the variations of ψ_m with α is shown. In Fig. 2, three different values of $n_i^{(0)}$ ($=0.5, 0.6$ and 0.7) are considered with constant values of U_0 , $u_i^{(0)}$ and $\phi^{(0)}$ whereas in Fig.3, four different values of $u_i^{(0)}$ are considered with constant values of U_0 , $n_i^{(0)}$ and $\phi^{(0)}$. In both the situations, amplitudes are increases as α increases.

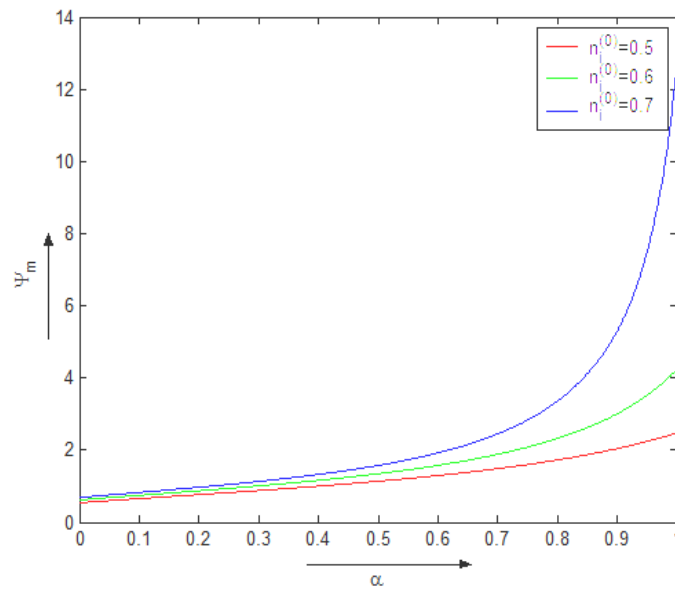


Figure 2: Variation in the solitary wave amplitude ψ_m with the nonthermal electron parameter α with $U_0=1$, $u_i^{(0)}=0.2$ and $\phi^{(0)}=0.5$ for different values of n_0 .

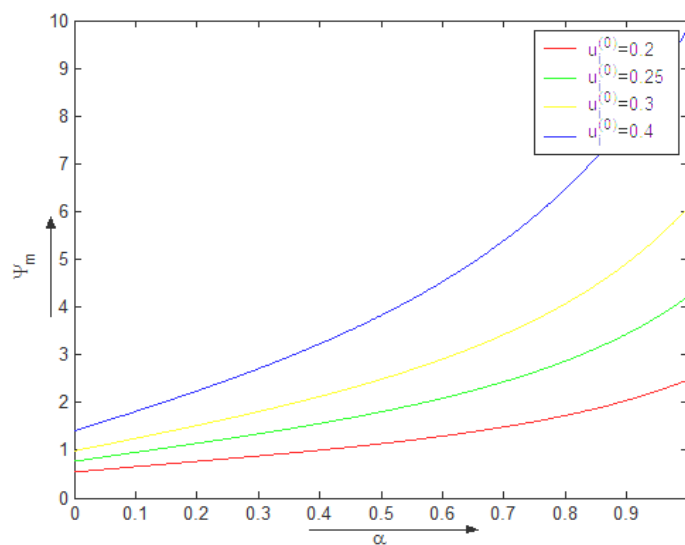


Figure 3: Variation in the solitary wave amplitude ψ_m with the nonthermal electron parameter α with $U_0=1$, $n_i^{(0)}=0.5$ and $\phi^{(0)}=0.5$ for different values of $u_i^{(0)}$ ($=0.2, 0.25, 0.3$ and 0.4)

The dependence of soliton width ω_1^{-1} on nonthermal electron parameter α is observed in Fig.4 with parameters $U_0 = 1$, $n_i^{(0)} = 0.5$, $u_i^{(0)} = 0.5$ and $\phi^{(0)} = 0.5$. It shows that as α increases, the soliton width ω_1^{-1} is also increase.

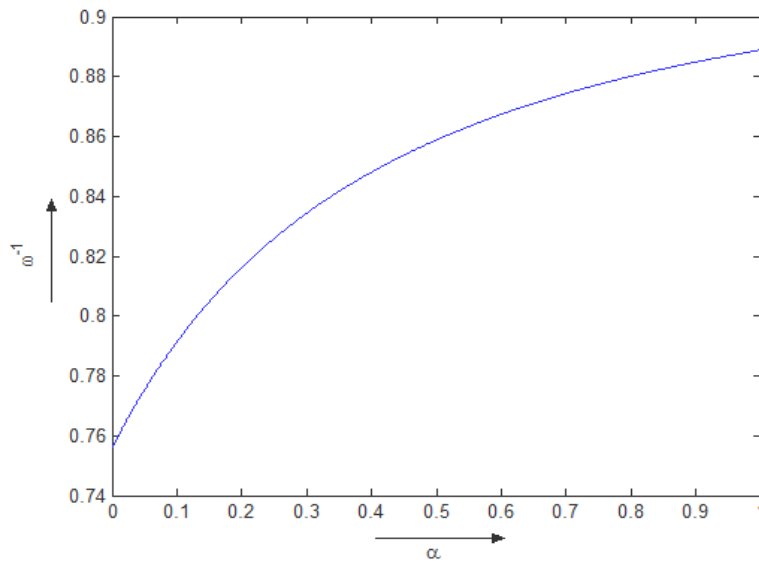


Figure 4: Variation in the solitary wave width ω_1^{-1} with the nonthermal electron parameter α with $U_0=1$, $n_i^{(0)}=0.5$, $u_i^{(0)}=0.5$ and $\phi^{(0)}=0.5$.

Conclusion

In this present paper we have studied the evolution of solitons in inhomogeneous plasma that has nonthermal electron. Our main focus has been on the contribution of nonthermal parameter to the soliton propagation characteristic. A pseudo potential approach has been applied to study the effect of nonthermal parameter in the amplitude and width of the solitons. Finally it was found that the soliton evolves with smaller amplitude and larger width under the effect of nonthermal electron parameter.

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