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Maintenance Planning For The Reliability Maximization In A Large System With Vagueness In The Reliability Values Of Some Components

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Abstract

This paper tackles the problem to single out the maintenance actions to be executed on a production system during the planned stop of given length. In detail, from a reliability viewpoint the system is assimilated as a series-parallel multi-component system and the maintenance actions need to maximize the system reliability up to the next planned stop. Moreover, it is assumed that some components reliability values are affected by vagueness within a given range. To solve the considered problem, an exact dynamic programming algorithm suitable to quickly point out the maintenance scheduling is developed and, moreover, it is formulated a proper parameter able to express the robustness of the obtained optimal solutions. Finally, a numerical example with reference to a complex system composed by a large number of components is reported.

Keywords: Maintenance scheduling optimization; series–parallel systems; vagueness on reliability; dynamic programming algorithm.

1. Introduction

In the last few years, many researchers have faced some maintenance problems in the multi-component systems field. From a reliability point of view, these systems are constituted by several components arranged in a series disposition, some of which are in their turn parallel or parallel-series components. A wide overview about the multi-unit system maintenance models developed up to 1991 is presented by Cho and Parlar (1991), while a complete updated survey can be found in Dekker *et al.* (1997) and Nicolai and Dekker (2006). A comprehensive literature study on maintenance policies of deterioration systems are reported in Lupo (2014a).

In the literature, the most used measure in evaluating the maintenance policy performance is the stationary availability of a production system. The various models differ for the considered objective function, that usually is the global maintenance cost, for the eventual constraints and for the resolution approach (Bris et al., 2003; Certa et al., 2012a; Tam, 2006). Nevertheless, for some systems the failure event can be dangerous, too expensive or even disastrous. For these reasons, being the reliability defines as the probability that the system operates without failure for a fixed period of time under stated conditions, for such systems a high reliability level is imposed. That being so, the reliability constitutes another meaningful parameter to assess the system performance (Certa et al., 2011; 2012b; Lupo, 2013).

This paper tackles the problem of singling out the maintenance actions to execute on a series-parallel system exploiting planned plant stops, for example due to production change, so as a given reliability value is warranted up to the next planned stop.

Cassady (2001) tackles the problem of identifying the set of elements on which to operate during a planned downtime between two missions, aiming to maximize the system reliability up to the next mission. The maintenance activities must be completed within a stated time and a fixed cost. The problem is formulated by a mathematical programming model and two numerical examples with 10 and 12 components respectively are reported.

Rajagopalan and Cassady (2006) deal with the reliability maximization for a system constituted by a series arrangement of subsystems, each one containing a set of identical parallel arranged elements. A constraint on maintenance time is considered. Elements have a constant failure rate and therefore maintenance action reduces to the replacement of those failed. The decision variable is the number of failed elements to be replaced for each subsystem. The problem is a non-linear knapsack problem and thus the Authors propose four improvements to speed up the total enumeration approach originally employed (Rice et al., 1998). Recently, also suitable statistical tools based on for attributes (Inghilleri, 2015; Lupo, 2015) or for variables (Lupo, 2014b) control charts are considered to support the choice of maintenance actions to be performed on a multi-component system.

When lots of failure data are available for some components or they are monitored, the failure probability estimation can be considered reliable. On the contrary, for other components, estimates could be the more vague the longer their operating time is. For the decision maker it is important to take into account such vagueness in order to single out the best solution. In detail, such a solution must be robust, that is scarcely sensitive to possible differences between the supposed component reliability and the real one. Therefore, the decision maker could be interested in having more optimal solutions for the problem previously formulated, obtained by assuming, at least for few components, different reliability values inside some vagueness range. In this way, he/she can verify how much robust a solution is in relation to various possible scenarios. This analysis could drive the decision maker in choosing a solution characterized by a lower level of system reliability than the optimal one obtainable under the hypothesis of sure reliability data, but more robust in that case a state of uncertainty exists (Certa et al., 2013).

Summing up, in the present paper, it is supposed to handle with uncertain reliability data. Consequently, the decision maker has to evaluate the most robust solution for different possible scenarios. To this purpose, a parameter is proposed for representing the solution robustness. Moreover, an exact algorithm is presented for selecting the system elements to be maintained. Such algorithm is an adaptation of a previous one designed by the same authors (Galante and Passannanti, 2009), of which a short description is given in section 3.

In the next section, the faced problem is justified and it is analytically expressed. After the resolution algorithm presentation, the fourth paragraph proposes a parameter for the evaluation of solution robustness. Lastly, a numerical example explains the whole procedure and final remarks conclude the paper.

2. Problem Formulation

The system taken into consideration is a system operating for process. The plant is periodically stopped for production change and system setup. During these stops, maintenance actions are carried out in order to maximize the system reliability. As a matter of fact, for this type of system, failure is an event that must be avoided because it is too expensive and/or too dangerous for workers. A time constraint is also introduced: maintenance must take place during the planned plant stop to execute system setup.

System reliability up to the end of the next utilization period can be calculated by the reliability values of its components. These values depend on the execution or not of maintenance actions during the considered plant stop. They can be regarded as sure values for some elements, in particular those ones that have cumulated a short use time or have been maintained. As a matter of fact, the mission time is assumed to be short as to the element life and the maintenance action is a “as good as new” type. Other elements characterized by sure reliability values are those working under monitored conditions. On the contrary, the reliability of other elements is affected with uncertainty. In particular, it is supposed that the real value of reliability of each of these elements falls into a range that can be estimated. The uncertainty arises from a poor knowledge of the actual system operative conditions and then it can be supposed that a high correlation exists among the reliability values affected with uncertainty. Consequently, in any case, all the real reliability values of these elements are in the same position of their vagueness range.

Each maintenance action involves both a resource engagement time and a spare part cost. If more maintenance crews can simultaneously operate and the interventions do not present precedence constraints, then the global time required to carry out the interventions singled out to be optimal is given by:

$$T = \sum_{i \in I} t_i / n \quad (1)$$

where I is the set of elements selected to be maintained, t_i is the maintenance time on the element i (i=1, ..., N) and n is the number of crews.

The previous problem can be formulated as follows:

$$max\{R\} \quad (2)$$

subject to:

$$T \leq T^* \quad (3)$$

where R is the system reliability and T^* is the planned downtime length for the system maintenance. In order to calculate the system reliability it is necessary to know the reliability values for all constituting elements. As some values are doubtful, a definite value must be fixed inside each uncertainty range. To this aim, each continuous range is substituted by S equidistant values and then each element is characterized by S different scenarios. As said before, concerning the reliability, it can retain that the same conditions occur for all elements and so the system will operate in a scenario in which all elements have a reliability value given by their own first scenario, or by the second, or by the S^{th} scenario. To sum up, S scenarios need to be considered and, for each of them, the elements to be maintained have to be singled out by solving the previously formulated problem.

Such a problem is a NP-hard combinatorial problem (Rice et al., 1998) and, even if it could be easily expressed in terms of mathematical programming, the presence of both Boolean variables and a non-linear objective function makes this approach the more difficult the bigger the problem dimension is.

3. Proposed optimization algorithm

Consider a system constituted by series components, some of them, in their turn, constituted by elements in parallel-series disposition. Regarding to the series components, constituted by only one element, they are ordered in a list however drawn out. The algorithm matches the two possible maintenance states of the first element with the two possible maintenance states of the second one. Among the four obtained sequences, the algorithm eliminates those dominated, if they exist. A sequence s_1 dominates a sequence s_2 if:

$$R_{s1} > R_{s2} \text{ and } T_{s1} \leq T_{s2} \quad (4)$$

or

$$R_{s1} = R_{s2} \text{ and } T_{s1} < T_{s2} \quad (5)$$

The survived sequences are matched with the two possible states of the next element in the list. The procedure continues until the last element is considered.

At each step, in order to reduce the number of partial sequences to be considered at the next step, two further elimination criteria can be introduced. In fact, each partial sequence *s* evolves to a maintenance time included between two extreme values. The first one, a Lower Bound value, $LB_T(s)$, is obtained if no maintenance is executed on the remaining elements. The other, an Upper Bound value $UB_T(s)$, is obtained hypothesizing that all the remaining components are maintained. Comparing these two time values with the constraint value, two cutting criteria are defined. LB criterion: if $LB_T(s)>T^*$, then the partial sequence *s*, even if non dominated, can be eliminated because it can not respect the constraint (3). UB criterion: if a partial sequence verifies the condition $UB_T(s)\leq T^*$, then all the others having a lower reliability are removed. Moreover, the survived one does not require to be further branched and it is completed by maintaining all the remaining components.

For each component constituted by elements parallel-series, each branch is considered separately and it is analyzed in the same way of the series systems. That is, the algorithm preliminarily eliminates the dominated sequences of a branch, saving those non dominated. The parallel is subsequently solved by considering all the possible combinations of the survived sequences of the first branch with those ones of the second branch and, as before, only the non dominated partial sequences are saved. The method iteratively continues by adding the non dominated sequences of the next branch to those saved at the previous step.

Firstly the algorithm solves the components constituted by the parallel-series elements, obtaining for each of them the non dominated sequences, and later it analyzes the overall system in which the parallel-series components will be considered as a series component characterized by the survived sequences, representing alternative states, rather than the two states maintenance yes or not.

4. Measure of the robusteness

After solving the problem expressed by equations (1) to (3) for all considered scenarios, the following quantities can be calculated:

- $R(i)$, system reliability with respect to the optimal solution obtained for the scenario *i*;
- $R(j|i)$, system reliability when the implemented solution is that one determined for the scenario *i* but that occurring is *j*.

Then, $L(j|i) = R(j) - R(j|i)$ measures the loss of reliability when the scenario *j* happens while the scenario *i* is erroneously considered to point out the maintenance planning. An expected loss value can be evaluated by:

$$L(i) = \sum_{j=1}^{S} L(j \mid i) / S \tag{6}$$

It is obvious that such summation should be minimized and, at the same time, it is opportune to relate it to the reliability value of the selected solutions. After all, the robustness of a solution determined for the specific scenario *i* can be defined by using the following ratio:

$$Rob(i) = R(i)/L(i) \quad (7)$$

that takes into account both the good quality of a solution, by means of the related reliability, and the possible reliability loss if a different scenario from that assumed occurs.

Finally, after the optimal solution has been found out for each hypothetic scenario and values of robustness have been calculated, the solution to be selected will be that one having the maximum value of robustness.

5. Numerical example

The procedure has been applied to a numerical example involving a system constituted by 44 components in series. Five components are macro-component constituted by parallel-series elements, while the others are constituted by just one element. If the macro-components are codified in the form $j(k; m_1,.. m_r,.. m_k)$, where *j* is the generic component, *k* indicates the number of branches and m_r the elements in series in the branch *r*, then the encoding of the macro-components is: 1(3; 6,6,6); 2(2; 4,4); 3(2; 1,1); 4(3; 3,3,3); 5(2; 2,2); so the total number of elements that constitute the system is eighty. The input data are reported in Table 1. $R_{i,b}$ is the reliability ($*10^8$) of a generic element *i* at the end of the next mission if it is not maintained, $R_{i,a}$ is the reliability under the hypothesis of maintenance and t_i is its execution time. If two values are reported for $R_{i,b}$, then the element reliability is doubtful: the two values represent the uncertainty range and they coincide with the values assigned to the extreme scenarios 1 and *S*.

In the example the uncertainty range has been further divided into ten scenarios (S=10). The maintenance time constraint has been changed acting on the parameter *w* in the relation $T^* = wT_{max}$, being *w* a positive number smaller than 1 and

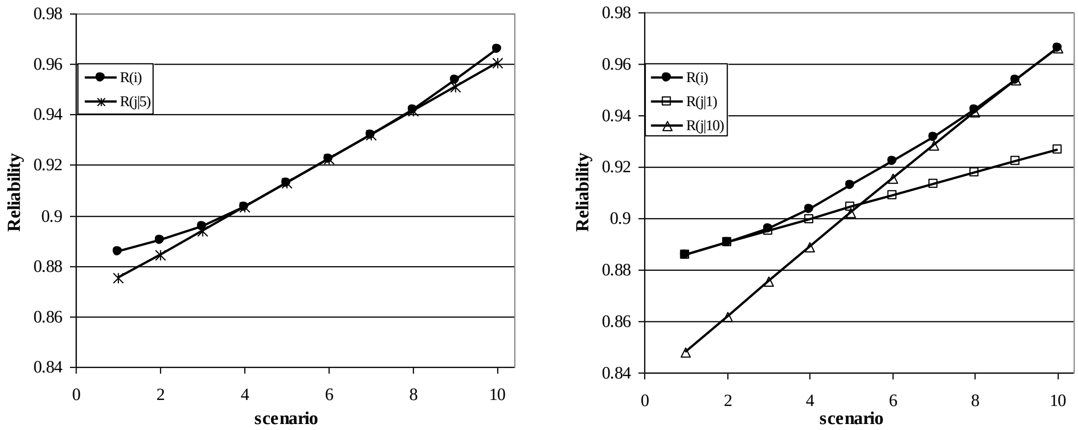
$$T_{\max} = \sum_{i=1}^{N} t_i / n \quad (8)$$

the time required to maintain all system elements.

Figs. 1a and 1b show, just for example, the reliability values that allow to calculate the robustness of the different solutions obtained by setting the parameter *w* equal to 0.5. The curve *R*(*i*) is the same in both Figs. and it is relevant to the reliability values of the optimal solutions obtained varying the scenario. Curves *R*(*j*|*i*) express the reliability that the solution individuated for the supposed scenario (1, 10 or 5) should have under the hypothesis of realization of the scenario *j*.

Table 1: Input data

i	$R_{i,b}$	$R_{i,a}$	t_i	i	$R_{i,b}$	$R_{i,a}$	t_i
1	91930489	99009072	12.9	41	91517631	99305850	9.9
2	91916844	99017004	12.0	42	99187777	99980093	7.5
3	91924971	99016226	11.3	43	99186848	99982467	13.4
4	91924935 – 98574989	99015988	8.5	44	99204631 – 99859641	99980020	10.5
5	91926427	99017716	10.5	45	99181823	99982357	9.1
6	91923929	99016700	9.5	46	99194409	99982572	8.9
7	91923803 – 98574789	99017301	6.9	47	99194705	99981993	9.4
8	91930728	99016298	8.8	48	99188859 – 9856857	99977393	11.6
9	91925076	99013058	10.1	49	99180396	99985579	10.2
10	91922546	99015311	11.6	50	99200604	99977563	8.5
11	91933422 – 98576486	99018656	7.2	51	99178013 – 99854943	99977251	6.9
12	91922546	99013912	12.1	52	99201034	99979810	9.3
13	91931273 – 98576107	99018048	6.1	53	99201814	99978317	9.8
14	91915307 – 98573289	99020536	10	54	99179145 – 99855143	99978999	9.7
15	91924484		8	55	99187814	99979287	8.6
16	91915365	99020339	12.5	56	99183147 – 99855849	99978902	14.1
17	91929715	99014839	12.1	57	99199266	99980844	8.7
18	91928219	99017534	10.7	58	99192017 – 99857415	99983186	10.7
19	95797552 – 99258392	99626280	9	59	99193540	99979081	9.4
20	95787842 – 99256678	99631849	9.6	60	99180335 – 99855353	99982996	10.5
21	95795488	99631002	12.4	61	99199992	99982115	8.4
22	95790784	99625609	7.4	62	99202238 – 99859218	99980961	8.9
23	95794939	99625553	10.5	63	99180862 – 99855446	99976535	10.9
24	95794122 – 99257786	99631856	6.9	64	99196064	99978833	11.2
25	95793123	99625423	11.1	65	99209178	99978564	11
26	95792950	99628841	6.5	66	99188961 – 99856875	99980975	8.2
27	83393649 – 97069467	98435609	12.6	67	99205810	99980256	10.2
28	85118183	98436647	12.8	68	99175949 – 99854579	99976166	9.8
29	84149720 – 97202892	98075810	7.7	69	99196575	99982187	13
30	84181065	98077443	7.8	70	99207745	99976730	11.2
31	84185946	98077900	12.4	71	99194642 – 99857878	99975020	13.3
32	84150043 – 97202949	98073956	8.5	72	99212815	99984006	8.2
33	84165735	98084021	5.6	73	99192962 – 99857582	99975324	10.2
34	84165296	98078055	12.1	74	99201034	99973021	12.8
35	84158305 – 97204407	98080317	11	75	99199718 – 99858774	99982010	11.9
36	84181451	98079127	8.8	76	99199693	99978684	8.9
37	84188387	98080376	14.1	77	99207021 – 99860063	99981753	6.1
38	91587808 – 98515496	99315673	7.9	78	99209486 – 99860498	99976429	9
39	91528130	99313566	11.3	79	99211086	99982102	12.5
40	92515189	99310754	8.4	80	99201792 – 99859140	99982140	9.2



(a) (b)

Fig. 1: *R*(*i*) and *R*(*j*|*i*) when *w* = 0.5

The reliability values graphically represented in these Figs. allow to evaluate the solution robustness for the scenarios 1, *5* and 10. Fig. 2 shows the robustness values for all scenarios (Eq. 7).

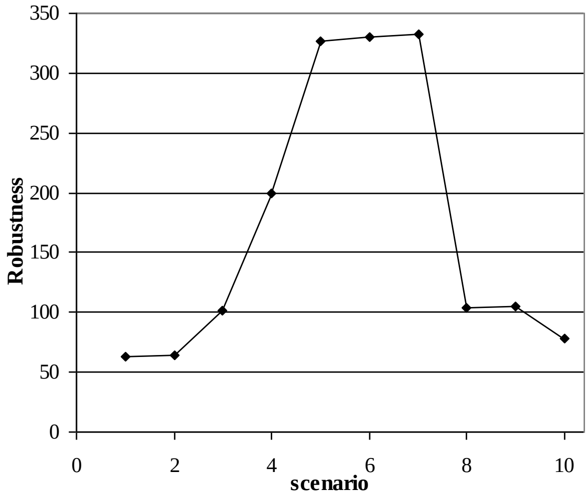


Fig. 2: Robustness when *w* = 0.5

The solution to be selected is that having the greatest robustness, hence that one obtained for the scenario *s*=7. Anyway, the scenario corresponding to the greatest robustness requires some consideration. When uncertain data are available, but precise values must be utilized in order to optimize some objective function, the prudence could induce to assume mean values inside the variability ranges: this choice ought to minimize the opportunity loss. That seams to be confirmed by curves in Fig. 1: the scenario *s*=5 gives rise to the lowest losses of reliability, but losses for *s*=7 are less meaningful as to the respective reliability value and then the solution obtained for *s*=7 is more robust. Curves in Fig. 3, that concern various constraints about the available maintenance time, confirm that a prudent choice is not always the best: if the most robust solution is obtained with *s*=5 when *w*=0.1 or *w*=0.9, the best solution is obtained with the scenario *s*=6 if *w*=0.05 and with *s*=8 if *w*=0.7.

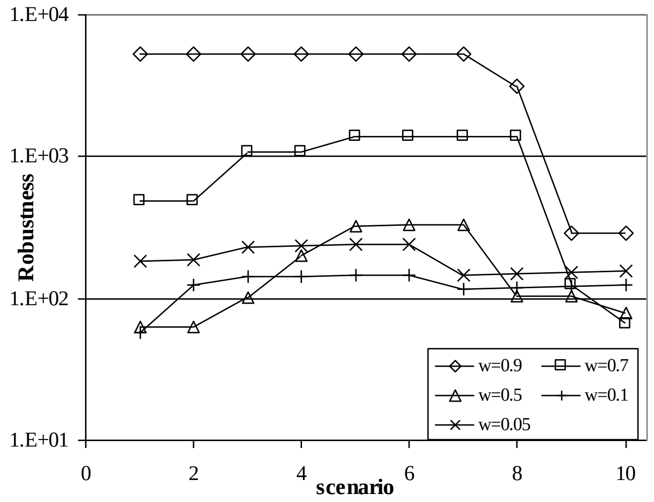


Fig. 3: Robustness versus scenario for some values of the parameter *w*

After all, it can be stated that prudence does not pay. In the same way, it can be affirmed that either optimism or pessimism do not pay: see the robustness falls for scenarios with high indexes when *w*=0.7 and *w*=0.9, and the low values obtained for *s*=1 and *s*=2 when *w*=0.5. The tuned procedure is able of determining the best solution whatever the decision maker mood is.

Fig. 4 points out that the most robust solutions are obtained when T^* is very low or approaching T_{max}.

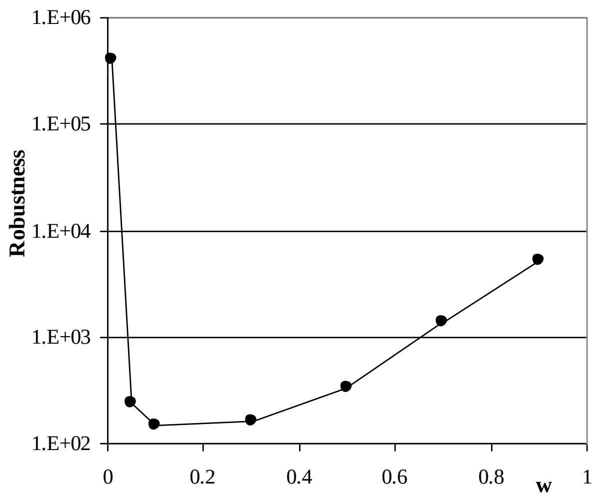


Fig. 4: Maximum robustness versus *w*

This fact can be justified by the following considerations.

By varying the considered scenario, the optimal solutions are generally different, hence reliability losses, *L*, (see Eq. 6) occur. Whenever *w* takes a high (low) value, the set of elements *I* that can be maintained within *T** is very numerous (very scanty), consequently the optimal solutions for the different scenarios are very similar among them since the eventual differences are restricted to the few elements excluded from (included in) *I*. If *w* approaches 1 (approaches 0), all elements (no elements) are maintained and all solutions are identical whatever scenario may come true: the robustness goes to infinity. On the contrary, if *w* assumes an intermediate value then the optimal solutions variability with relation to the scenario is high. These conditions bring to a low robustness value.

Lastly, about the efficiency of the proposed optimization procedure, the global run time, from the determination of the optimum solution for all the scenarios up to the singling out of the most robust one for given value of *w*, requires less than 15 seconds.

6. Conclusions

The present paper has been focused on a system reliability maximization problem by maintaining the components of a multi-component system during planned intervals of given length. Since some components reliability values can be affected by vagueness within a given range, the proposed procedure allows to single out a solution that is not only optimal in a mathematical point of view but also robust, *i.e.* not very sensitive to changing of operative conditions of the system. A suitable parameter is proposed for measuring such robustness which calculation requires that the optimization problem is solved several times. This is possible thanks to the developed optimization algorithm allowing to solve such problem to optimality and in a very short time, even for complex series-parallel systems.

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