

An Effective Approach For The Maintenance Scheduling In Large Systems With Required Reliability Level: A Case Study

A.Certa^{*}, G. Galante^{*}, T. Lupo^{*1} and G. Passannanti^{*}

^{}Dipartimento di Ingegneria Chimica,
Gestionale, Informatica, Meccanica (DICGIM)
Università degli Studi di Palermo
Viale delle Scienze, 90128, Palermo, Italy*

¹Corresponding Authoremial address toni.lupo@unipa.it

Abstract

This paper deals with the problem of the maintenance scheduling in a multi-component system for which a required reliability level has to be warranted until the next planned stop for maintenance. Particularly, the tackled problem concerns both the determination of the elements set on which to perform preventive maintenance and the optimal number of maintenance crews in order to warranty the required reliability level at the minimum maintenance cost. The problem is formulated as a mathematical programming model that becomes very hard to solve for large practical systems. For such reason, a new effective approach based on a constrained genetic algorithm is herein proposed and tested with reference to a naval diesel propulsion unit.

Keywords: Maintenance scheduling optimization; series–parallel systems; constrained genetic algorithm; mathematical programming model.

1.Introduction

Modern production systems are ever more complex and capital intensive than in the past, thus they must be daily operative for more than a shift per day. This result can be obtained by a proper plant design and realizing a proper maintenance policy in order to maintain over the time the system performance. The most used measure in evaluating the system performance is its stationary availability, defined as the probability that the system is working at a generic time t or also as the expected percentage of time in which the system is working. For some continuous operating systems (chemical processing facilities, power plants, aircrafts, ships, etc.) the failure

event can be dangerous, too expensive or even disastrous. For these reasons, a high level of reliability is required. The reliability is another meaningful parameter to assess the system performance and it expresses the probability that the system operates without failure for a fixed period of time under stated conditions. The evolution of the system reliability depends on the system structure as well as on the reliability function of its own elements. System reliability is strongly affected by maintenance activities and, in order to reach the related required value by the minimum maintenance cost, proper maintenance policies must be applied.

Maintenance policies are usually classified into two fundamental categories: corrective and preventive maintenance. The latter consists of actions that improve the condition of the system elements before they fail. The growing importance of preventive maintenance has been leading an increase in studies concerning with the optimal maintenance models. Basically, they can be classified into two fundamental classes: single-unit and multi-unit systems models (Certa et al 2013). This paper deals with a multi-unit system and therefore a short review of the literature on multi-component preventive maintenance optimization models is hereafter reported.

Cho and Parlar(1991) gave the following definition of multi-unit maintenance models: “Multi-component maintenance models are concerned with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other (economically/stochastically/structurally). The Authors presented a wide overview about the multi-unit system maintenance models developed up to 1991 while Dekker et Nicolai (1997) gave an overview of the literature on multi-component maintenance optimization models published after 1991. Wang (2002) made a survey on maintenance policies of deterioration systems that summarizes, classifies and compares various existing maintenance policies for both single-unit and multi-unit systems. A comprehensive literature study on maintenance policies of deterioration systems are reported in Lupo (2014a).

One of the most critical problems in preventive maintenance optimization is to set the optimal maintenance frequency and/or to establish the optimal grouping of elements on which perform the maintenance actions.

Tam et al. (2006), proposed simple models to assist managers of small to medium production plants to determine the optimal maintenance intervals with different managerial requirements, namely minimum reliability requirement, maintenance budget and minimum total cost. For systems constituted by many components, the grouping problem is a complicated combinatorial optimization problem. For this reason heuristic search algorithms are needed in order to find a good solution. Among them, genetic algorithms (GAs) have been proven to be effective optimization tools (Lupo, 2014a).

Muñoz et al. (1997) were probably the first ones that proposed the use of the GA as an optimization tool for maintenance scheduling activities. Marseguerra and Zio (2000) proposed an approach to the optimal maintenance and repair strategies of an industrial plant based on the coupling of GAs and Monte Carlo techniques. The Authors showed that the search procedure by GAs allows to efficiently perform the analysis of a realistic system in reasonable computing time. Lapa et al.

(2006) developed a model to find preventive maintenance policies which provide a high level of reliability with low cost and they proposed the GA as tool to search the optimum maintenance policy. Levitin and Lisnianski(2000) generalized a preventive maintenance optimization problem to a multi-state systems with a range of performance levels. The possible preventive maintenance actions are characterized by their ability to affecton effective age of component. A universal generating function technique and a GAis used to solve the problem of the respect of the system performance by the minimum maintenance cost. Recently, also suitable statistical tools based on for attributes (Inghilleri, 2015; Lupo, 2015) or for variables (Lupo, 2014b) control charts are considered to support the choice of maintenance actions to be performed on a multi-component system.

2. Problem formulation

The system herein considered consists of components, some of which constituted by parallel-series sub-systems. It is supposed that the maintenance actions, also on the elements in parallel, must be carried out within the planned system downtime. As well known, for some continuous operating systems the stop for failure implies high production loss and maintenance costs as well as risks for the workers. Furthermore, the downtime is required as short as possible and in any case smaller than the planned interval. In fact, out of this interval, the cost of system unavailability and the cost of the maintenance crews increase considerably. For these reasons, the failure probability must be lower that an accepted value, that is the reliability of the system between the restart and the next planned stop must be higher than a fixed value.

The maintenance actions on the elements are carried out by maintenance crews that can not overcome a fixed number. The maximum number of crews arises from the maximum resources availability or from a constraint on the number of resources that can operate simultaneously on the system to be maintained. Crews must be reserved for the planned maintenance period, within which the maintenance actions should be performed, unless exceptional cases. Therefore the unit cost of such resources is time dependent and in particular it presents three values: the first occurs for the period in which the resource is actually employed, the second, lower than the first, for the time within the planned interval in which the resource eventually is not employed and the third, higher than the first, for the employment over the planned interval if it occurs. A maintenance action on a specific element implies a cost of spare parts and it makes the element as good as new. The system downtime, which implies an unavailability cost, is calculated as the ratio between the sum of time to perform all maintenance actions and the number of all employed crews.

The tackled problem concerns the determination of both the set of elements on which to perform preventive maintenance and the optimal number of maintenance crews warranting the required reliability level at the minimum maintenance cost. The problem can be formulated as:

$$\min C \tag{1}$$

with:

$$C = \sum_{i \in I} C_i + (1-y) \cdot [(c_s + c_c \cdot n) \cdot \frac{\sum_{i \in I} t_i}{n} + c'_c \cdot n \cdot (t_p - \frac{\sum_{i \in I} t_i}{n})] + y \cdot [(c_s + c_c \cdot n) \cdot t_p + (c'_s + c''_c \cdot n) (\frac{\sum_{i \in I} t_i}{n} - t_p)] \quad (2)$$

Subject to:

$$n \leq n_{max} \quad (3)$$

$$R_s \geq R_s^* \quad (4)$$

where:

C_i = cost of spare parts for the element i ;

n = integer variable representing the number of the maintenance crews;

I = set of components on which to perform the maintenance actions;

t_i = time to perform the maintenance actions on the component i ;

t_p = planned maintenance interval;

c_s = cost for unit time for system unavailability within the planned interval;

c'_s = cost for unit time for system unavailability out of the planned interval;

c_c = crew cost for unit time for the period in which the resource is actually employed within the planned interval;

c_c = crew cost for unit time for the time within the planned interval in which the resource is not employed;

c''_c = cost for unit time of a crew out of the planned interval;

y = binary variable that takes value 1 if the time to perform maintenance actions is greater than t_p and 0 otherwise;

n_{max} = maximum number of crews that may work on the system simultaneously;

R_s^* = minimum accepted value of reliability.

As mentioned before, the system considered consists of several components in series, some of them constituted by parallel-series sub-systems. System reliability can be expressed in terms of reliability of the K components, by:

$$R_s = \prod_{j=1}^K R_j \quad (5)$$

The reliability of each sub-system is calculated on the base of the reliability R_i of each element and on the base of the reliability relations among them. Given the reliability functions of each element and called T_i the starting time of the maintenance planned interval, it is possible to calculate the reliability of the elements at time T_{i+1} , known the instant of the last action on the element and the information about the state

of the element (operating or not) at time T_i . Therefore it is possible to calculate the system reliability at time T_{i+1} . If, as it is to be expected, the value is lower than the level required, the elements on which to perform the maintenance actions must be determined. The actions will imply an increase of each element reliability and thus of the system reliability.

3 Mathematical programming model

The tackled problem can be formulated as a mathematical programming model. Eq. (2), to be minimized, becomes:

$$C = \sum_{i=1}^N C_i \cdot x_i + (1 - y) \cdot [(c_s + c_c \cdot n) \cdot \frac{\sum_{i=1}^N t_i \cdot x_i}{n} + c'_c \cdot n \cdot (t_p - \frac{\sum_{i=1}^N t_i \cdot x_i}{n})] + y \cdot [(c_s + c_c \cdot n) \cdot t_p + (c'_s + c''_c \cdot n) (\frac{\sum_{i=1}^N t_i \cdot x_i}{n} - t_p)] \tag{6}$$

subject to constraints expressed by Eqs. (3) and (4).

In Eq. 6, the variables x_i are Boolean variables equal to 1 if the component i belongs to the maintenance group I and 0 otherwise. Furthermore, the Boolean variable y is subject to the two following constraints:

$$\left(\frac{\sum_{i=1}^N t_i \cdot x_i}{n} - t_p \right) \leq \left(\frac{\sum_{i=1}^N t_i}{n} - t_p \right) \cdot y \quad \left(\frac{\sum_{i=1}^N t_i \cdot x_i}{n} - t_p \right) \geq -t_p \cdot (1 - y) \tag{7}$$

In order to simplify the model, the constraint (4) can be linearly expressed by:

$$\ln R_s = \sum_{j=1}^K \ln R_j \geq \ln R_s^* \tag{8}$$

The reliability R_j of the component j at time T_{i+1} depends on the maintenance actions on the elements belonging to it. The logarithmic increase of each element reliability due to a maintenance action does not imply the same increase of the parallel-series components reliability. In order to maintain the linearity of the constraint, the following procedure, described by a simple example, has been implemented. Suppose to have a component constituted by two elements A and B in parallel. The maintenance actions on this component may involve the element A only, the element B only or both. This last condition is expressed by introducing a further Boolean variable x_{AB} . The logarithm of the component reliability turns into:

$$\ln R_j = \ln R'_j + x_A \cdot \Delta \ln R_{j,A} + x_B \cdot \Delta \ln R_{j,B} + x_{AB} \cdot \Delta \ln R_{j,AB} \quad (9)$$

where R'_j is the component reliability with no actions and $\Delta \ln R_{(j,i)}$ is the reliability increase of the component j due to the action on i ($i = A, B, AB$). To express that only one of the three conditions can be realized, the following constraint is introduced:

$$x_A + x_B + x_{AB} \leq 1 \quad (10)$$

It is clear that, by increasing the number of elements it implies a greater number of variables to treat and therefore the problem becomes rapidly intractable (Certa et al., 2012a). In order to overcome the limits of the mathematical programming approach, the GA described in the following section is proposed.

4 Genetic algorithm and optimal number of crews calculation

A chromosome binary-coding is used to encode a generic solution. According to this encoding methodology, the chromosome size is N , that is the number of the system elements, and the generic site i is equal to 1 if the maintenance action on element i is performed, 0 otherwise.

The choice of parents is randomly determined by using the roulette wheel method and the employed crossover operator is a classical two point crossover. The mutation operator simply exchanges two randomly selected genes of the chromosome. The fitness function is the inverse of the cost function expressed by Eq. (2).

Since the considered model is constrained, during the evolutionary process it is possible to produce infeasible solutions. There are three possible approaches to the management of infeasible solutions: (a) repair the infeasible solutions to force it to be feasible through some repair procedure; (b) discard infeasible solutions and continue the evolutionary process until a feasible solution is produced; and (c) penalize infeasible solutions by using a specific penalty function. It was shown that the GA optimization can be more effective when action (a) or (c) are carried out (Certa et al., 2012b).

In this work the first approach is utilized, by adding to the evolution operators a deterministic repair procedure. The repair operator changes some genes 0 into 1 until the reliability constraint is respected. The choice of the repair sites is randomly determined assigning to each gene 0 a probability to be selected, proportional to the ratio between the system reliability increment, due to the action on the gene, and the related cost. The values of the GA parameters are: crossover and mutation probabilities 1 and 0.15 respectively; population dimension 50; number of generations 60.

For sake of simplicity, the variable n , i.e. the number of the maintenance crews, is not considered in the chromosome encoding and therefore, in order to calculate the fitness value, the number of crews must be determined. Known the elements on which to perform the maintenance actions and calculated the related

maintenance time, the optimal number of crews can be calculated by the following procedure.

If, employing the maximum number of crews, the real maintenance time overcomes the planned interval, n_{max} represents the optimal value, since the cost function (Eq. 2) decreases by increasing the value of n . Otherwise, the number of crews that minimizes the cost can be calculated by fixing the derivative of the cost function (Eq. 2) equal to 0, with $y = 0$:

$$\frac{dC}{dn} = -c_s \cdot \frac{\sum_{i \in I} t_i}{n^2} + c'_c \cdot t_p = 0 \tag{11}$$

and thus:

$$n = \sqrt{\frac{c_s \cdot \sum_{i \in I} t_i}{c'_c \cdot t_p}} \tag{12}$$

Because n must be an integer value, the cost function must be calculated for $\text{int}^+[n]$ and $\text{int}[n]$ and the value that implies the minimum cost will be chosen. Moreover, the following equation must be satisfied, as well:

$$n = \text{int}^+ \left[\frac{\sum_{i \in I} t_i}{t_p} \right] \tag{13}$$

Therefore, the optimal value of n is the maximum between the two values previously calculated if it is lower than n_{max} , otherwise it is n_{max} .

5. Case study

The GA previously developed is applied and tested with referenced to the optimal maintenance scheduling in a naval diesel propulsion unit. The latter represents a critical system for the ship availability, thus a very high reliability level is required.

The considered diesel propulsion unit is a medium-size system, thus both the mathematical programming approach and the GA previously developed can be adopted to optimize its maintenance scheduling. In such a way, the effectiveness of the proposed GA approach can be tested by comparing the obtained results. The considered diesel propulsion unit is composed by 34 components, some of them constituted by l branches in parallel with m elements in series for each branch. Such a component is indicated in Table 1 as (l,m) and the corresponding elements are listed reading the branches from the top to the bottom and from the left to the right. The table also shows the data that characterize each element. In particular, R_i is the element reliability without maintenance action and ΔR_i is the increment of reliability

due to maintenance action. Assumed c_c as cost unit, all other cost parameters are referred to it. The system reliability at time T_{i+1} , without maintenance actions at time T_i , results equal to 0,9109.

Table 1: Input data

Comp.	Elem.	C_i	t_i	R_i	ΔR_i	Comp.	Elem	C_i	t_i	R_i	ΔR_i
1(2,1)	1	50	11	0.98504	0.01495	11(2,1)	26	20	5	0.98684	0.01209
	2	60	7	0.98679	0.01320		27	20	5	0.98684	0.01209
2(2,1)	3	50	11	0.98544	0.01455	12	28	50	12	0.99703	0.00296
	4	60	7	0.98601	0.01239	13	29	30	6,67	0.99768	0.00231
3(2,1)	5	50	11	0.84745	0.15254	14	30	55	8,33	0.99785	0.00214
	6	60	7	0.94715	0.05125	15	31	41	8,33	0.99782	0.00216
4(2,1)	7	50	11	0.98543	0.01456	16	32	41	13,33	0.99784	0.00215
	8	60	7	0.98679	0.01161	17	33	100	10	0.99706	0.00293
5(2,2)	9	40	8	0.91724	0.06957	18	34	41	13	0.99797	0.00202
	10	50	9	0.91943	0.06941	19	35	30	8	0.99789	0.00210
	11	40	8	0.88733	0.09948	20	56	30	9	0.99768	0.00231
	12	50	9	0.90827	0.08057	21	37	40	7	0.99778	0.00221
6(2,2)	13	13	12	0.98677	0.01321	22	38	30	10	0.99781	0.00218
	14	97	10	0.98405	0.01276	23	39	40	12	0.99767	0.00232
	15	13	12	0.98679	0.01320	24	40	20	8,33	0.99782	0.00217
	16	97	10	0.98407	0.01274	25	41	50	10	0.99778	0.00221
7(2,1)	17	20	8	0.99541	0.00439	26	42	30	12	0.99785	0.00215
	18	20	8	0.99541	0.00439	27	43	20	13	0.99778	0.00221
8(2,1)	19	50	11	0.98104	0.01242	28	44	10	14	0.99784	0.00214
	20	50	11	0.98107	0.01239	29	45	15	10	0.99785	0.00205
9(2,1)	21	30	5	0.99596	0.00368	30	46	30	5	0.99767	0.00232
	22	30	5	0.98660	0.00980	31	47	35	6	0.99784	0.00214
10(3,1)	23	10	10	0.98662	0.00979	32	48	40	7	0.99767	0.00232
	24	10	10	0.98660	0.00980	33	49	50	8	0.99767	0.00232
	25	10	10	0.98659	0.00981	34	50	44	13	0.99789	0.00210

In order to test the effectiveness of the GA, two values are considered for each of the principal parameters that characterize the problem (t_p , R_S^* , n_{max} and c_s) and a full experimental design has been carried out, as shown in Table 3. In the same Table, the results obtained by the GA are compared with those obtained by mathematical programming approach. The latter has been performed by means of the commercial software LINGO 10. The other parameters that characterize the problem are reported in table 2.

Table 2: Other parameters

c_s	c_c	c_c''
$2 \cdot c_s$	0.7	1.5

Table 3: Solutions obtained

t_p	R_S^*	n_{max}	c_s	Genetic algorithm		Mathematical programming	$\Delta C\%$	
				cost	n	elements		
30	0.97	5	10	936.2	5	5,11,12,29,35,38,40,42,48	5,9,10,29,35,38,40,42,48	0.00%
			50	1976.90	5	6,9,11,29,31,35,38,40,45,49	6,11,12,28,29,31,33,35,37,40,45,48	0.57%
		10	891.20	8	5,9,10,29,35,38,40,42,48	5,11,12,29,35,38,40,42,48	0.00%	
	0.99	5	10	1454.30	10	6,9,11,29,35,38,40,44,49	5,11,12,28,29,36,40,45,49	0.73%
			50	1858.94	5	6,9,12,21,28,32,35,49	5,9,12,28,32,35,49	1.95%
		10	4262.69	5	6,9,12,28,31,33,35,49	6,9,12,28,31,33,35,49	0.00%	
100	0.97	5	10	1182.00	4	5,9,11,29,35,38,40,41,48	6,9,11,12,29,35,38,40,43,48	0.10%
			50	2221.90	5	6,9,11,29,31,35,38,40,45,49	6,11,12,28,29,31,33,35,37,40,45,48	0.51%
		10	1181.20	4	5,11,12,35,38,40,42,48	6,9,11,12,29,35,38,40,43,48	0.03%	
	0.99	5	10	1944.30	10	6,9,11,29,35,38,40,44,49	5,11,12,28,29,36,40,45,49	0.55%
			50	1834.12	5	5,11,12,15,28,29,31,49	5,9,12,28,29,31,32,35,50	1.75%
		10	3696.90	5	6,9,12,28,38,40,42,49	6,9,12,28,31,33,35,49	1.00%	
0.00%	10	10	1792.26	6	5,9,12,28,29,31,32,34,50	5,9,12,28,29,31,32,34,50	0.00%	
		50	2928.00	10	6,9,11,12,28,32,34,49	5,9,12,28-32,35,49	0.04%	

The crews number obtained by LINGO is the same as the one obtained by the GA. The results show the effectiveness of the GA developed. Just in two cases the difference between the costs obtained by the two methods exceeds 1%. In four cases the GA finds the optimum value: while in three cases the solutions of the two approaches are identical, in one case (the first row in table 3) the GA singles out a solution having the same cost and a greater reliability. In fact the two solutions differ for the branch of the component n.5 on which the maintenance actions are performed. The cost is equal because the elements of the two branches are identical but, having a different history, they have different reliability. The GA selects the branch with lower reliability and then maximizes the component reliability.

6. Conclusions

The paper deals with a maintenance optimization problem for a multi-component system for which the goal is to respect a fixed level of reliability at minimum maintenance cost. The tackled problem is formulated as a non-linear integer programming model and solved by a commercial software. Nevertheless, the presence of parallel-series components makes hard or even impossible to solve it for large real systems. In order to overcome the limits of the mathematical programming, a constrained genetic algorithm is developed and its effectiveness is proved by the comparison of the results obtained by the two approaches with reference to a case study of medium dimension. Future development can regard the employ of a penalty function to manage the infeasible solutions of the genetic algorithm. In fact, the

penalty function approach could lead to better results in the cases more constrained in which the considered repair procedure implies several actions on the genes.

References

1. Certa, A., Enea, M., Galante, G., Lupo, T., (2012a). A multi-objective approach to optimize a periodic maintenance policy. Proceedings - 18th ISSAT International Conference on Reliability and Quality in Design; Boston, MA; United States, pp. 379-383. Code 100093.
2. Certa, A., Enea, M., Galante, G., Lupo, T., (2012b). A multi-objective approach to optimize a periodic maintenance policy. *International Journal of Reliability, Quality and Safety Engineering*. 19(6). Article number 1240002.
3. Certa A., Enea, M., Galante G., Lupo T., (2013). A multi-decision makers approach to select the maintenance plan for a multi-component system, 19th ISSAT International Conference on Reliability and Quality in Design, RQD 2013; Honolulu, HI; United States; Code 100087. pp. 434-438.
4. Cho,D. and Parlar, M., (1991). A survey of maintenance models for multi-unit systems. *European Journal of Operational Research*,51,pp. 1-2.
5. Dekker, R., Schouten, F. A., and Wildeman R., (1997). A review of multi-component maintenance models with economic dependence”, *Mathematical Methods of Operations Research*, 45(3), pp. 411–435.
6. Inghilleri, R., Lupo, T. and Passannanti, G., (2015). An effective double sampling scheme for the c control chart. *Quality and Reliability Engineering International*. 31(2): pp. 205-216. DOI: 10.1002/qre.1572.
7. Lapa, C. M. F., Pereira, C. M. N.A. and Paes de Barros M., (2006). A model for preventive maintenance planning by genetic algorithms based in cost and reliability, *Reliability Engineering and system safety*, Vol.91, pp. 233-240.
8. Levitin, G., Lisnianski A., (2000). Optimization of imperfect preventive maintenance for multi-state systems. *Reliability Engineering and System Safety*, Vol. 67, pp.193-203.
9. Lupo, T., (2013). The optimization of a maintenance policy related to a global service contract. Proceedings - 19th ISSAT International Conference on Reliability and Quality in Design, RQD 2013; Honolulu, HI; United States; pp. 260-264; Code 100087.
10. Lupo, T., (2014a). A constrained genetic algorithm to optimize a maintenance global service. *International Journal of Applied Engineering Research*. 9(23), pp. 21459-21471.
11. Lupo, T., (2014b). Economic-Statistical Design Approach For a VSSI X-BAR Chart Considering Taguchi Loss Function and Random Process Shifts. *International Journal of Reliability, Quality and Safety Engineering*. Vol. 21, No. 2 (2014) 1450006. DOI: 10.1142/S0218539314500065.
12. Lupo, T., (2015). Economic Robustness Analysis of Adaptive Chart Schemes for Monitoring the Total Nonconformities Number in Inspection Units.

- International Journal of Reliability, *Quality and Safety Engineering*. Vol. 22(01) (2015) 1550002. DOI: 10.1142/S0218539315500023.
13. Marseguerra, M. and Zio E., (2000). Optimizing maintenance and repair policies via a combination of genetic algorithms and Monte Carlo simulation, *Reliability Engineering and system safety*, 68, pp.68-83.
 14. Muñoz, A. Martorell, S. and Serradell, V., (1997). Genetic Algorithms in Optimizing Surveillance and Maintenance of Components”, *Reliability Engineering and system safety*,57(2), pp.107–20.
 15. Tam, A. S. B., Chan, W.M. and Price, J.W. H., (2006). Optimal Maintenance Interval for a multi-component system. *Production Planning & Control*, 17(8), pp.769-779.
 16. Wang H. Z. (2002), A survey of maintenance policies for deterioration systems, *European Journal of Operational Research*, Vol.139, pp.469-489.

