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Design of FFT Based Hilbert Transform For Phase Detection

S. Suvethasri
School of Computing, SASTRA University, Thanjavur-613401, India
email: suvethasri6@gmail.com
R. Parameshwaran
School of Computing, SASTRA University, Thanjavur-613401, India
email: paramu32@gmail.com

Abstract

The Phase locked Loop (ADPLL) is a feedback system that dynamically reduces the frequency/Phase offset between a standard input signal and a feedback signal to certain minimum value. The ADPLL structure is a system that is implemented in pure digital circuitry. The main building of ADPLL is the phase detector that reduces the frequency/phase offset value. This project focuses on the phase detection part of ADPLL. The phase detection is realized using the discrete Hilbert transform followed by the cordic algorithm, which works only on the presence of the analytic signals. The hilbert transform produces 90 degree phase shift of the original signal irrespective of its frequency. The FFT based hilbert transform is designed for accurate phase detection. This approach improves the speed of the system by reducing the number of computations.

Keywords: Hilbert transform, Fast Fourier transform, phase detector, Decimation in time, decimation in frequency

Introduction

Phase locked loop: A Review

A control system structure that gives the phase difference between the original input clock signal and the feedback signal is the phase locked loop. The main components of the phase locked loop structure are the phase detector, low pass filter and the VCO as in Figure 1.

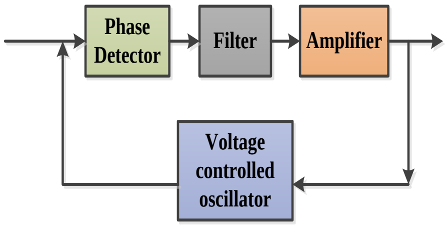

Figure 1: Phase Locked Loop

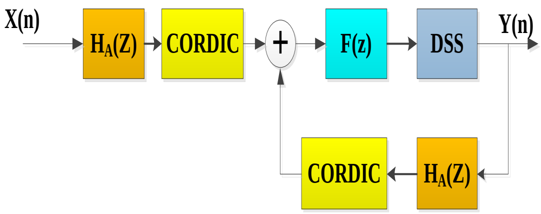

Figure 2: ADPLL

The phase detector just compares the feedback signal and the input reference signal and produces the corresponding error signal. The phase detector produces this error signal depending on their phase difference. This output signal is used to control the functionality of the VCO, so that a constant frequency signal is being generated. The high frequency components that generally affect the performance of the PLL are removed using low pass filters [9].

Phase Detector

The phase detection part of this PLL structure can be implemented using various methodologies. The most common implementations aremultiplier , J-K flipflop, Exclusive or(EXOR)gate, Digital Phase-Frequency detector. The simplest method is the use of multiplier as a phase detector, where the sum and the difference of the input signals are being calculated and the difference value is taken for phase detection. At low operating frequency this method tends to provide a good performance. The next method is the sequential circuit based phase detection. The basic structure is this phase detection is based on the flipflop. The next method is based on the exclusive OR gate phase detector [6]. In the digital Phase-Frequency detector part both the phase and the frequency difference of the given input signal is noted to obtain the output.

Proposed Method of Phase Detection

The phase detection part of the phase locked loop can also be implemented using hilbert transform which is found to be more efficient than the other methods in terms of speed. The phase detection using hilbert transform is carried out as shown in Figure 2.The hilbert transform along with the cordic algorithm forms the phase detection part of the proposed system. The reference input signal and the feedback signals should be converted into analytic form for giving it as the input to the cordic block.

Hilbert Transform

Hilbert transform finds its application in many fields like signal processing, modulation etc., this hilbert transform can be used for obtaining the analytic signals for finding out the phase value of the desired input signal [2].

One of the most commonly used methodology is a filter based hilbert transform [3]. The filter based structure has their own advantages and disadvantages. The main drawbacks of filter based design are the implementation. In hilbert transform obtained through FIR system, the generated targeted signal is obtained by delaying the real part by N/2 clock cycles and the imaginary part is obtained through the filter response [10]. This results in attenuation of the imaginary part of the signal which is a band limited and the real part will remain as same as the attenuated signal without any attenuation. The implementation of FIR based hilbert transform is complex when compared to the IIR based structure [4]. They can be designed but they are sensitive to truncation of filter coefficient. Due to this nature they become unstable. The FIR based structure remains stable as they do not include any feedback structure and they are less sensitive to truncation of the filter coefficients. Since the FIR based structure is complex to implement an alternative approach is chosen, where hilbert transform is being implemented through FFT structure.

Analytic Signal

Hilbert transform is a system that gives the relationship between the real and imaginary part of a signal as: the imaginary part of a signal is obtained from the real input signal using hilbert transform, as in Figure 3.

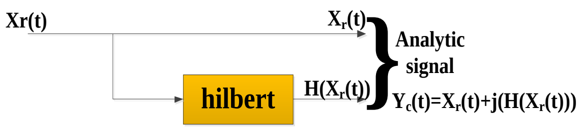

Figure 3: Analytic signal

This analytic signal can be represented by

$$Y_c\ t\ = X_r\ t\ + jH\ \ X_r\ t \tag{1}$$

The analytic signals are those signals that don't have any negative frequency components. The real signals are those that have both real and imaginary part. The real part of the real signal has both negative and positive frequency elements where they are mirror images of each other. Its imaginary part has conjugate symmetry with each other[7]. The complex signals are those that are not restricted to conjugate symmetry. An analytic signal is special type of signal that doesn't have negative frequency components that are in conjugate symmetry to the real part. As the do not have the negative frequency components their bandwidth utilization becomes more, which produces optimized results. The complex signals are not used for instaneous phase detection because when the input signal is bounded then this will produce infinite instaneous phase values, so analytic signals can be used for phase detection. The property of this analytic signal can be proved as follows:

The positive and negative frequency component is given by

$$x_{pi}\ \ t\ \ = e^{iwt} \quad (2)$$

$$x_{ni}\ \ t\ \ = e^{-iwt} \quad (3)$$

By performing shifting operation as per the hilbert transform on (2) and (3) following new wave forms are obtained

$$y_{hpi}\ \ t\ \ = e^{iwt} \cdot e^{-i\pi/2} \quad (4)$$

$$y_{hni}(\mathrm{t}) = e^{-iwt} \cdot e^{i\pi/2} \quad (5)$$

The analytic signal obtained will be given by

$$z_{po}\ \ t\ \ = x_{pi}\ \ t\ \ + iy_{hpi}\ \ t \quad (6)$$

$$z_{no}\ \ t\ \ = x_{ni}\ \ t\ \ + iy_{hni}\ \ t \quad (7)$$

On substituting the deriving the values in (6) and (7)

$$z_{po}\ \ t\ \ = e^{iwt} + i(-ie^{iwt}) \quad (8)$$

$$z_{no}\ \ t\ \ = e^{-iwt} + i(\mathrm{i}\,e^{-iwt}) \quad (9)$$

So the negative frequency component will be removed leaving behind only the positive frequency components.

The phase of the signal is obtained by the formula

$$\theta\ \ t\ \ = \arctan \frac{f\ \ t}{\hat{f}\ \ t} \quad (10)$$

Using these basic functionalities the phase detection part in the PLL structure can be designed.

Hilbert Transform Implementation

The hilbert transform can be obtained through the steps shown in Figure 4. The input signal sequence is converted to spectral domain through the discrete Fourier transform process, using DIF algorithm. The input signal sequence is given without bit reversing in DIF method. Then the obtained sequence is multiplied with ±j. The negative frequency component is multiplied with +j and the positive frequency component is multiplied with –j so that input sequence is shifted in such a way that the sequence doesn’t have any negative frequency components.

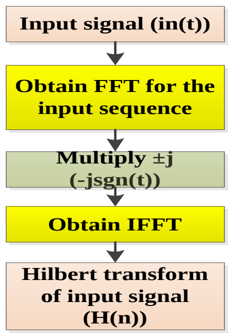

Figure 4: Design activity flow

Now the shifted version of the input sequence is again converted to original domain through inverse FFT process.

Hilbert Transform From Fourier Transform

The concept of hilbert transform can be obtained from the Fourier transform .The hilbert transform is a process of obtaining the analytic signals, which doesn't have any negative frequency components. So the input signal that is given time domain format is converted to frequency domain where negative frequency components are eliminated by shifting and finally converting it into another time domain signal. On the whole the hilbert transform is a special type of transform that converts an input signal into another signal (both in time domain) in an optimized manner without any lose in the information of the signal. The relationship between the Fourier transform and the hilbert transform is by the following statement.

In the time domain format the hilbert transform is represented as a convolution between the kernel convolution and the input signal. The kernel convolution output is given by the format $\frac{1}{\pi t}$.So the hilbert transform is represented by the format

$$H(n) = \frac{1}{\pi t} * \text{in}(t) \tag{12}$$

Where in(t) is represented as the input signal. H(n) represents the hilbert transform output.

As known the convolution in one domain will be converted as multiplication in another domain. Taking Fourier transform of the above signal the following results are obtained.

$$F(H(n)) = F\left(\frac{1}{\pi t}\right).F(in(t)) \tag{13}$$

Where $F\left(\frac{1}{\pi t}\right) = -j\,\text{sgn}(t)$ (14)

So the above equation now gets converted to the format

$$F \;\; H \;\; n \;\; = -j\,\mathrm{sgn}\;\; t \;\; .F \;\; in \;\; t \tag{15}$$

Taking inverse Fourier transform for the above equation

$$H \;\; n \;\; = IFT \;\; -j\,\mathrm{sgn}\;\; t \;\; .F \;\; in \;\; t \tag{16}$$

Fourier Transform

The most important fundamental operations that is generally been used in digital signal processing are the linear filtering and the Fourier transform. On the other hand there computational requirements makes a heavy burden in most of the applications. The order of complexity for them is N^2, N is the transform size. So in mind with the reduction of the complexity Cooley-Tukey algorithm is considered, which has a complexity of $N \log N$ iterations. Fast Fourier Transform has found to be widely used in all digital signal processing systems. In the recent year further improvement of the algorithm has resulted in further reduction in the arithmetic complexity of the algorithm [7]. The complexity can be further reduced by designing pipelined FFT structures [5]. In this paper radix-2 butterfly structure is used for the formation of the analytic signals. These algorithms mainly work on the divide and conquer technique, in which it involves decimation in time and decimation in frequency concepts.

Radix2 and radix 4 algorithms

Since most of the signal processing systems mostly involve the signals length mostly in the size of power 2, these algorithms are very popular. In this process first step is to consider the input sequence that is in powers of 2. Then divide that input sequence into odd and even numbered samples, thus forming two set of sequences.The basic structure of decimation in time is given in Figure 5.

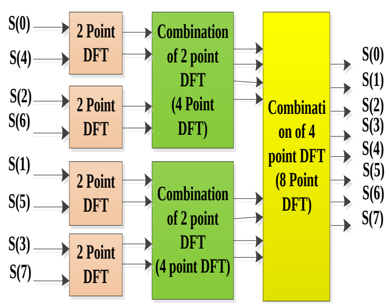

Figure 5: 8 point DIT FFT

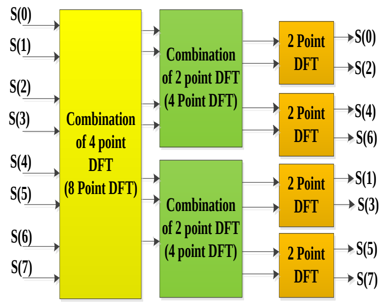

Figure 6: 8 point DIF FFT

The other representation is the decimation in frequency which is represented as in Figure 6.The butterfly structure of the above implementation for decimation in time is given as in Figure 7.

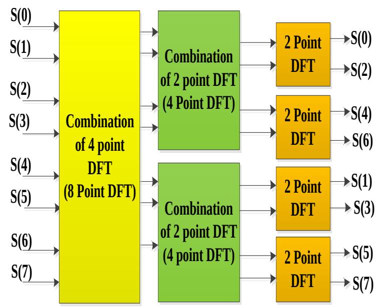

Figure 6: 8 point DIF FFT

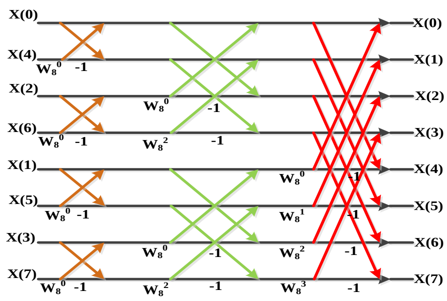

Figure 7: 8 point DIT FFT

This butterfly structure will be the basic module in the FFT based Hilbert transform structure.

Results and Discussion

The synthesis report and layout using Synopsys tool for the proposed system has been tabulated below

Table 1: Synthesis Report

Area	Power	Timing (minimum period)	Frequency (maximum frequency)
Total area: 367441.498331μm^2	Total Dynamic Power: 3.9793 mW Cell Leakage Power: 1.2357 Mw	21.772ns	45.930MHz

From the obtained value optimized results are generated compared to the earlier systems

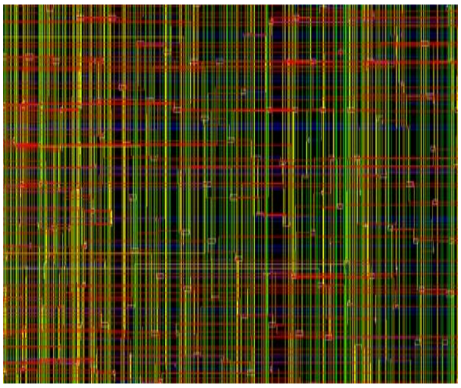

Figure 9: Layout

Conclusion

In this paper various hilbert transform methodology has been studied and an efficient method for obtaining the analytic signal has being discussed.

Further this structure can be pipelined for obtaining more efficiency. This method can also be effectively used for envelope detection, demodulation and many more applications

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