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Semiprimary Ideals In Duo Ternary Semigroups

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Abstract

In this paper, the termssemiprimary ideal, semiprimary ternary semigroup and natural ordering in ternary semigroups are introduced.It is proved that in a ternary semigroup T,1) every left primary ideal is a semiprimary ideal, 2) every lateral primary ideal is a semiprimary ideal, 3) every right primary ideal is a semiprimary ideal. It is proved that if A is a semiprime ideal of a ternary semigroup T, then the following are equivalent 1) A is a prime ideal, 2) A is a primary ideal, 3) A is a left primary ideal, 4) A is a lateral primary ideal, 5) A is a right primary ideal and 6) A is a semiprimary ideal. Finally it is proved that a ternary semi group T is semiprimary if and only if prime ideals of T form a chain under set inclusion.If T is a semiprimary duo ternary semigroup, then globally idempotent principal ideals form a chain under set inclusion. It is proved that, if E is the set of all idempotent elements of ternary semigroup T. Then the natural ordering is a partial ordered relation, In a semiprimaryduo ternary semigroupt, the idempotents in T forms a chain under natural ordering. Further it is proved that, the principal ideals of ternary semigroup T form a chian iff ideals in T form a chain . It is proved that, If T is a regular duo ternary semigroup with identity , then i) every principal ideal of T generated by an idempotent, ii) principal ideals of T form a chain if and only if idempotents of T form a chain under natural ordering. Furter it is proved that, in a semisimple ternary semigroupthe following are equivalent1. Every ideal in T is a prime ideal.2. T is a primary ternary semigroup. 3. T is a left primary ternary semigroup.4. T is a lateral primary ternary semigroup. 5. T is a right primary ternary semigroup.6. T is a semiprimary ternary semigroup. 7. Prime ideals of T form a chain.
Further it is proved that, In a semisimple duo ternary semigroup T with identity, then the following are equivalent.1. Every ideal in T is a prime ideal.2. T is a primary ternary semigroup.3. T is a left primary ternary semigroup.4. T is a lateral primary ternary semigroup.5. T is a right primary ternary semigroup.6. T is a semiprimary ternary semigroup.7. Prime ideals of T form a chain8. Ideals of T form a chain.9. Principle ideals of T form a chain.10. Idempotents of T forms a chain under natural ordering. Further it is

proved that, Every ideal of a ternary semigroupT is prime if and only if T is semisimple primary ternary semigroup. It is also proved that, in a ternary semigroup T, the following are equivalent.1. Every ideal in T is a prime ideal. 2. T is a semisimple and ideals of T form a chain.3. T is a semisimple and prime ideals of T form a chain. Finally it is proved that, in a semisimpleduo ternary semigroup T, the following are equivalent.1. Every ideal in T is a prime ideal.2. T is a regular primary ternary semigroup.3. T is a regular semiprimary ternary semigroup.4. T is a regular ternary semigroup and idempotents of T form a chain under natural ordering.

Key Words: Semiprimary ideal, semiprimary ternary semigroup, natural ordering in duo ternary semigroups.

Introduction

The ideal theory in commutative semigroups was developed by SATYANARAYANA [7], [8]. The ideal theory in general semigroups was developed by ANJANEYULU [1], [2], [3], GIRI and WAZALWAR [4]. SANTIAGO [9] developed the theory of ternary semigroups. In this paper we introduce the notion of semiprimary ideals in ternary semigroups and charecterise semiprimary ideals in duo ternary semigroups.

Priliminaries

Definition 2.1

Let T be a non-empty set. Then T is said to be a ***ternary semigroup*** if there exist a mapping from T×T×T to T which maps $(x_1, x_2, x_3) \to x_1x_2x_3$ satisfying the condition : $\left[x_1x_2x_3 \ \ x_4x_5 \right] = \left[x_1 \ \ x_2x_3x_4 \ \ x_5 \right] = \left[x_1x_2 \ \ x_3x_4x_5 \right] \ \forall \ x_i \in$ T, $1 \le i \le 5$.

Note 2.2

For the convenience we write $x_1x_2x_3$ instead of $x_1x_2x_3$

Note 2.3

Let T be a ternary semigroup. If A,B and C are three subsets of T , we shall denote the set ABC = $abc : a \in A, b \in B, c \in C$.

Definition 2.4

A ternary semigroup T is said to be ***commutative*** provided *abc* = *bca* = *cab* = *bac* = *cba* = *acb*for all *a*,*b*,*c*$\in$ T.

Definition 2.5

A ternary semigroup T is said to be ***quasi commutative*** provided for each *a*, *b*, *c*∈ T, there exists a natural number *n* such that $abc = b^n ac = bca = c^n ba = cab = a^n cb$.

Definition 2.6
A nonempty subset A of a ternary semigroup T is said to be ***left ternary ideal*** or ***left ideal*** of T if $b, c \in$ T, $a \in$ A implies $bca \in$ A.

Note 2.7
A nonempty subset A of a ternary semigroup T is a left ideal of T if and only if TTA $\subseteq$ A.

Definition 2.8
A nonempty subset of a ternary semigroup T is said to be a ***lateral ternary ideal***or simply ***lateral ideal*** of T if $b, c \in$ T , $a \in$ A implies $bac \in$ A.

Note 2.9
A nonempty subset of A of a ternary semigroup T is a lateral ideal of T if and only if TAT $\subseteq$ A.

Definition 2.10
A nonempty subset A of a ternary semigroup T is a ***right ternary ideal***or simply ***right ideal*** of T if $b, c \in$ T , $a \in$ A implies $abc \in$ A

Note 2.11
A nonempty subset A of a ternary semigroup T is a right ideal of T if and only if ATT $\subseteq$ A.

Definition 2.12
A nonempty subset A of a ternary semigroup T is a ***two sided ternary ideal***or simply ***two sided ideal*** of T if $b, c \in$ T , $a \in$ A implies $bca \in$ A, $abc \in$ A.

Note 2.13
A nonempty subset A of a ternary semigroup T is a two sided ideal of T if and only if it is both a left ideal and a right ideal of T .

Definition 2.14
A nonempty subset A of a ternary semigroup T is said to be ***ternary ideal*** or simply an ***ideal*** of T if $b, c \in$ T , $a \in$ A implies $bca \in$ A, $bac \in$ A, $abc \in$ A.

Note 2.15
A nonempty subset A of a ternary semigroup T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T .

Definition 2.16
An ideal A of a ternary semigroup T is said to be a ***proper ideal*** of T if A is different

from T.

Definition 2.17
An ideal A of a ternary semigroup T is said to be a ***trivial ideal*** provided T\ A is singleton.

Definition 2.18
An ideal A of a ternary semigroup T is said to be a ***maximal ideal*** provided A is a proper ideal of T and is not properly contained in any proper ideal of T.

Theorem 2.19
If T is a ternary semigroup with unity 1 then the union of all proper ideals of T is the unique maximal ideal of T.

Definition 2.20
An ideal A of a ternary semigroup T is said to be ***a principal ideal*** provided A is an ideal generated by a for some $a \in$ T. It is denoted by J (a) (or) <a>.

Notation 2.21
Let T be a ternary semigroup. If T has an identity, let T^1 = T and if T does not have an identity , let T^1 be the ternary semigroup T with an identity adjoined usually denoted by the symbol 1.

Notation 2.22
Let T be a ternary semigroup. if T has a zero, let T^0 = T and if T does not have a zero, let T^0 be the ternary semigroup T with zero adjoined usually denoted by the symbol 0.

Definition2.23
An ideal A of a ternary semigroup T is said to be a ***completely prime ideal*** of T provided $x, y, z \in$ T and $xyz \in$ A implies either $x \in$ A or $y \in$ A or $z \in$ A.

Definition 2.24
An ideal A of a ternary semigroup T is said to be a ***prime ideal*** of T provided X,Y,Z are ideals of T and XYZ $\subseteq$ A $\Rightarrow$ X $\subseteq$ A or Y $\subseteq$ A or Z $\subseteq$ A.

Definition 2.25
If A is an ideal of a ternary semigroup T , then the intersection of all prime ideals of T containing A is called ***prime radical*** or simply ***radical*** of A and it is denoted by $\sqrt{A}$ or *rad* A.

Definition 2.26
If A is an ideal of a ternary semigroup T , then the intersection of all completely prime ideals of T containing A is called ***completely prime radical*** or simply ***complete***

radical of A and it is denoted by *c.rad* A.

Corollary 2.27

If $a \in \sqrt{A}$, then there exist a positive integer n such that $a^n \in$ A for some odd natural number $n \in$ N.

Corollary 2.28

If A is an ideal of a commutative ternary semigroup T, then *rad* A = *c.rad* A.

Theorem 2.29

An ideal Q of ternary semigroup T is a semiprime ideal of T if and only if $\sqrt{Q}$ =Q.

Theorem 2.30

If A, B and C are any three ideals of a ternary semigroup T , then

1. **A $\subseteq$ B** $\Rightarrow \sqrt{A} \subseteq \sqrt{B}$
2. **if A ∩ B∩ C ≠ ∅ then** $\sqrt{ABC} = \sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$
3. **iii)** $\sqrt{\sqrt{A}} = \sqrt{A}$.

Definition 2.31

A ternary semigroup T is said to be a ***left duo ternary semigroup*** if every left ideal of T is both right and lateral ideal of T.

Definition 2.32

A ternary semigroup T is said to be a ***right duo ternary semigroup*** if every right ideal of T is both left and lateral ideal of T.

Definition 2.33

A ternary semigroup T is said to be a ***duo ternary semigroup*** if it is both a left duo ternary semigroup and a right duo ternary semigroup.

Theorem 2.34

A ternary semigroup T is a duo ternary semigroup if and only if $xT^1T^1 = T^1T^1x = T^1xT^1$ for all $x \in$ T.

Definition 2.35

A ternary semigroup T is said to be ***normal*** if abT = Tab = aTb for all a, $b \in$ T.

Theorem 2.36

Let A be an ideal of a duo ternary semigroup T and a, b, $c \in$ T. Then $abc \in$ A if and only if $\langle a\rangle\langle b\rangle\langle c\rangle \subseteq$ A.

Corollary 2.37
Let A be an ideal of a duo ternary semigroup T and a_1 , a_2 , a_3 ,. . . .$a_n \in$ T. Then $a_1 a_2$ a_3 .....$a_n \in$ A if and only if $<a_1>< a_2>< a_3>........< a_n> \subseteq$ A.

Corollary 2.38
Let A be an ideal of a duo ternary semigroup T and $a \in$ T. Then for any odd natural number n,$a_n \in$ A if and only if $<a>_n \subseteq$ A.

Theorem 2.39
If T is a duo ternary semigroup and $a \in$ T, then the following are equivalent.

1. a is regular .
2. a is left regular .
3. a is right regular.
4. a is semisimple.

Definition2.40:
An ideal A of a ternary semi group T is said to be a ***left primary ideal*** if

1. X, Y, Z are three ideals of T such that XYZ $\subseteq$ A and Y $\nsubseteq$A, Z $\nsubseteq$ A, impliesX $\subseteq \sqrt{A}$.
2. $\sqrt{A}$ is a prime ideal.

Definition 2.41
An ideal A of a ternary semi group T is said to be a ***lateral primary ideal*** if

1. X, Y, Z are three ideals of T such that XYZ $\subseteq$ A and X $\nsubseteq$A, Z $\nsubseteq$A, implies Y $\subseteq \sqrt{A}$.
2. $\sqrt{A}$ is a prime ideal.

Definition 2.42
An ideal A of a ternary semi group T is said to be a ***right primary ideal*** if

1. X, Y, Z are three ideals of T such that XYZ $\subseteq$ A and X $\nsubseteq$A, Y $\nsubseteq$A, implies Z $\subseteq \sqrt{A}$.
2. $\sqrt{A}$ is a prime ideal.

Definition 2.43
An ideal A of a ternary semigroup T is said to be a ***primary ideal*** if A is left primary, lateral primary and right primary ideal of T.

Theorem 2.44
Let A be an ideal A in a ternary semigroup T. X, Y, Z are three ideals of T such that XYZ $\subseteq$ A and Y $\nsubseteq$A, Z $\nsubseteq$A, implies X $\subseteq \sqrt{A}$ if and only if $x, y, z \in$ T, $<x><y><z> \subseteq$ A and $y \notin$ A, $z \notin$ A implies $x \in \sqrt{A}$.

Theorem 2.45
Let A be an ideal A in a ternary semigroup T. X, Y, Z are three ideals of T such that XYZ$\subseteq$A and X $\nsubseteq$A, Z $\nsubseteq$A, implies Y $\subseteq \sqrt{A}$ if and only if $x, y, z \in$ T,<x><y><z>$\subseteq$A and $x \notin$ A, $z \notin$ A implies $y \in \sqrt{A}$.

Theorem 2.46
Let A be an ideal A in a ternary semigroup T. X, Y, Z are three ideals of T such that XYZ$\subseteq$A and X $\nsubseteq$A, Y $\nsubseteq$A, implies Z $\subseteq \sqrt{A}$ iff $x, y, z \in$ T, <x><y><z>$\subseteq$A and $x \notin$ A, $y \notin$ A implies $z \in \sqrt{A}$.

Definition 2.47
A ternary semigroup T is said to be a ***left primary*** provided every ideal in T is a left primary ideal.

Definition 2.48
A ternary semigroup T is said to be a ***lateral primary*** provided every ideal in T is a laterly primary ideal.

Definition 2.49
A ternary semigroup T is said to be a ***right primary*** provided every ideal in T is a right primary ideal.

Definition 2.50
A ternary semigroup T is said to be a ***primary*** provided every ideal in T is a primary ideal.

Semi Primary Ideals

Definition 3.1
An ideal A of a ternary semigroup T is said to be ***semiprimary*** if $\sqrt{A}$ is a prime ideal

Definition 3.2
A ternary semigroup T is said to be ***semiprimary ternary semigroup*** if every ideal of Tis a semi primary ideal.

Theorem 3.3
In a ternary semigroup T the following conditions are equivalent
1. Every left primary ideal of a ternary semigroup is a semiprimary ideal
2. Every lateral primary ideal of a ternary semigroup is a semi primary ideal
3. Every right primary ideal of a ternary semigroup is a semiprimary ideal

Proof:

By the definition of 3.1, every left primary ideal is a semiprimary ideal.
By the definition of 3.2, every lateral primary ideal is a semiprimary ideal.
By the definition of 3.3, every right primary ideal is a semiprimary ideal

Theorem 3.4

Let T be a ternary semigroup (not necessarily with identity) and let A be an ideal of T with $\sqrt{A}$ is a maximal ideal of T. Then A is a semi primary ideal.

Proof

If there is no proper prime ideal P containing A, then every prime ideal is equal to T. Then the intersection of all prime ideals of T containing A $\Rightarrow \sqrt{A} =$ T. Since $\sqrt{A}$ is a maximal ideal, $\sqrt{A}$ must be a proper ideal. Therefore there exists a proper prime ideal P containing A. Now $\sqrt{A} \subseteq$ P $\subseteq$ T and $\sqrt{A}$ is maximal, we have $\sqrt{A}$ = P. Therefore $\sqrt{A}$ is a prime ideal and hence A is a semiprimary ideal.

Theorem 3.5

If A is a semiprime ideal of a ternary semigroup T, then the following are equivalent.

1. A is a prime ideal
2. A is a primary ideal
3. A is a left primary ideal
4. A is a lateral primary ideal
5. A is a right primary ideal
6. A is a semi primary ideal.

Proof

(1)$\Rightarrow$(2) $\Rightarrow$(3) $\Rightarrow$(6), (1)$\Rightarrow$(2) $\Rightarrow$(4) $\Rightarrow$(6) and (1)$\Rightarrow$(2) $\Rightarrow$(5) $\Rightarrow$(6) are clear.

(6)$\Rightarrow$(1) : Suppose that A is a semiprime ideal. Then $\sqrt{A}$ is a prime ideal. Since A is semiprimary, $\sqrt{A}$=Awhich is a prime ideal of T. Thus all of the above are equivalent.

Theorem 3.6

A ternary semigroup T is semiprimary if and only if prime ideals of T form a chain under set inclusion

Proof

Suppose that T is a semi primary ternary semigroup. Let A,B be three prime ideals of T. By theorem 2.30, $\sqrt{A \cap B} = \sqrt{A} \cap \sqrt{B}$ = A $\cap$ B. Therefore by theorem 2.29, A $\cap$ B is semiprime. Since T is a semiprimary ternary semigroup it follows that A $\cap$ B is semiprimary. By theorem 3.5,A $\cap$ B is prime. Suppose if possible A$\not\subseteq$Band B$\not\subseteq$A. Then there exists $a \in$ A\B, $b \in$ B\A. Now <*a*><*b*><*b*>$\subseteq$ A $\cap$ B and *a, b* $\notin$ A $\cap$ B. It is a contradiction. Therefore prime ideals of T forms a chain under set inclusion.

Conversely suppose that prime ideals of T form a chain under set inclusion
For every ideal A, $\sqrt{A} = \cap P_\alpha$, where intersection is over all prime ideals P_α containing A yields $\sqrt{A} = P_\alpha$ for some α. So that A is a semi primary ideal. Therefore T is a semiprimary ternary semigroup.

Theorem 3.7
If T is a semiprimary ternary semigroup, then globally idempotent principal ideals form a chain under set inclusion.

Proof
Suppose that $<x>$, $<y>$ are two globally idempotent principal ideals in T. Since T is a semiprimary ternary semigroup, we have $\sqrt{<x>}$ and $\sqrt{<y>}$ are prime ideals of T. By theorem 3.6, either $\sqrt{<x>} \subseteq \sqrt{<y>}$ or $\sqrt{<y>} \subseteq \sqrt{<x>}$. If $\sqrt{<x>} \subseteq \sqrt{<y>}$,then $x \in \sqrt{<y>}$ and hence $x^n \in <y>$. Therefore $<x>^n \subseteq <y>$ for some odd natural number n. Since $<x>$ is a globally idempotent principal ideal, $<x>^n = <x>$ and hence $<x> \subseteq <y>$. Similarly we can show that if $\sqrt{<y>} \subseteq \sqrt{<x>}$, then $<y> \subseteq <x>$. Therefore globally idempotent principal ideals forms a chain under set inclusion.

Definition 3.8
Let T be a ternary semigroup and $a, b \in$ T. The ***natural ordering*** between a and b is defined as $a \leq b$ if $a = abb = bba$.

Theorem 3.9
Let T be a ternary semigroup and E be the set of all idempotent elements T. Then the natural ordering is a partial ordered relation.

Proof
Let E be the set of all idempotent elements of a ternary semigroup T.

1. Reflexive : Let $x \in$ E, then $x^3 = x$.
   Therefore $xxx = x \Rightarrow x \leq x$. Hence E is reflexive.
2. Antisymmetric : Let x, y $\in$ E such that $x \leq y$ and $y \leq x$.
   Now $x \leq y \Rightarrow x = xyy = yyx$. Similarly, $y \leq x \Rightarrow y = yxx = xxy$.
   Therefore $x = xyy = xxxyy = xyy = y$. Thus $x = y$. Therefore $\leq$ is antisymmetric.
3. Transitive : Let $x, y, z \in$ E such that $x \leq y$ and $y \leq z$.
   Now $x \leq y \Rightarrow x = xyy = yyx$. Similarly, $y \leq z \Rightarrow y = yzz = zzy$.
   Therefore $x = xyy = xyyzz = xzz$ and $x = yyx = zzyyx = zzx$.
   Therefore $zzx = xzz = x$. Thus $x \leq z$. Therefore $\leq$ is transitive.
   Hence $\leq$ is a partial ordered relation on E.

Theorem 3.10
Let T be a semiprimary duoternary semigroup. Then the idempotents in T forms a

chain under natural ordering.

Proof

Suppose that e and f are two idempotents in T. Then $\langle e\rangle$, $\langle f\rangle$ are globally idempotent principal ideal of T. By theorem 3. 7, $\langle e\rangle \subseteq \langle f\rangle$ or $\langle f\rangle \subseteq \langle e\rangle$. If $\langle e\rangle \subseteq \langle f\rangle$, then e $\in \langle f\rangle$. Since T is a duo ternary semigroup, $e \in \langle f\rangle = f\,TT$ = TTf , implies that $e = fst = pqf$ for some $s, t, p, q \in$ T. So $eff = pqfff = pqf = e$ and $ffe = fffst = fst = e$. Therefore $e \leq f$. Similarly if $\langle f\rangle \subseteq \langle e\rangle$, then we get $f \leq e$. Therefore the idempotents in T form a chain under natural ordering

Theorem 3.12

Let T be a ternary semigroup. Then the principal ideals of T form a chian iff ideals in T form a chain .

Proof

Suppose that principal ideals of T form a chain. Let A, B be two ideals of T. Suppose if possible A $\not\subseteq$ B and B$\not\subseteq$ A. Then there exists $a \in$A\B and $b \in$B\A. Now $a \in$A $\Rightarrow \langle a\rangle \subseteq$ A and $b \in$ B $\Rightarrow \langle b\rangle \subseteq$ B. Since principal ideals form a chain, either $\langle a\rangle \subseteq \langle b\rangle$ or $\langle b\rangle \subseteq \langle a\rangle$. If $\langle a\rangle \subseteq \langle b\rangle$ then $a \in \langle b\rangle \subseteq$ B, which is not true. If $\langle b\rangle \subseteq \langle a\rangle$ then $b \in \langle a\rangle \subseteq$ A, which is not true. It is contradiction. Hence either A $\subseteq$B or B$\subseteq$ A. Therefore ideals of T form a chain. Conversely suppose that ideals of T form a chain. Then clearly principal ideals of T form a chain.

Theorem 3.13

If T is aregular duo ternary semigroup with identity e. Then every principal ideal of T generated by an idempotent.

Proof

Suppose that T is a regular duo ternary semigroup with identity e. Let $\langle a\rangle$ be a principal ideal of T. Since $a \in$ T, T is regular, there exists $x \in$ T such that $axaxa = a$. consider $y = axe$, then $y^3 = axeaxeaxe = axaxaeexe = aeexe = axe = y$. Therefore y is an idempotent element in T. Now $a = axaxa = axeeaxa = axeaxea = yya \subseteq \langle y\rangle$. Thus $\langle a\rangle \subseteq \langle y\rangle$. Now $y = axe \in \langle a\rangle$, implies that $\langle y\rangle \subseteq \langle a\rangle$. Hence $\langle y\rangle = \langle a\rangle$. Therefore every principal ideal of T generated by an idempotent clement.

Theorem 3.14

Let T be a regular duo ternary semigroup with identity 1. Then principal ideals of T form a chain if and only if idempotents of T form a chain under natural ordering.

Proof

Let T be a regular duo semigroup with identity 1. Suppose that principal ideals of T form a chain. Let e and f be two idempotents in S. Then $\langle e\rangle$, $\langle f\rangle$ are globally idempotent principal ideal of T. Since principal ideals of T form a chain, $\langle e\rangle \subseteq \langle f\rangle$ or $\langle f\rangle \subseteq \langle e\rangle$. If $\langle e\rangle \subseteq \langle f\rangle$, then e $\in \langle f\rangle$. Since T is a duo ternary semigroup, $e \in \langle f\rangle = f\,TT$ = TTf , implies that $e = fst = pqf$ for some $s, t \in$S. Now$eff = pqfff = pqf = e$ and ffe

= *fffst* = *fst* = *e*. Therefore $e \leq f$. Similarly if $<f> \subseteq <e>$, then we get $f \leq e$. Therefore the idempotents in T form a chain under natural ordering. Conversely suppose that idempotents in T form a chain under natural ordering. Let $<a>$, $<b>$ be two principal ideals of S. By theorem 3.10, $<a> = <e>$ and $<b> = <f>$ for some idempotent elements *e*, *f* in S. Since *e* and *f* are idempotents in T and the idempotents in T form a chain under natural ordering , either $e \leq f$ or $f \leq e$. If $e \leq f$, then $e = ffe = eff \in <f>$ and hence $<e> \subseteq <f>$ i.e.,$<a> \subseteq <b>$. If $f \leq e$, then $f = eef = fee \in <e>$ and hence $<f> \subseteq <e>$ i.e.,$<b> \subseteq <a>$. Therefore principal ideals of T form a chain.

Theorem 3.15

If T is a semisimple ternary semigroup, then the following are equivalent.

1. Every ideal in T is a prime ideal.
2. T is a primary ternary semigroup.
3. T is a left primary ternary semigroup.
4. T is a lateral primary ternary semigroup.
5. T is a right primary ternary semigroup.
6. T is a semiprimary ternary semigroup.
7. Prime ideals of T form a chain

Proof

Suppose that T is a semisimple ternary semigroup. Therefore $<x> = <x>^3$ for all $x \in$ T. Let A be any ideal of T. Suppose that $x \in$ T and $<x>^3 \subseteq$ A. Then $x \in <x>^3 \subseteq$ A and hence $x \in$ A. Therefore A is a semiprime ideal of T. Therefore every ideal of T is a semiprime ideal of T. By theorem 3.5, (1) to (6) are equivalent.

By the theorem 3.6, (6) and (7) are equivalent. Hence (1) to (7) are equivalent.

Theorem 3. 16

If T is a semisimple duo ternary semigroup with identity e, then the following are equivalent.

1. Every ideal in T is a prime ideal.
2. T is a primary ternary semigroup.
3. T is a left primary ternary semigroup.
4. T is a lateral primary ternary semigroup.
5. T is a right primary ternary semigroup.
6. T is a semiprimary ternary semigroup.
7. Prime ideals of T form a chain
8. Ideals of T form a chain.
9. Principle ideals of T form a chain.
10. Idempotents of T forms a chain under natural ordering.

Proof

By theorem 3.15, (1) to (7) are equivalent. By theorem 3.12, (8) and (9) are equivalent. By theorem 3.14, (9) and (10) are equivalent. Clearly, (8)$\Rightarrow$(7) is true. By theorem 3.6, (6)$\Rightarrow$(10) is true. Therefore (1) to (10) are equivalent.

Theorem 3.17
Every ideal of a ternary semigroupT is prime if and only if T is semisimple primary ternary semigroup.

Proof
Suppose that every ideal of T is prime. Then by theorem 3.6, every ideal of T is a primary ideal. Therfore T is a primary ternary semigroup. Let $a\in$ T. Now $<a><a><a>\subseteq<a>^3$ and since $<a>^3$is prime, implies that $<a>\subseteq<a>^3$. So a is semisimple and therefore T is a semisimple primary ternary semigroup.Conversely suppose that T is a semisimple primary ternary semigroup. By thorem 3. 6, every ideal in T is prime.

Theorem 3.18
If T is a ternarysemigroup, then the following are equivalent.
1. Every ideal in T is a prime ideal.
2. T is a semisimple and ideals of T form a chain.
3. T is a semisimple and prime ideals of T form a chain.

Proof
(1)⇒ (2): Suppose that every ideal of a ternary semigroup T is prime. By theorem 3.7, T is a semisimple primary semigroup. By theorem 3.6, ideals in T form a chain. Hence T is a semisimple ternary semigroupand ideals in T form a chain. (2) ⇒ (3) is clear. (3)⇒ (1): Suppose that T is a semisimple and prime ideals of T form a chain. Then By theorem 3.6, every ideal of T is prime.

Theorem 3. 19
If T is a semisimple duo ternary semigroup. Then the following are equivalent.
1. Every ideal in T is a prime ideal.
2. T is a regular primary ternary semigroup.
3. T is a regular semiprimary ternary semigroup.
4. T is a regular ternary semigroup and idempotents of T form a chain under natural ordering.

Proof
(1)⇒(3): Suppose that every ideal of T is prime. Let A be an ideal of T. Then A is prime. By theorem 3.4, T is primary ternary semigroup. Let $a\in$ T. Now $<a><a><a>\subseteq<a>^3$ and $<a>^3$ is a prime ideal of T, we have $<a>\subseteq<a>^3$ and hence $a\in<a>^3$. So a is semisimple for all $a\in$ T. Since T is duo semisimple, by theorem 2.39, T is regular. Hence T is a regular primary ternary semigroup.

(2)⇒(3): Suppose that T is a regular primary ternary semigroup. By theorem 3.6, T is semi primary ternary semigroup. Therefore T is a regular semiprimary ternary semigroup.

(3)⇒(4) : Suppose that T is regular semiprimary ternary semigroup. Since T is semiprimary, by theorem 3. 10, the idempotents of T forms a chain under natural ordering. Hence T is regular and idempotents of T forms a chain under natural ordering.

(4)⇒(1) : Suppose that T is a regular ternary semigroup and idempotents of T form a chain under natural ordering. By theorem 3. 18, every ideal of T is prime.

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