International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 7 (2015) pp. 18721-18735

http://www.ripublication.com

Performance Analysis of OFDM System WithChannel Estimation And Phase Noise Reduction Using EKF

Mangayarkkarasi M.
Assistant Professor, Department ofECE
Sri Muthukumaran Institute of Technology
chikkarayaburam, India.
cmmangai@gmail.com
Joseph Gladwin S.
Assistant Professor, Department of ECE
SSN College of Engineering
Kalavakkam – 603110, India.
josephs@ssn.edu.in

Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is widely used in wireless communications. The phase noiseimpact on the system performance is important to evaluate. In this paper, an extended Kalman filter (EKF)algorithm for phase noise reduction is proposed and further channel estimation was performed using the Least Mean Square (LMS) estimator, Recursive Least Square (RLS) estimatorand Least Square (LS) estimator. The performance of OFDM system over a multipath frequency selective fading channel is analyzed, by Mean Square Error (MSE), Bit Error Rate (BER) and Mean of Error with phase noise and without phase noise. The analysis shows a reduction in the MSE, BER and Mean of Error. From the graphs of BER, MSE and Mean of Error, against Signal to Noise Ratio (SNR) we can evaluate the performance degradation with phase noise and without phase noise. The simulation results using MatLab shows the comparison of the performance analysis with and without phase noise (MSE, BER and Mean of Error) of the LMS, RLS and LS algorithms.

Index Terms: OFDM, phase noise, multipath Rayleigh fading channel, EKF, LMS, RLS, LS, MSE, BER, Mean of Error.

Introduction

OFDM is a multi-carrier modulation technique making use of orthogonal subcarriers and divides the entire bank of frequency into sub bandsis one of the main advantage of

OFDM system.When compared to single carrier systems,OFDM has many advantages including its resistance to frequency selective fading [1].

The performance of an OFDM system was evaluated under the combined influence of phase noise, CFOand timing jitter over a rayleigh fading channel. The closed form expression for the SINR is derived and the combined outcomes of these synchronization impairments were exhibited by BER performance of a BPSK-OFDM system over Rayleigh fading channel. Results shows that OFDM system suffers significant SINR penalty due to CFO and Jitter, however, the effect of phase noise is the dominant one. From this we find out CFO model, Phase Noise model, Timing jitter model, OFDM system model and exact SINR expression for the combined effect and the performance of the system was analyzed [2].

A precise numerical technique used for calculating the effect of the CFO and phase noise on the BER or Symbol Error Rate (SER) in an OFDM system. Beaulieu Series was used to derive the SER expression. From this we find out the OFDM signaling, CFO, Phase Noise model and the exact expression for the probability of error presented [3].

Phase noise should be carefully considered while designing an OFDM based communication system. An accurate prediction on the tolerable phase noise leads to the system and RF engineers to relax the specifications. From this we find out OFDM, Phase Noise and Phase Noise effect in the quality of an OFDM signal [4].

For frequency down conversion at the movablereceiverwe can use the phase noise of the oscillator. For this reason, the evaluation of the impact of phase noise on the system performance is important. For performance analysis different modulation schemes are compared. From this we find out the System Model and the performance evaluation using BPSK and QPSK [5].

The performance of OFDM systems with joint effects of phase noise and channel estimation errors in Rayleigh fading is presented. The average probability of Bit error with channel estimation error is obtained. The phase noise impairments can cause severe system performance degradation even at high SNRs. The added effect of channel estimation errors further deteriorates the system performance [6].

The effects of phase noise on the OFDM system performance and provide the corresponding solution. In order to perform the analysis, we have derived an exact closed-form expression for SINR, with which system behavior can clearly be judged for any phase noise levels. In the presence of phase noise, the quantitative revaluations of critical parameters to system performance, such as phase noise line width, number of subcarriers, transmission data rate, and SNR is clearly presented in mathematical functions [7].

In this paper the Extended Kalman filter is proposed as a means for estimating phase noise and its suppression in OFDM transmission. The performance of extended Kalman filter algorithm was studied in terms of the Bit Error Rate and SNR degradation [8].

The concept of the adaptive filter algorithms that are implemented with FIR filter structures and their variety of applications in those systems where minimal information is available about the incoming signal. He discussed several applications of adaptive filters based on FIR filter structures such as system identification, adaptive equalization for data transmission, echo cancellation for speech-band data of transmission, linear predictive coding of speech signals and array processing.

The concept to design an adaptive equalizer using the mean square error(MSE) criterion and also explained the concept to minimize the cost function adaptively by applying the least mean square(LMS) algorithm are discussed in Werner et al.[9].

LS method offers exact estimation of channel. Here, LS estimator was used for initial channel estimation. For better accuracy of channel estimation, LMS estimator was added to receiver which includes a feedback of output and improves the BER performance of the system. The channel estimation with LMS algorithm can be closed to LS method with a careful choice of λ. In the performed simulations it is observed that with a careful choice of λ, the channel estimation will be similar to the methods of estimating LS [10].

For data transmission, wireless channel is modeled as Rayleigh channel which can be used as a medium. The time varying nature of received envelop of the flat fading signal was described by Rayleigh distribution[11].In this paper, the performances of an OFDM system is evaluated with the influence of the phase noise impairment. Fig.1. shows the Phase noise impaired OFDM system.

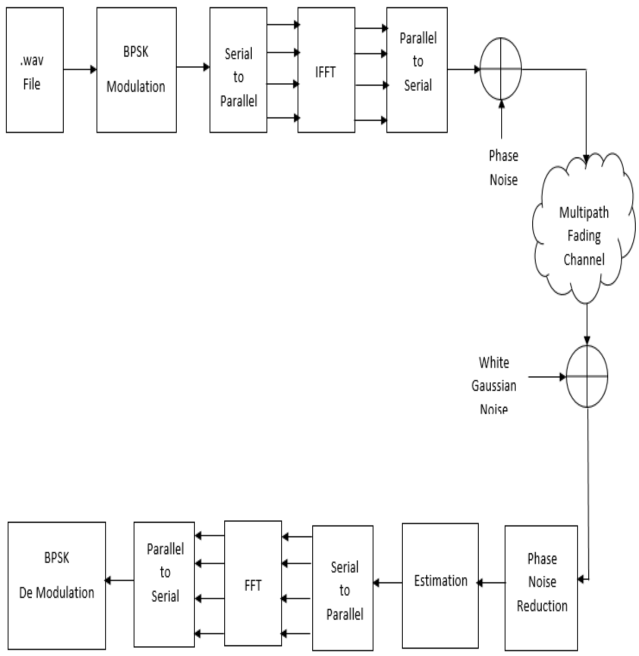

Figure 1: Oscillator Phase Noise Impaired OFDM System (Basic Block Diagram)

Here Binary Phase Shift Keying (BPSK) modulation is used to modulate the input signal. The serial form of the modulated signal can be converted to parallel form. The parallel signal is converted to time domain signal by Inverse Fast Fourier Transform (IFFT) operation. After the parallel to serial conversion, the received signal is called OFDM signal [12]. Then, Phase noise can be added at the transmitter end of the

OFDM system. ThePhase Noise added OFDM signal passes through the multipath fading channel. The phase noise can be reduced using Extended Kalman Filter (EKF).
The channel was estimated using the Least Mean Square (LMS) estimator, Recursive Least Square (RLS) estimator and Least Square (LS) estimator. Demodulation of the received signal by using FFT demapping the data to retrieve the original data.
This paper contains the following sections: In Section II, OFDM system modelandphase noise process was analyzed. In Section III,The phase noise added at the transmitter end is reduced using Extended Kalman Filter (EKF). In Section III,The channel was estimated using the Least Mean Square (LMS) estimator, Recursive Least Square (RLS) estimator and Least Square (LS) estimator. Section V describe the result analysis of system performance and finally Section VI concludes the paper.

System Model

A. Phase Noise Model

For autonomous oscillators, as $t \rightarrow \infty$, the PN (t) becomes asymptotically a Gaussian process with variance$\sigma^2 = ct$that linearly increases with time, the rate of the variance is $c$that value depends on the form of oscillator used [8]. By sampling the continuous time counterpart (t), a discrete Wiener process (nTs) is obtained. It is typically given as

$$\theta \ n = \sum_{i=0}^{n} \varepsilon \ i \qquad (1)$$

where, by definition of the Wiener PN process, $\theta(0) = \varepsilon(0) = 0$ and $\varepsilon(i) = \theta(i) - \theta(i - 1)$ are the independent increments drawn from a zero mean Gaussian distribution with variance[13].

$$\sigma^2 = c \times T_s = \frac{c}{f_{sub} N_c} = \frac{4\pi f_{3dB}}{f_{sub} N_c} \qquad (2)$$

B. Frequency Selective Fading Channel Model

The transmitted signal is propagate through a frequency selective fading channel. Rayleigh model is the best approximates a practical frequency selective fading channel.In a frequency selective fading channelthe coherence bandwidth is less than the bandwidth of the signal [14].

$$R(t) = \sum_{k}^{l} ak(t) * S(t - \tau k) \qquad (3)$$

Where, τ is variable delay (phase shift)

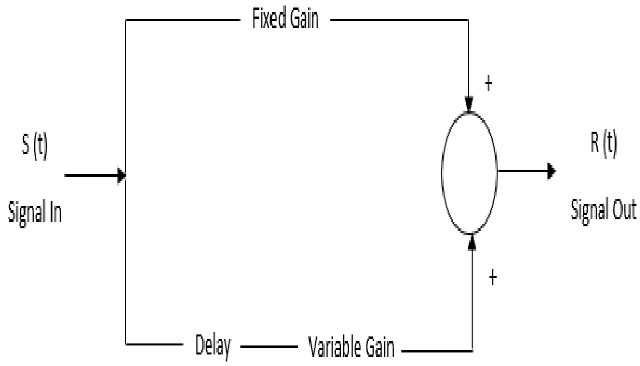

Figure 2: Simple Two Path Rayleigh Fading Channel

The Fig.2 shows the passing of an input signal S(t) through a simple two path Rayleigh fading channel. A part of the signal flows through the path with fixed gain, while the other path, has a delay with the variable gain. The outputs from both the paths are added to get the output signal R(t) to which the white Gaussian noise is added .

Phase Noise Reduction

A. Extended Kalman Filter (EKF)

1. Phase Noise Estimation Scheme

To estimate the quantity ϕ (n) using an EKF in each ofdm signal, the state equation is built as

$$\phi(n) = \phi(n-1) \quad (4)$$

In this case we are estimating an unknown constant ϕ. This constant is distorted by a non-stationary process X (n), an observation of which is the preamble symbols preceding the data. The observation equation is

$$y(n) = \left[X(n) * h(n)\right]e^{j\phi(n)} + w(n) \quad (5)$$

Where, y(n) is the received signal distorted in the channel, X(n) is the input signal ieofdm signal, h(n)vis the channel response and w(n) is the AWGN.

2. EKF Formulation

The Extended Kalman Filter (EKF) can be formulated using the following equations

Initialize $p(n), \hat{\phi}(0)$

For $n = 1, 2, ...N_p compute$

$$H(n) = \frac{\partial y(X)}{\partial x} \quad (6)$$

$$H(n) = x(n)\, e^{j\hat{\phi}(n-1)} \quad (7)$$

$$K(n) = \frac{p(n-1)H^*(n)}{\left[H(n)p(n-1)H^*(n) + \sigma^2\right]} \quad (8)$$

$$\hat{\phi}(n) = \hat{\phi}(n-1) + \mathrm{Re}\left\{K(n)\left[y(n) - x(n)e^{j\hat{\phi}(n-1)}\right]\right\} \quad (9)$$

$$p(n) = \left[1 - K(n)H(n)\right]p(n-1) \quad (10)$$

The output of the phase noise estimation is

$$\text{Estimated output} = y(n)\, e^{-j\hat{\phi}(n)} \quad (11)$$

As the estimation of the phase noise by the EKF scheme is pretty efficient and accurate, it is expected that the performance will be mainly influenced by variation of the AWGN [8].

Channel Estimation

The channel was estimated using Adaptive filtering and Least Square estimator.

A. Adaptive Filtering

Adaptive filters are used in variety of applications like acoustic echo cancellation, radar guidance systems, and wireless channel estimation. Here we consider Adaptive filter for wireless channel Estimation.

I) Least Mean Square (LMS) algorithm

Here the adaptive algorithm is LMS algorithm which iteratively updates the coefficient and feeds it to the FIR filterwhich is shown in Figure.3.The FIR filter uses the coefficient w(n) along with x(n) is the input signal to generate the output signal y(n).To generate an error signal the output signal y(n) is subtracted from the desired signal d(n). The error signal which is used by the LMS algorithm to compute the next set of coefficients.

This is a classical Weiner filtering problem for which the solution can be iteratively found using the LMS algorithm[15].

The a mathematical relation for the transfer function of the system is

$$y(n) = W_n^T X(n) \quad (12)$$

Here, the input signal is x(n), the impulse response is w(n) and the output signal is y(n).

Then the input signal x(n) is defined as

$$X(n) = \left[x(n), x(n-1), x(n-2), ...x(n-(N-1))\right] \qquad (13)$$

Where, $W_n^T = \left[w_0(n), w_1(n), ...w_{N-1}(n)\right]$ (14)

are the weight coefficients for an N[th] order FIR filter. Using an estimate of the ideal cost function the following equation can be derived.

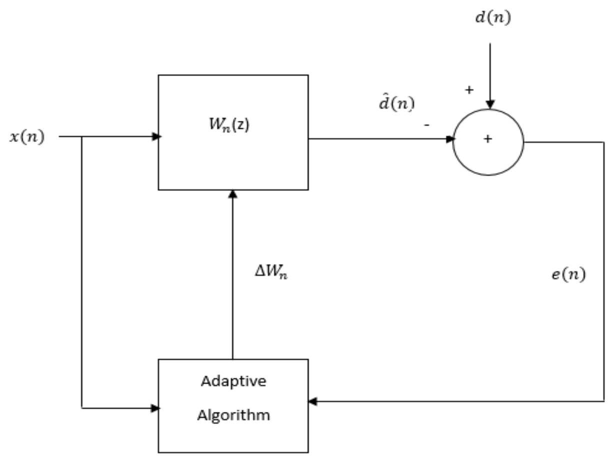

Figure 3: Block Diagram of AnAdaptive Filter

$$W(n+1) = w(n) - \mu\Delta_{E[e^2(n)]} \qquad (15)$$

From equation 15 we know the new coefficient values for the next time interval. The scaling factor is μ and ideal cost function is $\Delta_{E[e^2]}(n)$ with respect to vector w(n).

$$W(n+1) = w(n) - \mu e(n)x(n) \qquad (16)$$

The equation 16 is the estimate for ideal cost function.

Where, $e(n) = d(n) - y(n)$ (17)

and $y(n) = x^T(n)w(n)$ (18)

From equation 16 μ is sometimes multiplied by 2, but here we will assume it is absorbed by the μ factor.

II)Recursive least squares (RLS) algorithm

This algorithm is contrast to the least mean squares (LMS) algorithm that aim to reduce the mean square error. The input signals are considered deterministic in RLS algorithm, while for the LMS and similar algorithm they are considered stochastic. Compared to most of its competitors, the RLS exhibits extremely fast convergence.

Let us consider the design of an FIR adaptive wiener filter and find the filter coefficients

$$W_n = \left[w_n(0), w_n(1), ..., w_n(p) \right]^T \quad (19)$$

The equation 19 minimize, at time n, the weighted least square error is

$$\varepsilon(n) = \sum_{i=0}^{n} \lambda^{n-i} \left| e(i) \right|^2 \quad (20)$$

An exponential weighting (forgetting) factor is $0 < \lambda \le 1$ and
The error signal is

$$e(i) = d(i) - y(i) = d(i) - W_n^T X(i) \quad (21)$$

In equation 21 X(i) is the input signal and the error difference between the desired signal d(i) and the filtered output at time i, is e(i) using the least set of filter coefficients, $w_n(k)$.
Initialization:

$$w_0 = 0$$

$$P(0) = \delta^{-1} I$$

Computation:
For $n = 1, 2, ..., compute$

$$z(n) = P(n-1) X^*(n) \quad (22)$$

The gain vector g(n) is

$$g(n) = \frac{1}{\lambda + X^T(n) z(n)} z(n) \quad (23)$$

Then the filter coefficient is defined as

$$W_n = W_{n-1} + \alpha(n) g(n) \quad (24)$$

where, $\alpha(n) = d(n) - W_{n-1}^T X(n)$ (25)

The equation 25 is the error difference between d(n) and the estimate of d(n) that is formed by applying the previous set of filter coefficients, W_{n-1}, to the new data vector, X(n).

Then, $P(n) = \frac{1}{\lambda} \left[P(n-1) - g(n) z^H(n) \right]$ (26)

From the above equations P is the filter order, λ is the exponential weighting factor and δ is used to initialize P(0).
The current set of filter coefficients are close to their optimal values When α(n) is small. The current set of filter coefficients are not performing well in estimating d(n)when α(n) is larger [16].

B. Least Squares (LS)

To estimate the system h[m] by minimizing the squared error between estimation and detection the Least Squares Error (LSE) estimation method can be used which is shown in Fig. 4 [8].

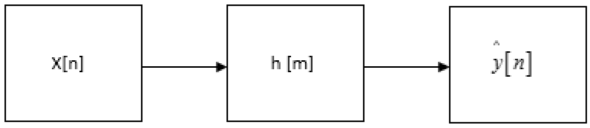

Figure 4: Least Square Estimation

In matrix form, it can be written as

$$y = Xh \qquad (27)$$

So the error 'e' can be defined as

$$e = y'' - y \qquad (28)$$

Where y" is the expected output.
The squared error (S) can be defined as

$$S = |e|^2 \qquad (29)$$

$$S = \ y'' - y\ ^{2} \qquad (30)$$

$$S = \ y'' - y\ *\ y'' - y\ ^{t} \qquad (31)$$

Where superscript't' stands for complex transpose of a matrix.

$$S = \ y'' - Xh\ *\ y'' - Xh\ ^{t} \qquad (32)$$

This equation can be minimized by taking its derivative w.r.t 'h' and equating it equal to zero. The final equation we get is:

$$h'' = \ X^{t}X\ ^{-1}\ X^{t}y \qquad (33)$$

Which can be written as

$$h'' = X^{-1}y \ = \ h_{ls} = X^{-1}y \qquad (34)$$

Performance Analysis

The performance of the OFDM system for BPSK modulation scheme over a Frequency Selective Rayleigh fading channel is investigated by means of simulation. The system with phase noise and without phase noise performance is measured in terms of BER, MSE and Mean of Error Vs SNR. The parameters used for this analysis is shown in Table I.

Table 1: System and Channel Parameters For Simulation

Number ofsamples	128128
Modulation	BPSK
Constellations	M=2 for BPSK
Channel Type	Multipath Rayleigh fading channel(Two path)(Frequency selective)
Phase NoiseReduction	Extended Kalman Filter (EKF)
Channel Estimation	LMS, RLS and Least Square Estimators
Input SNR	5 to 20 in dB

C. Bit Error Rate (BER)

It's defined as the ratio of number of error bit occurred in the system to the total number of bits transmitted in the system. The bit error rate can be calculated using the formula:

$$\text{BER}= \frac{Numberoferrorbits}{TotalnumberofbitsTransfered} \tag{35}$$

There is some probability of errors introduced in the system during the data transmitted through the channel.If the channel is good and signal to noise ratio is high, then the BER will be very small and it does not affect the overall system performance. If the noise can be detected then the BER will be considered. Theoretically the BER is computed as

$$P_b = BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{36}$$

Where E_b/N_0 is the Signal to Noise Ratio (SNR) and Q is the quadratic function this can also be expressed as follows:

$$Q\ \ x\ = \frac{1}{2\pi}\int_{x}^{\infty} e^{-t^2/2}\, dt \tag{37}$$

For phase noise impaired OFDM system the error can be calculated using demodulated data and data source.

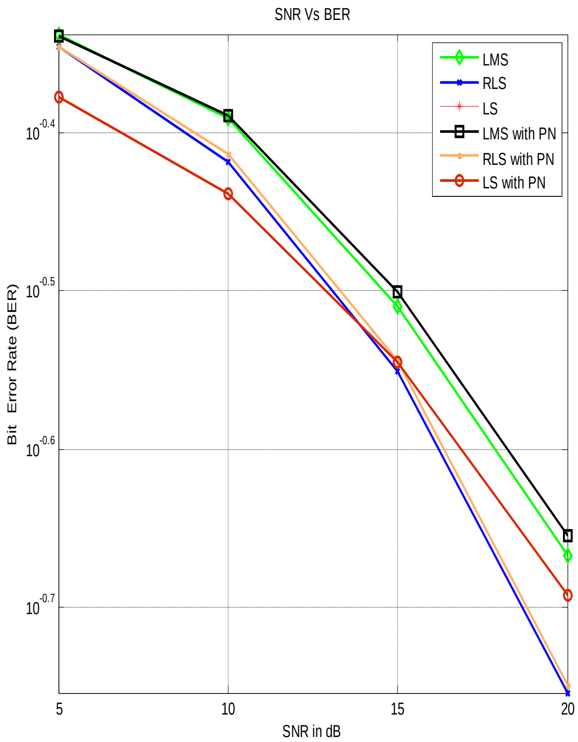

Figure 5:SNR Vs BER

Table 2: Quantitative Values of Simulation Result

SNR	LMS with Phase Noise	LMS without Phase Noise	RLS with Phase Noise	RLS without Phase Noise	LS with Phase Noise	LS without Phase Noise
5	0.4581	0.4593	0.4508	0.4513	0.4194	0
10	0.4083	0.4061	0.3859	0.3812	0.3640	0
15	0.3156	0.3091	0.2854	0.2814	0.2852	0
20	0.2213	0.2151	0.1782	0.1763	0.2032	0

The performance of BER will decrease when SNR values increases. This can be analyzed from the figure 5 and table II. Here the LS estimator without phase noise give better performance compared to RLS and LMS algorithms with and without phase noise because there is no error between the demodulated data and data source.

D. Mean Square Error (MSE)

Here, MSE can be defined as the ratio of the squared error difference between the demodulated data and data source to the product of the length of the demodulated data and data source.

$$\text{MSE} = \frac{\sum_{M,N} \left[I_1 \;\; m,n \;\; - I_2 \;\; m,n \;\; \right]^2}{M * N} \qquad (38)$$

Where, I_1 is the data source and I_2 is the demodulated data. M is the length of the data source and N is the length of the demodulated data.

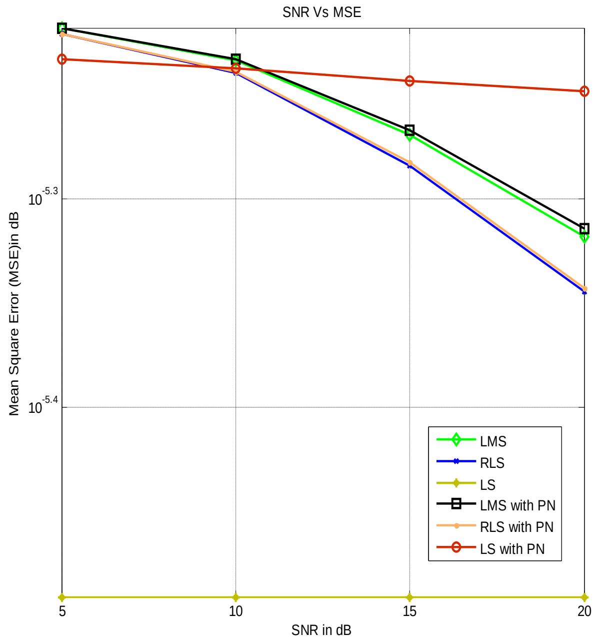

Figure 6: SNR Vs MSE

Table 3: Quantitative Values of Simulation Result

SNR	LMS with Phase Noise	LMS without Phase Noise	RLS with Phase Noise	RLS without Phase Noise	LS with Phase Noise	LS without Phase Noise
5	$6.051*10^{-6}$	$6.051*10^{-6}$	$6.009*10^{-6}$	$6.012*10^{-6}$	$5.841*10^{-6}$	$3.230*10^{-6}$
10	$5.846*10^{-6}$	$5.838*10^{-6}$	$5.763*10^{-6}$	$5.754 *10^{-6}$	$5.783*10^{-6}$	$3.230*10^{-6}$
15	$5.406*10^{-6}$	$5.379*10^{-6}$	$5.218*10^{-6}$	$5.194*10^{-6}$	$5.702*10^{-6}$	$3.230*10^{-6}$
20	$4.846*10^{-6}$	$4.805*10^{-6}$	$4.541*10^{-6}$	$4.525*10^{-6}$	$5.643*10^{-6}$	$3.230*10^{-6}$

The performance of MSE will decrease when SNR values increases. This performance was analyzed from figure 6 and table III. Here we can conclude that the RLS algorithm without phase noise has a better performance when compared to the LMS with and without phase noise. By observing the performance of the LS algorithm without phase noise, we find that it has a superior performance compared to the RLS and LMS algorithms.

E. Mean of Error (ME)

The Mean of Error (ME) can be defined as the mean of error difference between the data source and received signal.

Mean of Error (ME) $= I_1 \quad m,n \quad - I_2 \quad m,n$ (39)

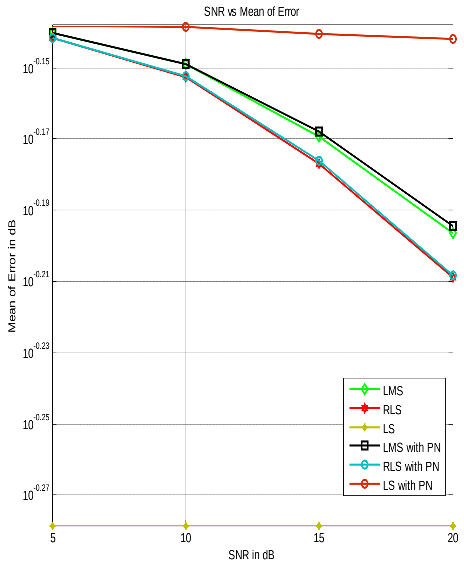

Figure 7: SNR Vs Mean of Error

Table 4: Quantitative Values of Simulation Result

SNR	LMS with Phase Noise	LMS without Phase Noise	RLS with Phase Noise	RLS without Phase Noise	LS with Phase Noise	LS without Phase Noise
5	0.7243	0.7239	0.7219	0.7219	0.5266	0.7276
10	0.7098	0.7097	0.7041	0.7038	0.5266	0.7247
15	0.6793	0.6774	0.6669	0.6654	0.5266	0.7238
20	0.6390	0.6363	0.6191	0.6181	0.5266	0.7224

When SNR values increase, the performance of Mean of Error will decrease. We can conclude that the RLS algorithm has a better performance of ME in comparison to the

LMS. By observing the performance of the LS algorithm, we find that it has a superior performance of ME, compared to the RLS and LMS algorithms.

Conclusion

The performance of an OFDM systems have been analytically evaluated with phase noise and without phase noise. In this work, the BER, MSE and Mean of Error performance of an OFDM system over frequency selective fading channel was analyzed. The phase noise introduced at the transmitter end, prior to the channel, is reduced using the Extended Kalman Filter (EKF). By observing the performance of the LS algorithm, it is observed that it has a superior performance than RLS and LMS algorithms.

Acknowledgment

The authors would like to thank the anonymous reviewers for their careful revision and important suggestions which significantly helped to improve the presentation of this paper.

References

[1] Ying Shan Li and Jin-Soo Park, “Non-linear Analysis of thePhase Noise in the OFDM Communication System,” IEEETransactions on Consumer Electronics, Vol. 50, No.1, February 2004.

[2] S. Mallick and S. Majumder, “Performance analysis of an OFDM system in the presence of carrier frequency offset, phase noise and timing jitter over Rayleigh fading channels,” Dec.2008.

[3] K. Sathananthan and C. Tellambura, “Performance analysis of an OFDM system with carrier frequency offset and phase noise,” 2001.

[4] Garcia Armada, “Understanding the effects of phase noise in orthogonal frequency division multiplexing OFDM,” June 2001.

[5] L. Tomba, “On the effect of Wiener phase noise in OFDM systems,” May 1998.

[6] M. Jalloh, M. Al-Gharabally, and P. Das, “Performance of OFDM systems in Rayleigh fading channels with phase noise and channel estimation errors,” in Proc. IEEE Military Commun. Conf., Oct. 2006.

[7] W. Songping and Y. Bar-Ness, “OFDM systems in the presence of phase noise: consequences and solutions,” IEEE Trans. Commun., vol. 52, no. 11, pp. 1988–1996, Nov. 2004.

[8] MadhurendraBensan and Vishnu Pratap Singh Kirar and Kavita Burse “Phase Noise Reduction and Tracking in OFDM using Extended Kalman Filter,” inFourth International Conference on Computational Intelligence and Communication Networks, 2012.

[9] S. Haykin, “Adaptive Filter Theory, 3/e”, PrenticeHall, 1996.

[10] Aboozarlashkari, Babakehyaee, Naviddaryasafar, “Channel estimation in MIMO-OFDM systems based on comparative methods by LMS algorithm,” IJCSIInternational Journal of Computer Science Issues, vol. 9, issue 3 no 3, May 2012.

[11] T. Schenk, RF Imperfections in High-Rate Wireless Systems. Springer, 2008

[12] PramodMathecken, and TaneliRiihonen, May 2011, “Performance Analysis of OFDM with Wiener Phase Noise and Frequency Selective Fading channel,” in Proc.IEE Transaction on communication Vol.59, No.5.

[13] S. Wu and Y. Bar-Ness, “OFDM systems in thepresence of phase noise: consequences and solutions,”IEEE Trans. Commun., Oct. 2004.

[14] Mohd Al Hafiz Bin Osman, Modelling Rayleigh Propagation and effect in Wireless Communication channel using matlab, May, 2008.

[15] Statistical Digital Signal Processing and Modeling by Monson H. Hayes.

[16] Mohammad Ali, RehanElahi, Ali Saljuk, SheheryarBukhari,March 2008, ”Channel Estimation of MIMO OFDM System.