International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 7 (2015) pp. 18691-18708

http://www.ripublication.com

An Efficient Bernstein Operational Matrix Based Algorithm For A Few Boundary Value Problems

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Abstract

In this paper, we have established an efficient Bernstein operational matrix algorithm (BOMA) for solving a few boundary value problems (BVPs) arising in science and engineering. The main idea for obtaining the numerical solutions for these equations is essentially developed by reducing the differential equations with their initial and boundary conditions to a system of linear or nonlinear algebraic equations in the unknown exponential coefficients. Some numerical examples are given to demonstrate the validity and applicability of the proposed method. Numerical results obtained are comparing favorably with the exact known solutions. The proposed method in general is easy to implement, and yields good results. The power of the manageable method is confirmed.

Keywords: Bernstein operational matrices; differential equations; boundary value problems; collocation method

1.Introduction

In recent years, mathematical modeling of complex processes is a major challenge for contemporary scientists. In contrast to simple classical system, where the theory of

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integer order differential equations is sufficient to describe their dynamics. Most realistic mathematical models cannot be solved through the traditional methods providing an excellent means to put across the underlying theory; instead, they must be dealt with the computational methods that produce approximate solutions. Polynomials are useful mathematical tools as they are pretty defined, fast calculation on a modern computer system and can represent a great variety of functions. Also differentiation and integration is very simle. In recent years Bernstein operational matrices of differentiation have attracted the attention of many researchers.

With the advent of computer graphics, Bernstein polynomial in the interval $x \in (0,1)$ becomes an important in the form of Bezier curves [1,2]. Bernstein polynomials have many useful properties such as the positivity, the continuity, recursive relation, symmetry and unity partition of the basis set over the interval (0, 1). For this reason, they have been studied in an enormous number of publications, and are frequently used both in approximation theory and computer aided geometric design [2]. Farouki [3] had introduced the Legendre-Bernstein basis transformations. Farouki and Goodman [4] had introduced the optimal stability of the Bernstein basis. Solving Fredholm integral equations [5,6], Volterra integral equations [7], Bhatta and Bhatti [8] have been used modified Bernstein polynomials for solving KdV equation. Amit K. Singh et al. [9] established the B-polynomials have been first orthonormalized by using Gram-schmidt ortho normalization process and then the operational matrix of integration has been acquired. By the expansion of B-polynomials in terms of Taylor basis and Yousefi and Behroozifar [10] found the operational matrices of integration and product of B-polynomials. The same author [11] implemented the Bernstein operational matrix method for solving the parabolic type PDEs. Bhatti and Bracken [12] have given solutions of linear and non-linear differential equations with linear combinations of Bernstein polynomials, and their coefficients have been determined by Galerkin method. Doha and et al. [13,14] have proved new formulas about derivatives and integrals of Bernstein polynomials, and have used the Galerkin and Petrov-Galerkin methods based on Bernstein polynomials for solving high even-order differential equations. Isık et al. [15,16] have introduced a new method to solve high order linear differential equations with initial and boundary conditions. The method is numerically based on rational interpolation and Bernstein series solution depending on collocation method. Ordokhani et al.[17] used Bernstein polynomial for solving differential equations. Integral equations [18], partial differential equations [19,20]. Yousefi et al. [21] implemented the Ritz-Galerkin method for solving an initial boundary value problem that combines Neumann and integral condition for the wave equation. Bhattacharya et al.[22]implemented the algorithm for integro differential equations.

Hariharan and his coworkers [23-25] established the Haar wavelet method (HWM) for solving a few partial differential equations arising in engineering. Recently, Saadatmandi [26], have developed the fractional order differential equations by the Bernstein operational matrix. Recently, Ahmed [27] established the numerical solutions of the 2nd-order linear differential equations subject to Dirichlet boundary conditions.

The aim of the present paper is to apply Bernstein operational matrix algorithm (BOMA) for a few boundary value problems. The main characteristic behind the approach using this technique is that it reduces these problems to those of solving a system of algebraic equations thus greatly simplifying the problem. By several nonlinear boundary value problems, it is clearly indicate that the Bernstein operational matrix Method show its validity and efficiency.

This paper is organized as follows. In section 2, the basic properties of Bernstein polynomials are presented. Function approximation is illustrated in section 3. Some numerical examples are explored in section 4. Concluding remarks are given in section 5.

2. Properties of Bernstein Polynomials [26]

The Bernstein basis polynomials of degree n are defined in [0,1] as [10,11]

$$b_i^n(x) = \binom{n}{i} x^i (1-x)^{n-i}, i = 0,1,2,...n \tag{1}$$

These Bernstein polynomials form a complete basis on [0,1]. Using the recursive definition to generate these polynomials

$$b_i^n(x) = (1-x) b_i^{n-1}(x) + x b_{i-1}^{n-1}(x), i = 0,1,...,n$$

Where $b_{-1}^{n-1}(x)$ = 0 and $b_n^{n-1}(x)$=0 . Since the power basis forms a basis for the space of polynomials of degree less than or equal to *n*, any Bernstein polynomial of degree *n* can be written in terms of the power basis. This can be directly calculated using the binomial expansion of

$(1-x)^{n-i}$, one can show that $b_i^n(x) = \sum_{j=1}^{n} (-1)^{j-i} \binom{n}{i} \binom{n-i}{j-i} x^j, i = 0,1,...,n$ (2)

The dual basis is defined by the property

$\int_0^1 b_i^n(x) d_j^n(x) = \begin{cases} 1\, for i = j \\ 0\, for i \neq j \end{cases}$ for i, j =0,1,2,...,n

A function $f(x)$, square integrable in [0,1] , may be expressed in terms of Bernstein basis [10,11]. In practice, only we choose the first (*n* + 1) term Bernstein polynomials are considered.

Therefore we can write,

$$f(x) \approx \sum_{i=0}^{n} c_i b_i^n(x) = C^T B(x) \tag{3}$$

Where C is the Bernstein coefficient vector and *B*(*x*) is Bernstein vector

$$C^T = [c_0, c_1, ..., c_n],$$
$$B(x) = [b_0^n(x), b_1^n(x), ..., b_n^n(x)]^T, \tag{4}$$

Then

$$c_i = \int_0^1 f(x) d_i^n(x) dx, i = 0,1,...,n$$

Author of [28] has derived explicit representations,

$$d_j^n(x) = \sum_{k=0}^{n} \lambda_{jk} b_k^n(x), \; j = 0,1,...,n$$

for the dual basis functions, defined by the coefficients

$$\lambda_{jk} = \frac{(-1)^{j+i}}{\binom{n}{j}\binom{n}{k}} \sum_{i=0}^{\min(j,k)} (2i+1)\binom{n+i+1}{n-j}\binom{n-i}{n-j}\binom{n+i+1}{n-k}\binom{n-i}{n-k} for j,k = 0,1,...,n \quad (5)$$

Bernstein polynomials and Legendre polynomials both span the same spaces and the transformation between Legendre and Bernstein polynomials is comparatively well-conditioned. The Bernstein polynomials are advantageous for practical computations, on account of its intrinsic numerical stability [4]. One of the useful property of Bernstein basis polynomials is that they all vanish at end points of the interval, except the first and the last one, which are equal to one at x=0 and x=1 respectively. This gives greater flexibility in which to impose boundary conditions at the end points of the interval [12]. Also, Bernstein polynomials have two main properties: their sum equals 1 and every $b_i^n(x)$ is positive for all real *x* belonging to the interval $x \in (0,1)$. Moreover, as pointed by [21], the Bernstein basis polynomials have the following properties:

1. $b_i^n(x)$ has a root with multiplicity i at a point x=0 (note if *i* is 0 there is no root at 0).
2. $b_i^n(x)$ has a root with multiplicity n-i at a point x=1 (if *n* = *i* there is no root at 1).

While for the Legendre polynomials, no explicit formula of the roots is known.

2.1 The operational matrix of the derivative [26]

The derivative of the vector *B*(*x*) can be expressed by

$$\frac{dB(x)}{dx} = D^{(1)} B(x) \quad (6)$$

Where $D^{(1)}$ is the (n+1) x (n+1) operational matrix of derivative and is given in [10] as

$$D^{(1)} = AVB^{*} \quad (7)$$

Here

$$A=\begin{pmatrix} (-1)^0\binom{n}{0} & (-1)^1\binom{n}{0}\binom{n-0}{1} & . & . & . & (-1)^{n-0}\binom{n}{0}\binom{n-0}{n-0} \\ 0 & (-1)^0\binom{n}{i} & . & . & . & (-1)^{n-i}\binom{n}{i}\binom{n-i}{n-i} \\ . & 0 & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & .0 & . & . & 0 & (-1)^0\binom{n}{n} \end{pmatrix}_{(n+1)(n+1)}$$

$$V=\begin{pmatrix} 0 & 0 & 0 & . & . & . & 0 \\ 1 & 0 & 0 & . & . & . & 0 \\ 0 & 2 & 0 & . & . & . & 0 \\ . & . & . & & . & & . \\ . & . & . & & . & & . \\ . & . & . & & . & & . \\ 0 & 0 & 0 & . & . & . & n \end{pmatrix}_{(n+1)(n)} , \quad B=\begin{pmatrix} A_{[1]}^{-1} \\ A_{[2]}^{-1} \\ A_{[3]}^{-1} \\ . \\ . \\ . \\ A_{[n]}^{-1} \end{pmatrix}_{(n)(n+1)}$$

Where $A_{[k]}^{-1}$ is the k-th row of A^{-1} for k= 1,2,...,n

$$D^{\alpha} b_i^n(x) \approx \left[\sum_{j=\lceil\alpha\rceil}^{n} w_{i,j,0}, \sum_{j=\lceil\alpha\rceil}^{n} w_{i,j,1}, \ldots, \sum_{j=\lceil\alpha\rceil}^{n} w_{i,j,n} \right] B(x), i=0,1,\ldots,n \qquad (8)$$

This leads to the desired result.

Generalized Bernstein operational matrix of differentiation

$$\frac{d^n}{dx^n}\phi(x)=\left[D^{(1)}\right]^n B(x) \text{ , n=1,2,…}$$

$$B(x)=\left[b_0^n(x), b_1^n(x), b_2^n(x), \ldots, b_n^n(x)\right]^T$$

$$b_i^n(x)=\binom{n}{i} x^i (1-x)^{n-i} \text{ , i=0, 1, 2,…, n}$$

Where $b_{-1}^{n-1}(x)=0$ and $b_n^{n-1}(x)=0$

For example if n=2 we get

$$B(x)=\left[b_0^2(x), b_1^2(x), b_2^2(x)\right]$$

$$b_0^2(x) = \binom{2}{0} x^0 (1-x)^{2-0} = (1-x)^2$$

$$b_1^2(x) = \binom{2}{1} x^1 (1-x)^{2-1} = 2x(1-x)$$

$$b_2^2(x) = \binom{2}{2} x^2 (1-x)^{2-2} = x^2$$

Therefore for n=2 $B(x) = \begin{pmatrix} (1-x)^2 \\ 2x(1-x) \\ x^2 \end{pmatrix}$

Similarly we can define B(x) for any value of n.

$$\frac{d}{dx} B(x) = D^{(1)} B(x)$$

$D^{(1)}$ is the (n+1)x(n+1) operational matrix.

$D^{(1)} = AVB^*$

Where A is the (n+1) x (n+1) matrix, V is the (n+1) xn matrix , B^* is nx (n+1) matrix

For example if n=2

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}_{3x3} \qquad V = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}_{3x2} \qquad A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^* = \begin{bmatrix} A_{[1]}^{-1} \\ A_{[2]}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & 1 \end{bmatrix}_{2x3}$$

Therefore by using $D^{(1)} = AVB^* = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 2 & 0 & -2 \\ 0 & 1 & 2 \end{bmatrix}$

Similarly we can find $D^{(1)}$ for any value of n.

By equation $D^{(1)} = AVB^*$, it is clear that

$$\frac{d^n \Phi(x)}{dx^n} = \left(D^{(1)}\right)^n B(x) \tag{9}$$

Where $n \in \mathrm{N}$ and the superscript, in $D^{(1)}$, denotes matrix powers. Thus

$$D^{(n)} = \left(D^{(1)}\right)^n \text{ , n=1,2,...} \tag{10}$$

Theorem 1. **[26]** *Let* $B(x)$ *be Bernstein vector defined in (4) and also suppose* $\alpha > 0$ *then*

$$D^{\alpha}B(x) \cong D^{(\alpha)}B(x). \tag{11}$$

3.Function Approximation

Consider the differential equation

$$F\left(x, y(x), Dy(x), D^2 y(x),... D^p y(x)\right) = 0 \tag{12}$$

With the initial or boundary conditions

$$H_i\left(y(\xi_i), y^{'}(\xi_i),... y^{(p)'}(\xi_i)\right) = d_i \quad i = 0,1,2\ldots,p \tag{13}$$

$$\xi_i \in [0,1] \ , \ i = 0,1,2\ldots,p$$

H_i are linear combination of $y(\xi_i), y^{'}(\xi_i),\ldots y^{(p)'}(\xi_i)$ and $y(x) \in L^2[0,1]$. It should be noted that F can be nonlinear in general. In order to use Bernstein polynomials for this problem.

In general, we approximate any function y(x) with first n+1 polynomials as

$$y(x) \approx \sum_{i=0}^{n} c_i b_i^n(x) = C^T B(x) \tag{14}$$

Where vector $C = [c_0,..., c_n]^T$ is unknown vector, $B(x) = \left[b_0^n(x), b_1^n(x),..., b_n^n(x)\right]^T$

Using Eqs. (10) and (14) we have

$$Dy(x) \approx C^T D^{(1)} B(x) \ , \ D^2 y(x) \approx C^T D^{(2)} B(x), \ldots \ , \ D^p y(x) \approx C^T D^{(p)} B(x) \tag{15}$$

By substituting these equations in Eq. (12) we get

$$F\left(x, C^T B(x), C^T D^{(1)} B(x), C^T D^{(2)} B(x),.. C^T D^{(P)} B(x)\right) = 0 \tag{16}$$

Similarly, substituting Eq. (14) in Eq. (13) yields

$$H_i\left(C^T B(\xi_i), C^T D^{(1)} B(\xi_i),..., C^T D^{(p)} B(\xi_i)\right) = d_i, i = 0,1,..., p \tag{17}$$

To find the solution $y(x)$, we first collocate Eq. (16) at $(n-p)$ points. For suitable collocation points we use

$$x_i = \left(\frac{1}{2}\right)\left(\cos\left(\frac{i\pi}{n}\right) + 1\right), i = 1,..., n-p$$

These equations together with equation (17) generate $(n+1)$ algebraic equations which can be solved to find unknown coefficients c_i , i=0,1,...,n. Consequently the unknown function $y(x)$ given in Eq. (14) can be calculated.

4.Numerical Examples

Example 4.1

Consider the following boundary value problem

$$y'' + \frac{\alpha}{x} y' - \beta x^{\alpha+\beta-2}\left[(\alpha+\beta-1)+\beta x^{\beta}\right] y = 0 \ , \tag{18}$$

with boundary conditions

$$y(0)=1, y(1)=e \tag{19}$$

Eq.(18) has the exact solution of the initial value problem for $\alpha = 1, \beta = 3$ is $y(x) = e^{x^{\beta}} = e^{x^3}$.

Applying the method developed in section 2 and 3 for M=2

We have $y(x) = c_0 b_0^2(x) + c_1 b_1^2(x) + c_2 b_2^2(x) = C^T B(x)$

Here we have $y(x) = C^T B(x)$, $y'(x) = C^T DB(x)$, $y''(x) = C^T D^2 B(x)$

Using the above values and put $\alpha = 1, \beta = 3$ in Eq.(18) , we obtain

$$C^T D^2 B(x) + \frac{1}{x}\left(C^T DB(x)\right) - C^T B(x)\left[6x^2 + 9x^5\right] = 0 \tag{20}$$

From Eq.(10) we have

$$D^{(1)} = \begin{pmatrix} -2 & -1 & 0 \\ 2 & 0 & -2 \\ 0 & 1 & 2 \end{pmatrix} \quad D^{(2)} = \begin{pmatrix} 2 & 2 & 2 \\ -4 & -4 & -4 \\ 2 & 2 & 2 \end{pmatrix}$$

By collocating Eq(20) at $x_1 = \frac{1}{2}$ we obtain

$$-57c_0 - 562c_1 + 455c_2 = 0 \tag{21}$$

Applying the boundary conditions from Eq.(19) we get

$$c_0 = 1 \tag{22}$$

$$c_2 = 2.7183 \tag{23}$$

Eqs.(21) – (23) can be solved for the unknown coefficients of the vector C ,we obtain

$c_0 = 1$, $c_1 = 2.0993$, $c_2 = 2.7183$

Consequently

$$y(x) = (1, 2.0993, 2.7183)\begin{pmatrix} (1-x)^2 \\ 2x(1-x) \\ x^2 \end{pmatrix} = (-0.4803)x^2 + (2.1986)x + 1$$

For M=3,

we have $c_0 = 1, c_1 = 2.12334$, $c_2 = 0.89074$, $c_3 = 2.7183$

then $y(x) = (5.4161)x^3 - (7.06781)x^2 + 3.37001\,x + 1$

For M=4, $y(x) = (12.3088)x^4 - (17.40976)x^3 - (8.5137)x^2 - 1.69448\,x + 1$

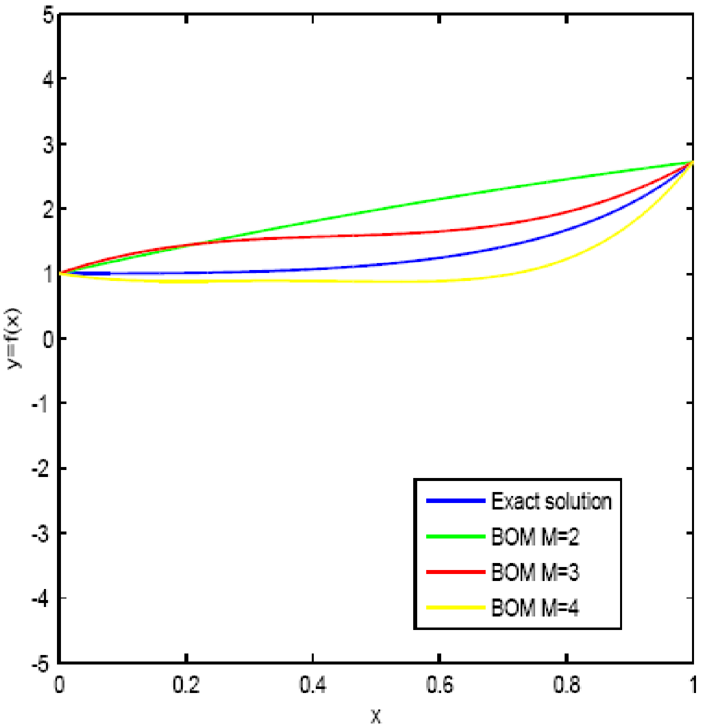

Figure 1: Numerical Solutions of Example 4.1 For Various Values of M

Table 1: Comparison Between The Exact and BOM For Example 4.1

x	Exact	BOM M=2	BOM M=3	BOM M=4
0.1	1.001000500	1.215057	1.2717390	0.89951012
0.2	1.008032086	1.420508	1.4346184	0.88206800
0.3	1.027367803	1.616353	1.5211348	0.88752676
0.4	1.066092399	1.802592	1.5637848	0.88528064
0.5	1.133148453	1.979225	1.5950650	0.87426500
0.6	1.241102379	2.146252	1.6474720	0.88295632
0.7	1.409168762	2.303673	1.7535024	0.96937220
0.8	1.668625110	2.451488	1.9456528	1.22107136
0.9	2.073006564	2.589697	2.2564198	1.75515364
1	2.718281828	2.718300	2.7183000	2.71826000

Table 2: Errors Between The Exact and BOM For Example 4.1

x	BOM M=2	BOM M=3	BOM M=4
0.1	0.214056500	0.270738500	0.101490380
0.2	0.412475914	0.426586314	0.125964086
0.3	0.588985197	0.493766997	0.139841043
0.4	0.736499601	0.497692401	0.180811759
0.5	0.846076547	0.461916547	0.258883453
0.6	0.905149621	0.406369621	0.358146059
0.7	0.894504238	0.344333638	0.439796562
0.8	0.782862890	0.277027690	0.447553750
0.9	0.516690436	0.183413236	0.317852924
1	1.81715E-05	1.81715E-05	2.18285E-05

Example 4.2

Consider the above boundary value problem with $\alpha = 0.5, \beta = 5$

$$y'' + \frac{\alpha}{x} y' - \beta x^{\alpha+\beta-2}\left[(\alpha+\beta-1)+\beta x^{\beta}\right] y = 0 \tag{24}$$

with boundary conditions

$$y(0) = 1, y(1) = e \tag{25}$$

Eq.(24) has the exact solution of the initial value problem for $\alpha = 0.5, \beta = 5$ is $y(x) = e^{x^{\beta}} = e^{x^{5}}$.

Boundary value problem by applying the method described in section 2 and 3 with M=2,

We approximate solution as $y(x) = c_0 b_0^2(x) + c_1 b_1^2(x) + c_2 b_2^2(x) = C^T B(x)$

By collocating Eq(24) at $x_1 = \frac{1}{2}$ we obtain

$$-0.51437\, c_0 - 4.52874\, c_1 + 3.48563\, c_2 = 0 \tag{26}$$

Applying the boundary conditions from Eq.(25) we get

$$c_0 = 1 \tag{27}$$

$$c_1 = 2.7183 \tag{28}$$

Eqs.(26) – (28) can be solved for the unknown coefficients of the vector C ,we obtain

$$c_0 = 1, \qquad c_1 = 1.97861\ , \quad c_2 = 2.7183$$

Thus $y(x) = (-0.23902\ )x^2 + (1.95742\ )x + 0.9999$

For M=3,

$$y(x) = (5.20808)x^3 - (6.62192)x^2 + 3.13214x + 1$$

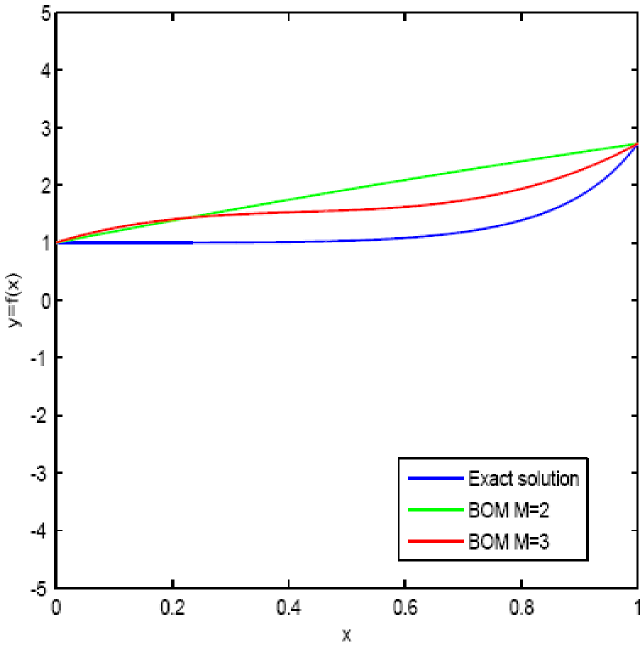

Figure 2: Numerical Solutions of Example4.2 For Various Values of M

Table 3: Comparison Between The Exact and BOM For Example 4.2

x	Exact	BOM M=2	BOM M=3
0	1.000000000	0.9999999	1.00000000
0.1	1.000010000	1.1933418	1.25220288
0.2	1.000320051	1.3819132	1.40321584
0.3	1.002432955	1.5657042	1.48428736
0.4	1.010292608	1.7447148	1.52666592
0.5	1.031743407	1.9189450	1.56160000
0.6	1.080863220	2.0883948	1.62033808
0.7	1.183019419	2.2530642	1.73412864
0.8	1.387744823	2.4129532	1.93422016
0.9	1.804872586	2.5680618	2.25186112
1.0	2.718281828	2.7183900	2.71830000

Table 4: Errors Between The Exact and BOM For Example 4.2

x	BOM M=2	BOM M=3
0	0.00001000	0
0.1	0.19333180	0.25219288
0.2	0.38159314	0.40289578
0.3	0.56327124	0.48185440
0.4	0.73442219	0.51637331
0.5	0.88720159	0.52985659
0.6	1.00753158	0.53947486
0.7	1.07004478	0.55110922
0.8	1.02520837	0.54647533
0.9	0.76318921	0.44698853
1	0.00010817	1.81715E-05

Example 4.3

Consider the non-linear boundary value problem

$$y''(x)+\frac{y'(x)}{x}+y(x)=4-9x+x^2-x^3 \tag{29}$$

with boundary conditions

$$y(0)=0,\ y(1)=0 \tag{30}$$

The exact solution for Eq.(29) is $y(x)=x^2-x^3$

By applying the method described in the section 2 and 3, the solution for various values of m are given below

M=2 the solution is $y(x)=-0.25x^2+0.25x$

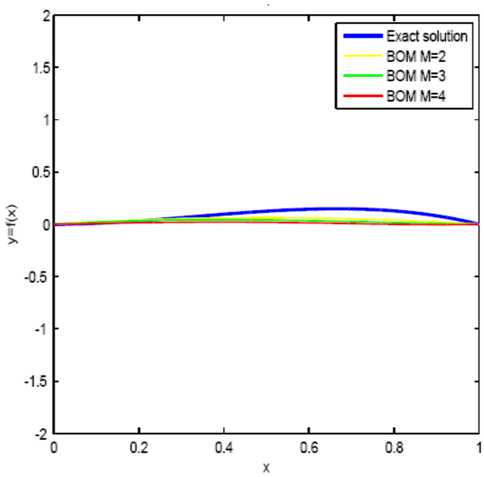

Figure 3: Numerical Solutions of Example 4.3 For Various Values Of M

Table 5: Comparison between the exact and BOM for Example 4.3

x	Exact	BOM M=2
0	0	0
0.1	0.0090	0.0225
0.2	0.0320	0.0400
0.3	0.0630	0.0525
0.4	0.0960	0.0600
0.5	0.1250	0.0625
0.6	0.1440	0.0600
0.7	0.1470	0.0525
0.8	0.1280	0.0400
0.9	0.0810	0.0225
1	0	0

Table 6: Errors Between The Exact and BOM For Example 4.3

x	BOM M=2
0	0
0.1	0.0135
0.2	0.0080
0.3	0.0105
0.4	0.0360
0.5	0.0625
0.6	0.0840
0.7	0.0945
0.8	0.0880
0.9	0.0585
1	0

Example 4.4

Consider the following initial value problem $4y'' - 2y'^2 + y = 0$ (31)

Subject to the initial conditions y(0)= -1 and y'(0)=-1 (32)

which has the following analytical solution $y(x) = \frac{x^2}{8} - 1$

We solve this initial value problem Eq.(31) by applying the method described in Section 2 and 3 using Bernstein operational matrices of derivatives with M=2 we have

$c_0 = -1, c_1 = -1, c_2 = -0.875$ and get

$y = c^T B(x) = \frac{x^2}{8} - 1$ which is the exact solution.

It is clear that in Example 4.4, our proposed algorithm is rapidly convergent to the exact value.

For a small value of M=2, we reach the exact solution .

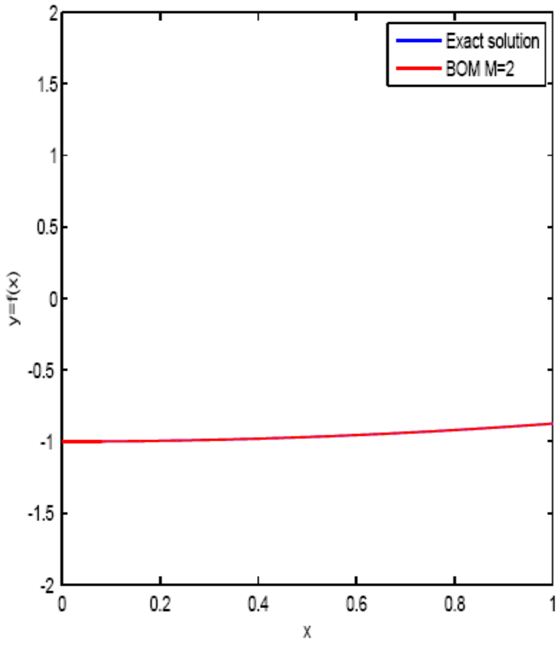

Figure 4: Numerical Solutions of Example-4 For Value Of M=2

Table 7: Comparison Between The Exact and BOM For Example 4.4

x	**Exact**	**BOM M=2**
0	-1	-1
0.1	-0.99875	-0.99875
0.2	-0.99500	-0.99500
0.3	-0.98875	-0.98875
0.4	-0.98000	-0.98000
0.5	-0.96875	-0.96875
0.6	-0.95500	-0.95500
0.7	-0.93875	-0.93875
0.8	-0.92000	-0.92000
0.9	-0.89875	-0.89875
1	-0.87500	-0.87500

Table 8: Errors between the exact and BOM for Example 4.4

x	**BOM M=2**
0	0
0.1	0
0.2	0
0.3	0
0.4	0
0.5	0
0.6	0
0.7	0
0.8	0
0.9	0
1	0

The accuracy of the results is estimated by error function $E = |y_{exact} - y_{BOM}|$. The results are shown in Tables (See Tables 1-8). In order to assess the advantages, efficiency and the accuracy of the BOM for solving the nonlinear differential equations, we use our method to solve another nonlinear differential equation, whose exact solutions are known. Results in the Tables 1-8 show that the BOM agrees with the exact solutions. When solving the non-periodic problems, the proposed method has the superiorities (the calculation is easy implementation, and the approximation effect is better) which the CAS wavelet cannot be compared with it. For larger M, the numerical results have good agreement with the exact solutions.

All the numerical experiments presented in this section were computed in double precision with some MATLAB codes on a personal computer System Vostro 1400 Processor x86 Family 6 Model 15 Stepping 13 Genuine Intel ~1596 Mhz.

5.Conclusion

We conferred a general formulation for the Bernstein operational matrix algorithm (BOMA) for solving boundary value problems arising in science and engineering. The numerical results given in the previous section demonstrate the good accuracy of these algorithms. For smaller m, we get the numerical results closer to the real values. The power of the manageable technique is confirmed. The obtained numerical values are comparing favorably with the analytic ones. It provides more realistic series solutions that converge very speedily in real physical problems.

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