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Existence of Fuzzy Solutions for Second Order Boundary Value Problems with Integral Boundary Conditions

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Abstract

In this paper, we prove the existence and uniqueness of fuzzy solutions for a class of second order nonlinear boundary value problems with integral boundary conditions by using the Banach fixed point theorem.

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1. Introduction

The theory of fuzzy sets, fuzzy valued functions and the necessary calculus of fuzzy functions have been investigated in the monograph by Lakshmikantham and Mohapatra [17] and the references cited therein. Recently, there have been new advances in the theory of fuzzy differential equations [29, 26, 27], fuzzy integral equations [23, 28, 30] and fuzzy integrodifferential equations [3, 6, 7, 25].

The topics of fuzzy integral equations which attracted growing interest in literature due to its application in relation to fuzzy control, have been developed in recent years.

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Most mathematical models used in many problems of physics, biology, chemistry and engineering are based on integral equations.

The fixed point theorems like the BanachFLs principle and the DarboFLs theorem were the tools used to prove, on one hand the existence and on the other hand the existence and uniqueness of the solution of fuzzy integral equations (see [5, 9, 8, 10, 21, 20, 22, 31]). Mordeson and Newman [18] started the study in the topic of fuzzy integral equations. Sufficient conditions for the boundedness of the solutions of fuzzy integral equations were obtained in [15, 19]. A distinct study on the existence of a unique solution for fuzzy Fredholm integral equations is carried out in [18]. Some applications of the fuzzy volterra equations to control models with fuzzy uncertainties are presented in [14].

The theory of boundary-value problems with integral boundary conditions for ordinary differential equations arises in different areas of applied mathematics and physics. For example, heat conduction, chemical engineering, underground water flow, thermo-elasticity, and plasma physics can be reduced to nonlocal problems with integral boundary conditions. Integral boundary conditions appear in population dynamics [13] and cellular systems [2].

Fuzzy boundary value problems with integral boundary conditions constitute a very interesting and important class of problems. They include two, three, multipoint and nonlocal boundary value problems as special cases. Benchohra et al [4] studied the existence of fuzzy solutions for multipoint boundary value problems. Ahmad and Nieto [1] analysed the existence results for nonlinear boundary value problems of fractional integrodifferential equations with integral boundary conditions. Belarbi and Benchohra [11] examined the existence results for nonlinear boundary-value problems with integral boundary conditions. Benchohra et al [12] determined the method of upper and lower solutions for second order differential inclusions with integral boundary conditions.

This paper is concerned with the existence and uniqueness of fuzzy solutions for more general boundary value problems for second order differential equations with integral boundary conditions of the form

$$x''(t) = f(t, x(t)), \quad \text{for all } t \in [0, 1], \tag{1}$$

$$x(0) - k_1 x'(0) = \int_0^1 h_1(x(s))ds, \tag{2}$$

$$x(1) + k_2 x'(1) = \int_0^1 h_2(x(s))ds, \tag{3}$$

where $f : [0, 1] \times E^n \to E^n$ is a continuous function, E^n is the set of all upper semi-continuous, convex, normal fuzzy numbers with α-level, $h_i : E^n \to E^n (i = 1, 2)$ are continuous functions and k_i $(i = 1, 2)$ are nonnegative constants. The conditions (2)–(3) used in this paper are of special interest in the study of boundary value problems than those considered in the previous literatures. Our approach here is based on the Banach contraction principle.

This paper is organized as follows: In Section 2, we will recall briefly some basic definitions and preliminary facts which will be used in the later sections. In Section 3, we prove the existence of fuzzy solutions for boundary value problems with integral boundary conditions. Finally, in Section 4, we give an example to show the advantage gained by the fuzzification of the differential operator in the differential equation to illustrate the theory presented in the previous sections.

2. Preliminaries

In this section, we introduce notations, definitions and preliminary facts which are used throughout this paper.

Definition 2.1. {fuzzy set} Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A : X \to [0, 1]$ and $A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

The value zero is used to represent complete non-membership, the value one is used to represent complete membership, and values in between are used to represent intermediate degrees of membership. The mapping A is called the membership function of fuzzy set A.

Example 2.2. The membership function of the fuzzy set of real numbers "close to one" can be defined as

$$A(t) = exp(-\beta(t-1)^2), \text{ where } \beta \text{ is a positive real number .}$$

Example 2.3. Let the membership function for the fuzzy set of real numbers "close to zero" defined as follows

$$B(t) = \frac{1}{1+x^3}$$

Using this function, we can determine the membership grade of each real number in this fuzzy set, which signifies the degree to which that number is close to zero. For instance, the number 3 is assigned a grade of 0.035, the number 1 a grade of 0.5 and the number 0 a grade of 1.

Let $CC(R^n)$ denotes the set of all nonempty compact, convex subsets of R^n. Denote by, $E^n = \{u : R^n \to [0, 1]$ such that they satisfy $(i) - (iv)$ mentioned below$\}$,

(i) u is normal i.e., there exists an $x_0 \in R^n$ such that $u(x_0) = 1$;

(ii) u is fuzzy convex, that is for $x, z \in R^n$ and $0 < \lambda \leq 1$

$$u(\lambda x + (1-\lambda)z) \geq \min\{u(x), u(z)\},$$

(iii) u is upper semicontinuous;

(iv) $[u]^0 = \overline{\{x \in R^n : u(x) > 0\}}$ is compact.

For $0 < \alpha \leq 1$, we denote $[u]^\alpha = \{x \in R^n : y(x) \geq \alpha\}$. Then from $(i) - (iv)$, it follows that the α-level sets $[u]^\alpha \in CC(R^n)$. If $g : R^n \times R^n \to R^n$ is a function, then by using Zadeh's extension principle we can extend g to $E^n \times E^n \to E^n$ by the equation

$$[g(u, v)(z) = \sup_{z=g(x,y)} \min \{u(x), v(y)\}.$$

It is well known that $[g(u, v)]^\alpha = g([u]^\alpha, [v]^\alpha)$ for all $u, v \in E^n, 0 \leq \alpha \leq 1$ and continuous function g. Further, we have $[u + v]^\alpha = [u]^\alpha + [v]^\alpha$, $[ku]^\alpha = k[u]^\alpha$, where $u, v \in E^n, k \in R, 0 \leq \alpha \leq 1$.

Let A, B be two nonempty bounded subsets of R^n. The distance between A and B is defined by the Hausdorff metric

$$H_d(A, B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} \|a - b\|, \sup_{b \in B} \inf_{a \in A} \|a - b\| \right\}$$

where $\|.\|$ denotes the usual Euclidean norm in R^n. Then $(CC(R^n), H_d)$ is a complete and seperable metric space [24]. We define the supremum metric d_∞ on E^n by

$$d_\infty(u, v) = \sup_{0<\alpha\leq 1} H_d([u]^\alpha, [v]^\alpha)$$

for all $u, v \in E^n$. (E^n, d_∞) is a complete metric space. The supremum metric H_1 on $C([0, 1], E^n)$ is defined by

$$H_1(x, y) = \sup_{t \in J} d_\infty(x(t), y(t)).$$

$(C([0, 1], E^n), H_1)$ is a complete metric space.

Definition 2.4. **[24]** A mapping $f : [0, 1] \to E^n$ is strongly measurable if, for all $\alpha \in [0, 1]$ the set-valued map $f_\alpha : [0, 1] \to CC(R^n)$ defined by $f_\alpha(t) = [f(t)]^\alpha$ is Lebesgue measurable when $CC(R^n)$ has the topology induced by the Hausdorff metric d.

Definition 2.5. **[24]** A map $f : [0, 1] \to E^n$ is called levelwise continuous at $t_0 \in [0, 1]$ if the multi-valued map $f_\alpha(t) = [f(t)]^\alpha$ is continuous at $t = t_0$ with respect to the Hausdorff metric d for all $\alpha \in [0, 1]$.

A map $f : [0, 1] \to E^n$ is said to be integrably bounded if there is an integrable function $h(t)$ such that $\|x(t)\| \leq h(t)$ for every $x(t) \in f_0(t)$.

Definition 2.6. Let $f : [0, 1] \to E^n$. The integral of f over [0, 1], denoted $\int_0^1 f(t)dt$ is defined by the equation

$$\begin{aligned}\left[\int_0^1 f(t)dt\right]^\alpha &= \int_0^1 f_\alpha(t)dt \\ &= \left\{\int_0^1 v(t)dt \;\middle|\; v : [0, 1] \to R^n \quad \text{is a measurable selection for} \quad f_\alpha\right\}\end{aligned}$$

for all $\alpha \in (0, 1]$.

A strongly measurable and integrably bounded map $f : [0, 1] \to E^n$ is said to be integrable over [0, 1], if $\int_0^1 f(t)dt \in E^n$.

If $f : [0, 1] \to E^n$ is strongly measurable and integrably bounded, then f is integrable.

Definition 2.7. A map $f : [0, 1] \to E^n$ is called differentiable at $t_0 \in [0, 1]$ if there exists a $f^{'}(t_0) \in E^n$ such that the limits

$$\lim_{h \to 0+} \frac{f(t_0 + h) - f(t_0)}{h} \quad \text{and} \quad \lim_{h \to 0+} \frac{f(t_0) - f(t_0 - h)}{h}$$

exist and are equal to $f^{'}(t_0)$. Here the limit is taken in the metric space (E^n, H_d). At the end points of [0, 1], we consider only the one-sided derivatives.

If $f : [0, 1] \to E^n$ is differentible at $t_0 \in [0, 1]$, then we say that $f^{'}(t_0)$ is the fuzzy derivative of $f(t)$ at the point t_0 or the Hukuhara derivative of $f(t)$ at t_0, usually denoted by $D_H f(t_0)$. For the concepts of fuzzy measurability and fuzzy continuity we refer to [16].

Definition 2.8. A map $f : [0, 1] \times E^n \to E^n$ is called levelwise continuous at point $(t_0, x_0) \in [0, 1] \times E^n$ provided, for any fixed $\alpha \in [0, 1]$ and arbitrary $\epsilon > 0$, there exists a $\delta(\epsilon, \alpha) > 0$ such that

$$H_d([f(t, x)]^\alpha, [f(t, x_0)]^\alpha) < \epsilon$$

whenever $|t - t_0| < \delta(\epsilon, \alpha)$ and $H_d([x]^\alpha, [x_0]^\alpha) < \delta(\epsilon, \alpha)$ for all $t \in [0, 1], \quad x \in E^n$.

3. The Main Result

In this section, we are concerned with the existence and uniqueness of solutions for the problem (1)–(3).

Definition 3.1. A function $x \in C^2([0, 1], E^n)$ is said to be the solution of (1)–(3) if x satisfies the equation $x^{''}(t) = f(t, x(t))$ on [0, 1] and the conditions (2)–(3).

We need the following auxiliary result. Its proof uses a standard argument.

Lemma 3.2. For any $\rho_1(t), \rho_2(t) \in C([0, 1], E^n)$, the nonhomogeneous linear problem

$$x^{''}(t) = f(t, x(t)), \quad \text{for all } \ t \in [0, 1],$$

$$x(0) - n_1 x^{'}(0) = \int_0^1 \rho_1(s)ds, \quad x(1) + n_2 x^{'}(1) = \int_0^1 \rho_2(s)ds,$$

has a unique solution $x \in C^2([0, 1], E^n)$ given by

$$x(t) = \Lambda(t) + h(t, x(t)) \int_0^1 G(t, s) f(s, x(s)) ds + \int_0^1 g(t, s, x(s)) ds,$$

where

$$\Lambda(t) = \frac{1}{1 + n_1 + n_2} \{(1 - t + n_2) \int_0^1 \rho_1(s) ds + (n_1 + t) \int_0^1 \rho_2(s) ds\}$$

is the unique solution of the problem

$$x''(t) = 0, \quad \text{for all} \quad t \in [0, 1],$$

$$x(0) - n_1 x'(0) = \int_0^1 \rho_1(s) ds, \quad x(1) + n_2 x'(1) = \int_0^1 \rho_2(s) ds,$$

and

$$G(t, s) = \frac{-1}{n_1 + n_2 + 1} \begin{cases} (n_1 + t)(1 - s + n_2), & 0 \le t < s \le 1, \\ (n_1 + s)(1 - t + n_2), & 0 \le s < t \le 1 \end{cases}$$

is the Green's function of the homogeneous problem.

Let us introduce the following hypotheses which are assumed hereafter:

Theorem 3.3. Assume that

(H1) There exists a constant d such that

$$H_d([h(t, x(t)) f(s, x(s))]^\alpha, h(t, y(t)) f(s, y(s))]^\alpha) \le d H_d([x(s)]^\alpha, [y(s)]^\alpha),$$

for all $t \in [0, 1]$ and all $x, y \in E^n$.

(H2) There exists a constant d_1 such that

$$H_d([h_1(x(s))]^\alpha, ([h_1(y(s))]^\alpha) \le d_1 H_d([x(s)]^\alpha, [y(s)]^\alpha).$$

(H3) There exists a constant d_2 such that

$$H_d([h_2(x(s))]^\alpha, ([h_2(y(s))]^\alpha) \le d_2 H_d([x(s)]^\alpha, [y(s)]^\alpha).$$

(H4) There exists a constant d_3 such that

$$H_d([g(t, s, x(s))]^\alpha, [g(t, s, y(s))]^\alpha \le d_3 H_d([x(s)]^\alpha, [y(s)]^\alpha).$$

If
$$\frac{1+n_2}{1+n_1+n_2}d_1 + d_2(1+n_1) + d \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)| < 1,$$
then the BVP (1) − (3) has a unique fuzzy solution on [0, 1].

Proof. Transform the problem into a fixed point problem. It is clear that the solutions of the problem (1)–(3) are fixed points of the operator $\Phi : C([0,1], E^n) \to C([0,1], E^n)$ defined by:

$$\Phi(x)(t) = \Lambda(x)(t) + h(t, x(t)) \int_0^1 G(t,s) f(s, x(s))ds + \int_0^1 g(t,s,x(s))ds$$

with

$$\Lambda(x)(t) = \frac{1}{1+n_1+n_2}(1-t+n_2)\int_0^1 h_1(x(s))ds + (n_1+t)\int_0^1 h_2(x(s))ds.$$

we shall show that Φ is a contraction operator. Indeed, consider $x, y \in C([0,1], E^n)$ and $\alpha \in (0, 1]$, then

$$\begin{aligned}
&H_d([\Phi(x)(t)]^\alpha, [\Phi(y)(t)]^\alpha)\\
&= H_d\Bigg(\Bigg[\frac{1-t+n_2}{1+n_1+n_2}\int_0^1 h_1(x(s))ds + (n_1+t)\int_0^1 h_2(x(s))ds + h(t,x(t))\\
&\quad\times \int_0^1 G(t,s)f(s,x(s))ds + \int_0^1 g(t,s,x(s))ds\Bigg]^\alpha,\\
&\quad\Bigg[\frac{1-t+n_2}{1+n_1+n_2}\int_0^1 h_1(y(s))ds + (n_1+t)\int_0^1 h_2(y(s))ds\\
&\quad + h(t,y(t))\int_0^1 G(t,s)f(s,y(s))ds + \int_0^1 g(t,s,y(s))ds\Bigg]^\alpha\Bigg)\\
&\le H_d\left(\left[\frac{1-t+n_2}{1+n_1+n_2}\int_0^1 h_1(x(s))ds\right]^\alpha, \left[\frac{1-t+n_2}{1+n_1+n_2}\int_0^1 h_1(y(s))ds\right]^\alpha\right)\\
&\quad + H_d\left(\left[(n_1+t)\int_0^1 h_2(x(s))ds\right]^\alpha, \left[(n_1+t)\int_0^1 h_2(y(s))ds\right]^\alpha\right)\\
&\quad + H_d\Bigg(\left[h(t,x(t))\int_0^1 G(t,s)f(s,x(s))ds\right]^\alpha,\\
&\qquad \left[h(t,y(t))\int_0^1 G(t,s)f(s,y(s))ds\right]^\alpha\Bigg)\\
&\quad + H_d\left(\left[\int_0^1 g(t,s,x(s))ds\right]^\alpha, \left[\int_0^1 g(t,s,y(s))ds\right]^\alpha\right)
\end{aligned}$$

$$\leq \frac{1-t+n_2}{1+n_1+n_2} H_d\left(\left[\int_0^1 h_1(x(s))ds\right]^\alpha, \left[\int_0^1 h_1(y(s))ds\right]^\alpha\right)$$
$$+ (n_1+t) H_d\left(\left[\int_0^1 h_2(x(s))ds\right]^\alpha, \left[\int_0^1 h_2(y(s))ds\right]^\alpha\right)$$
$$+ \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)| H_d\left(\left[\int_0^1 h(t,x(t)) f(s,x(s))ds\right]^\alpha,\right.$$
$$\left.\left[\int_0^1 h(t,y(t)) f(s,y(s))ds\right]^\alpha\right)$$
$$+ H_d\left(\left[\int_0^1 g(t,s,x(s))ds\right]^\alpha, \left[\int_0^1 g(t,s,y(s))ds\right]^\alpha\right)$$
$$\leq \frac{1-t+n_2}{1+n_1+n_2} \int_0^1 H_d([h_1(x(s))]^\alpha, [h_1(y(s))]^\alpha)ds$$
$$+ (n_1+t) \int_0^1 H_d([h_2(x(s))]^\alpha, [h_2(y(s))]^\alpha)ds$$
$$+ \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)| \int_0^1 H_d([h(t,x(t)) f(s,x(s))]^\alpha,$$
$$[h(t,y(t)) f(s,y(s))]^\alpha)ds + \int_0^1 H_d((x(s))^\alpha, (y(s))^\alpha)$$
$$\leq \frac{1+n_2}{1+n_1+n_2} d_1 \sup_{\alpha\in[0,1]} d_\infty(x(s),y(s)) + (n_1+1) d_2 d_\infty(x(s),y(s))$$
$$+ d \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)| d_\infty(x(s),y(s)) + d_3 d_\infty(x(s),y(s))$$
$$\leq \left(\frac{1+n_2}{1+n_1+n_2} d_1 + d_2(n_1+1) + d_3 + d \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)|\right) d_\infty(x(s),y(s))$$
$$\leq \left(\frac{1+n_2}{1+n_1+n_2} d_1 + d_2(n_1+1) + d_3 + d \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)|\right) H_1(x,y).$$

Hence,

$$H_1(\Phi(x), \Phi(y))$$
$$\leq \left(\frac{1+n_2}{1+n_1+n_2} d_1 + d_2(n_1+1) + d_3 + d \sup_{(t,s)\in[0,1]\times[0,1]} |G(t,s)|\right) H_1(x,y).$$

So, Φ is a contraction and thus, by Banach fixed point theorem, Φ has a unique fixed point which is the solution to (1)–(3). ■

4. Example

Example 4.1. In this section, we present an example to show the advantage gained by the fuzzification of the differential operator in the differential equation.

Consider the crisp initial value problem with unknown initial value x_0, that is,

$$x' = -x, \quad x(0) = x_0 \in [-1, 1] \tag{4}$$

The solution of problem (4) when restricted to the interval $[-1, 1]$ is

$$x(t) = [-e^t, e^{-t}], \quad t \geq 0.$$

The fuzzy differential equation corresponding to (4) in E^1 is

$$D_H x = -x \quad x(0) = x_0 = [-1, 1], \quad x_0 \in E^1. \tag{5}$$

Suppose that

$$[x]^\alpha = [x_1^\alpha, x_2^\alpha], [D_H x]^\alpha = \left[\frac{dx_1^\alpha}{dt}, \frac{dx_2^\alpha}{dt}\right]$$

are $\alpha-$level sets for $0 \leq \alpha \leq 1$. By extension principle, (5) becomes

$$\frac{dx_1^\alpha}{dt} = -x_2^\alpha, \frac{dx_2^\alpha}{dt} = -x_1^\alpha, \quad 0 \leq \alpha \leq 1. \tag{6}$$

The solution of (6) is given by

$$x_1^\alpha(t) = -e^t, \quad x_2^\alpha(t) = e^t, \quad t \geq 0$$

and therefore the fuzzy function $x(t)$ solving (5) is

$$x(t) = [-e^t, e^t], \quad t \geq 0,$$

which shows that the diameter $diam(x(t)) \to \infty$ as $t \to \infty$. This may be interpreted as the increasing of incertainty to go by the time, which is, infact, reasonable.

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