International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 7 (2015) pp. 17601-17614

http://www.ripublication.com

Some Striking Characterizations of Structural Properties of Intuitionistic Fuzzy h-Ideals in Γ-Hemiring

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Abstract

In this paper the concept of intuitionistic fuzzy h-ideals in Γ-hemiring is introduced and some striking characterizations of structural properties of intuitionistic fuzzy h-ideals in Γ-hemiring have been discussed.

Mathematical Subject Classification: 08A72, 16Y60, 16Y99

Keywords: Γ-hemiring, intuitionistic fuzzy h-ideal.

1. Introduction

Ideals of hemirings have a significant role to play in the structure theory and they are instrumental in fulfilling scores of purposes. But the specific issue is that, they do not, in general, coincide with the usual ring ideals. Many results in rings apparently have no analogues in hemi rings using only ideals. Informal applications, hemirings have their utilitarian importance in automation and formal languages. It is universally only known that the set of regular languages does form the "*star, semirings*". The introduction of fuzzy sets by L.A.Zadeh[15]triggered an academic revolution and the fuzzy set theory has become, over the years, the heart and soul of several applications in the royal domains of mathematics and other relevant fields. The idea of"Intuitionistic Fuzzy Set" was first published by K.T.Atanassov[1] as a generalization of the notion of fuzzy set. Jun and Lee [8]went a little further and applied the concept of fuzzy sets to the theory of Γ-rings. The notion of Γ-semiring was introduced by Rao[11]which, in course of time, gained momentum and included ternary semirings to provide algebraic home to non-positive cones of totally ordered rings. Henriksen[5], Lizuka [6] and La Torre [9]dwelled deep in the study of h-ideals and k-ideals in hemirings to amend the gap between ring ideals and semiring ideals. These concepts have been extended to Γ-semiring by Rao[11], Dutta and Sardar[2].Jun et al[7] to study the ideals in hemirings in terms of fuzzy subsets. A characterization of an h-hemiregular hemiring in terms of a fuzzy h-ideal had been discussed in detail by Zhan et al [16]. Some salient properties of fuzzy h-ideals in Γ-hemirings had been studied by Sujit Kumar et al[13].The notion of intuitionistic fuzzy h-ideals in Γ-hemirings had been discussed Ezhilmaran et al [4]in the light of the previous findings**.** In this Paper different structural characteristic properties of intuitionistfuzzy h- ideals in Γ-hemiring have been discussed and debated verbally.

2.Preliminaries

Definition 2.1

A hemiring (respectively semiring) is a nonemptyset S on which operations addition and multiplication have been defined such that (S, +) is a commutative monoid with identity 0, (S, .) is a semigroup (respectively monoid with identity 1_S) Multiplication distributes over addition from either side, $1_S \neq 0$ and $0_s = 0 = S_0$ for all $s \in S$.

Definition 2.2

Let S and Γ be two additive commutative semigroupswithzero. Then S is called a Γ-hemiring if there exists a mapping $S \times \Gamma \times S \to S\big((a, \alpha, b) \to a\alpha b\big)$ satisfying the following conditions:

1. $(a + b)\alpha c = a\alpha c + b\alpha c$,
2. $a\alpha(b + c) = a\alpha b + a\alpha c$,
3. $a(\alpha + \beta)b = a\alpha b + a\beta b$,
4. $a\alpha(b\beta c) = (a\alpha b)\beta c$,
5. $0_s\alpha a = 0_s = a\alpha 0_s$,
6. $a_\Gamma b = 0_s = b0_\Gamma a$,

for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$. For simplification we write 0 instead of $0_S$and 0_Γ.

Example 2.3

Let S be the set of all $m \times n$ matrices over $\boldsymbol{z_0^-}$ (the set of all non-positive integers) and Γ be the set of all $n \times m$ matrices over $\boldsymbol{z_0^-}$ then S forms a Γ-hemiring with usual addition and multiplication of matrices.

Definition 2.4

A left ideal A of a Γ-hemi ring S is called a left h-ideal if for any $x, z \in S$and $a, b \in A$, $x + a + z = b + z \Rightarrow x \in A$. A right h-ideal is defined analogously.

Definition 2.5

Let S be a Γ-hemi ring. A proper h-ideal I of S is said to be prime if for any two h-ideals H and K of S, $H\Gamma K \subseteq I$ implies that either $H \subseteq I \ or\ K \subseteq I$.

Definition 2.6 [13]

If I is an h- ideal of a Γ-hemi ring S then the following condition are equivalent:

i. I is a prime h- ideal of S.
ii. If $a\Gamma S\Gamma b \subseteq I$then either $a \in I \ or\ b \in I$ where $a, b \in S$.

Definition 2.7

Let μ and θ be two fuzzy sets of a Γ-hemi ring S define a generalized h-product of μ and θ by

$$\mu \circ_h \theta(x) = \begin{cases} sup\{min\{[\mu(a_i) \wedge \mu(c_i)], [\mu(b_i) \wedge \mu(d_i)]\}\} \ if \ x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z \\ 0 \ if \ x \ cannot \ be \ expressed \ as \ above \end{cases}$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma$ for $i = 1,2,3, \dots\dots n$.

Definition 2.8 Let μ be a non-empty fuzzy subset of a Γ-hemiring S (i.e.μ(x) ≠ 0 for some x ∈S).Then μ is called a fuzzy left ideal (fuzzy right ideal) of S if

i. $\mu(x + y) \geq min[\mu(x), \mu(y)]$ and
ii. $\mu(x\gamma y) \geq \mu(y)$ (respectively $\mu(x\gamma y) \geq \mu(x)$) for all x, y ∈ S,$\gamma \in \Gamma$.

A fuzzy ideal of a Γ-hemiring S is a non-empty fuzzy subset of S which is a fuzzy left ideal as well as fuzzy right ideal of S. Note that if μ is a fuzzy left or right ideal of a Γ-hemiring S, then$\mu(0) \geq \mu(x)$ for all $x \in S$.

3. Structures of Intuitionistic Fuzzy h- Ideals in Γ- Hemiring

Definition 3.1

Let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be two intuitionistic fuzzy subsets of a Γ –hemiring S.Then the intuitionistic sum of A and B is defined to be the intuitionistic fuzzy set $A \oplus B = \langle \mu_{A\oplus B}, \nu_{A\oplus B} \rangle$ in S given by

$$\mu_{A\oplus B}(x) = \begin{cases} \bigvee_{x+a+z=b+z} \{\mu_A(a) \wedge \mu_B(b)\} \ \text{if} \ x + a + z = b + z \\ 0 \ \text{otherwise} \end{cases}$$

$$\nu_{A\oplus B}(x) = \begin{cases} \bigwedge_{x+a+z=b+z} \{\nu_A(a) \vee \nu_B(b)\} \ \text{if} \ x + a + z = b + z \\ 1 \ \text{otherwise} \end{cases}$$

Definition 3.2

Let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be an two intuitionistic fuzzy sets of aΓ-hemiring S. Define h-product of A and B by

$$\mu_{A\Gamma_h B}(x) = \begin{cases} \vee\{\mu_A(a_1) \wedge \mu_A(a_2), \mu_B(b_1) \wedge \mu_B(b_2)\}, \text{if} \ x + a_1\gamma b_1 + z = a_2\delta b_2 + z \\ 0 \ , \text{if x cannot be expressed as above} \end{cases}$$ &

$$\nu_{A\Gamma_h B}(x) = \begin{cases} \wedge\{\nu_A(a_1) \vee \nu_A(a_2), \nu_B(b_1) \vee \nu_B(b_2)\} \text{if} \ x + a_1\gamma b_1 + z = a_2\delta b_2 + z \\ 0 \ , \text{if x cannot be expressed as above} \end{cases}$$

Where$x, z, a_1, a_2, b_1, b_2 \in S$and $\gamma_i, \delta_i \in \Gamma for i = 1,2,3, \dots\dots n$.

Definition 3.3

$$\mu_{A\circ_h B}(x) = \begin{cases} \vee\{[\mu_A(a_1) \wedge \mu_A(a_2)], [\mu_B(b_1) \wedge \mu_B(b_2)]\}, \text{if} \ x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z \\ 0 \ , \text{if x cannot be expressed as above} \end{cases}$$

$$\nu_{A\circ_h B}(x) = \begin{cases} \{\wedge[\nu_A(a_1) \vee \nu_A(a_2)], [\nu_B(b_1) \vee \nu_B(b_2)]\}, \text{if } x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z \\ 0 \qquad , \text{if x cannot be expressed as above} \end{cases}$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma$.

Lemma 3.4

Let A, B be two intuitionistic fuzzy h-ideal of a Γ-hemiring S. Then $A\Gamma_h B \subseteq Ao_h B \subseteq A \cap B \subseteq A, B$

Proof

$$\mu_{Ao_hB}(x) = \vee\left[\left\{\{\mu_A(a_i) \wedge \mu_A(c_i)\} \underset{i}{\wedge} \{\mu_B(b_i) \wedge \mu_A(c_i)\}\right\}\right]$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$and $\gamma_i, \delta_i \in \Gamma\, for\ i = 1,2,3, \dots \dots n.$

$$\geq \vee[\{\{\mu_A(a_1) \wedge \mu_A(a_2)\}\ \{\mu_B(b_1) \wedge \mu_A(b_2)\}\}], \qquad x + a_1\gamma b_1 + z = a_2\delta b_2 + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma\, for\ i = 1,2,3, \dots \dots n.$

$$= \mu_{A\Gamma_hB}(x)$$

There fore $\mu_{A\Gamma_hB}(x) \subseteq \mu_{Ao_hB}(x)$ ------------------------- (1)

$$\nu_{Ao_hB}(x) = \wedge\left[\left\{\{\nu_A(a_i) \vee \nu_A(c_i)\} \underset{i}{\vee} \{\nu_B(b_i) \vee \nu_A(c_i)\}\right\}\right]$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma\, for\ i = 1,2,3, \dots \dots n.$

$$\leq \wedge\{(\nu_A(a_1) \vee \nu_A(a_2)), ((\nu_B(b_1) \vee \nu_B(b_2))\}, \ x + a_1\gamma b_1 + z = a_2\delta b_2 + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma\, for\ i = 1,2,3, \dots \dots n..$

$$= \nu_{A\Gamma_hB}(x)$$

There fore $\nu_{A\Gamma_hB}(x) \subseteq \nu_{Ao_hB}(x)$--------------------------- (2)

$$\mu_{Ao_hB}(x) = \vee\left[\left\{\{\mu_A(a_i) \wedge \mu_A(c_i)\} \underset{i}{\wedge} \{\mu_B(b_i) \wedge \mu_A(c_i)\}\right\}\right]$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma\, for\ i = 1,2,3, \dots \dots n.$

$$\leq \vee\left[\underset{i}{\wedge} \{\mu_A(a_i) \wedge \mu_A(c_i)\}\right]$$

$$= \vee\left[(\underset{i}{\wedge}\mu_A(a_i)) \wedge (\underset{i}{\wedge}\mu_A(a_i))\right]$$

$$\leq \left[\left\{\mu_A\left(\sum_{i=1}^{n} a_i\gamma_i b_i\right) \wedge \mu_A\left(\sum_{i=1}^{n} c_i\delta_i d_i\right)\right\}\right], x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

$\leq \mu_A(x)$ -- (3)

$$\nu_{A o_h B}(x) = \wedge\left[\left\{\{\nu_A(a_i) \vee \nu_A(c_i)\} \underset{i}{\vee} \{\nu_B(b_i) \vee \nu_A(c_i)\}\right\}\right]$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma$ $for\ i = 1,2,3, \dots \dots n.$

$$\geq \wedge\left[\underset{i}{\vee}(\nu_A(a_i) \vee \nu_A(c_i))\right]$$

$$= \wedge\left[\left\{\underset{i}{\vee}\nu_A(a_i) \vee (\underset{i}{\vee}\nu_A(c_i)\right\}\right]$$

$$\geq \left[\left\{\nu_A\left(\sum_{i=1}^{n} a_i\gamma_i b_i\right) \vee \nu_A\left(\sum_{i=1}^{n} c_i\delta_i d_i\right)\right\}\right]\ x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

$\geq \nu_A(x)$ -- (4)

Since this is true for every representation of x, $\mu_{A o_h B}(x) \subseteq \mu_{A,} \nu_{A o_h B}(x) \subseteq \nu_A$

Similarly, We Can Prove that

$\mu_{A o_h B}(x) \subseteq \mu_B$ i.e $\mu_{A o_h B}(x) \subseteq \mu_A$ for all $x \in S$ -------- (5)

& $\nu_{A o_h B}(x) \subseteq \nu_B$ i. e $\nu_{A o_h B}(x) \subseteq \nu_B$ for all $x \in S$ --------------- (6)

Combining (3) and (5), we get,

$\mu_{A o_h B}(x) \leq \{\mu_A(x)\ \mu_B(x)\}$ for all $x \in S$

$= \mu_{A \cap B}(x)$ ----------------------------- (7)

$\nu_{A o_h B}(x) \geq \{\nu_A(x) \vee \nu_B(x)\}$ for all $x \in S$

$= \nu_{A \cap B}(x)$ --------------------------------- (8)

Therefore combining 1,2,3,4,5,6,7&

$$A\Gamma_h B \subseteq A o_h B \subseteq A \cap B \subseteq A, B$$

Hence the lemma.

Theorem 3.5

If A and B be two intuitionistic fuzzy left h-ideal of S then $\mu_{A o_h B}$and $\nu_{A o_h B}$ is an intuitionistic fuzzy left h-ideal of S.

Proof

$$\mu_{A\circ_h B}(x+y) = \vee\left\{\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_B(b_i) \wedge \mu_B(d_i)]\right\}\right\}$$

$$(x+y) + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

For $a_i, b_i, c_i, d_i, x, y, z \in S$ *and* $\gamma_i, \delta_i \in \Gamma$.

$$\geq \vee\left\{[\mu_A(a_i') \wedge \mu_A(c_i')] \underset{i}{\wedge} [\mu_B(b_i') \wedge \mu_B(d_i')] \underset{i}{\wedge} [\mu_A(a_i'') \wedge \mu_A(c_i'')] \underset{i}{\wedge} [\mu_B(b_i'') \wedge \mu_B(d_i'')]\right\}$$

$$(x+y) + \left(\sum_{i=1}^{n} a_i'\gamma_i' b_i' + \sum_{i=1}^{n} a_i''\gamma_i'' b_i''\right) + (z_1 + z_2) = \left(\sum_{i=1}^{n} c_i'\delta_i' d_i' + \sum_{i=1}^{n} c_i''\delta_i'' d_i''\right)(z_1 + z_2)$$

For $x, y, a_i', b_i', c_i', d_i', z_1, a_i'', b_i'', c_i'', d_i'', z_2 \in S$ and $\alpha_i', \beta_i', \gamma_i', \delta_i', \alpha_i'', \beta_i'', \gamma_i'', \delta_i'' \in \Gamma$.

$$\geq \vee\left\{[\mu_A(a_i') \wedge \mu_A(c_i')] \underset{i}{\wedge} [\mu_B(b_i') \wedge \mu_B(d_i')] \underset{i}{\wedge} [\mu_A(a_i'') \wedge \mu_A(c_i'')] \underset{i}{\wedge} [\mu_B(b_i{}') \wedge \mu_B(d_i{}')]\right\}$$

$$x + \sum_{i=1}^{n} a_i'\gamma_i' b_i' + z_1 = \sum_{i=1}^{n} c_i'\delta_i' d_i' + z_1, y + \sum_{i=1}^{n} a_i''\gamma_i'' b_i'' + z_2 = \sum_{i=1}^{n} c_i''\delta_i'' d_i'' + z_2$$

$x, y, a_i', b_i', c_i', d_i', z_1, a_i'', b_i'', c_i'', d_i'', z_2 \in S$ and $\alpha_i', \beta_i', \gamma_i', \delta_i', \alpha_i'', \beta_i'', \gamma_i'', \delta_i'' \in \Gamma$

$$\geq \left[\vee\left\{[\mu_A(a_i') \wedge \mu_A(c_i')] \underset{i}{\vee} [\nu_B(b_i') \wedge \mu_B(d_i')]\right\}\right]$$

$$\wedge \left[\vee\left\{[\mu_A(a_i'') \wedge \mu_A(c_i'')] \underset{i}{\wedge} [\mu_B(b_i{}') \wedge \mu_B(d_i{}')]\right\}\right]$$

$$x + \sum_{i=1}^{n} a_i'\gamma_i' b_i' + z_1 = \sum_{i=1}^{n} c_i'\delta_i' d_i' + z_1 y + \sum_{i=1}^{n} a_i''\gamma_i'' b_i'' + z_2 = \sum_{i=1}^{n} c_i''\delta_i'' d_i'' + z_2$$

$= \mu_{A\circ_h B}(x) \wedge \mu_{A\circ_h B}(y)$ and

$$\nu_{A\circ_h B}(x+y) = \wedge\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} [\nu_B(b_i) \vee \nu_B(d_i)]\right\}$$

$a_i, b_i, c_i, d_i, x, y, z \in S$ and $\gamma_i, \delta_i \in \Gamma$.

$$(x+y) + \sum a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

$$\leq \wedge\left\{[\nu_A(a_i') \vee \nu_A(c_i')] \underset{i}{\vee} [\nu_B(b_i') \vee \nu_B(d_i')] \underset{i}{\vee} [\nu_A(a_i'') \vee \nu_A(c_i'')] \underset{i}{\vee} [\nu_B(b_i{}') \vee \nu_B(d_i{}')]\right\}$$

$$(x+y) + \left(\sum_{i=1}^{n} a_i'\gamma_i' b_i' + \sum_{i=1}^{n} a_i''\gamma_i'' b_i''\right) + (z_1 + z_2)$$

$$= \left(\sum_{i=1}^{n} c_i'\delta_i' d_i' + \sum_{i=1}^{n} c_i''\delta_i'' d_i''\right) + (z_1 + z_2)$$

For $x, y, a_i', b_i', c_i', d_i', z_1, a_i'', b_i'', c_i'', d_i'', z_2 \in S$ and $\alpha_i', \beta_i', \gamma_i', \delta_i', \alpha_i'', \beta_i'', \gamma_i'', \delta_i'' \in \Gamma$

$$\leq \wedge\left\{[\nu_A(a_i') \vee \nu_A(c_i')] \underset{i}{\vee} [\nu_B(b_i') \vee \nu_B(d_i')] \underset{i}{\vee} [\nu_A(a_i'') \vee \nu_A(c_i'')] \underset{i}{\vee} [\nu_B(b_i{}') \vee \nu_B(d_i'')]\right\}$$

$$x + \sum_{i=1}^{n} a_i^{'}\gamma_i^{'}b_i^{'} + z_1 = \sum_{i=1}^{n} c_i^{'}\delta_i^{'}d_i^{'} + z_1 y + \sum_{i=1}^{n} a_i^{''}\gamma_i^{''}b_i^{''} + z_2 = \sum_{i=1}^{n} c_i^{''}\delta_i^{''}d_i^{''} + z_2$$

$x, y, a_i^{'}, b_i^{'}, c_i^{'}, d_i^{'}, z_1, a_i^{''}, b_i^{''}, c_i^{''}, d_i^{''}, z_2 \in S$ and $\alpha_i', \beta_i', \gamma_i', \delta_i', \alpha_i'', \beta_i'', \gamma_i'', \delta_i'' \in \Gamma$.

$$\leq \left\{\wedge\left\{[\nu_A(a_i') \vee \nu_A(c_i')] \underset{i}{\vee} [\nu_B(b_i') \vee \nu_B(d_i')]\right\}\right\}$$

$$\vee \left\{\wedge\left\{[\nu_A(a_i'') \vee \nu_A(c_i'')] \underset{i}{\vee} [\nu_B(b_i'') \vee \nu_B(d_i'')]\right\}\right\}$$

$$x + \textstyle\sum_{i=1}^{n} a_i'\gamma_i'b_i' + z_1 = \sum_{i=1}^{n} c_i'\delta_i'd_i' + z_1, y + \sum_{i=1}^{n} a_i''\gamma_i''b_i'' + z_2 = \sum_{i=1}^{n} c_i''\delta_i''d_i'' + z_2$$

$$= \nu_{Ao_hB}(x) \vee \nu_{Ao_hB}(y)$$

$$\mu_{Ao_hB}(x\gamma y) = \vee\left\{\left\{[\mu_A(p_i) \wedge \mu_A(\gamma_i)] \underset{i}{\wedge} [\mu_B(q_i) \wedge \mu_B(s_i)]\right\}\right\}$$

$$x\gamma y + \sum_{i=1}^{n} p_i\alpha_i q_i + z = \sum_{i=1}^{n} \gamma_i\beta_i s_i + z$$

$$\geq \vee\left\{\left\{[\mu_A(x\gamma a_i) \wedge \mu_A(x\gamma c_i)] \underset{i}{\wedge} [\mu_B(a_i) \wedge \mu_B(d_i)]\right\}\right\}$$

$$x\gamma y + \sum_{i=1}^{n} x\gamma a_i\gamma_i b_i + x\gamma z_1 = \sum_{i=1}^{n} x\gamma c_i\delta_i d_i + x\gamma z_1$$

Where $y + \sum_{i=1}^{n} a_i\gamma_i b_i + z_1 = \sum_{i=1}^{n} c_i\delta_i d_i + z_1$

$$\geq \vee\left\{\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_B(b_i) \wedge \mu_B(d_i)]\right\}\right\}$$

Where $y + \sum_{i=1}^{n} a_i\gamma_i b_i + z_1 = \sum_{i=1}^{n} c_i\delta_i d_i + z_1$

$$= \mu_{Ao_hB}(y)$$

$$\nu_{Ao_hB}(x\gamma y) = \wedge\left\{\left\{[\nu_A(p_i) \vee \nu_A(\gamma_i)] \underset{i}{\vee} [\nu_B(q_i) \vee \nu_B(s_i)]\right\}\right\}$$

$$x\gamma y + \sum_{i=1}^{n} p_i\alpha_i q_i + z = \sum_{i=1}^{n} \gamma_i\beta_i s_i + z$$

$$\leq \wedge\left\{\left\{[\nu_A(x\gamma a_i) \vee \nu_A(x\gamma c_i)] \underset{i}{\vee} [\nu_B(a_i) \vee \nu_B(d_i)]\right\}\right\}$$

$$x\gamma y + \sum_{i=1}^{n} x\Upsilon a_i\gamma_i b_i + x\gamma z_1 = \sum_{i=1}^{n} x\gamma c_i\delta_i d_i + x\gamma z_1$$

Where $y + \sum_{i=1}^{n} a_i\gamma_i b_i + z_1 = \sum_{i=1}^{n} c_i\delta_i d_i + z_1$

$$\leq \wedge\left\{\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} [\nu_B(b_i) \vee \nu_B(d_i)]\right\}\right\}$$

Where $y + \sum_{i=1}^{n} a_i\gamma_i b_i + z_1 = \sum_{i=1}^{n} c_i\delta_i d_i + z_1$

$$= \nu_{Ao_hB}(y)$$

Therefore A and B are intuitionistic fuzzy left ideal.

Now for h-ideal, suppose $x + a + z = b + z$ where $x, a, b, z \in S$. Then

$$\mu_{Ao_hB}(x) = \vee\left[\{\mu_A(a_i) \wedge \mu_A(c_i)\} \underset{i}{\wedge} \{\mu_B(b_i) \wedge \mu_B(b_i)\}\right]$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma$

$$\geq \vee\left[\left\{\{\mu_A(a) \wedge \mu_A(b)\} \underset{i}{\wedge} \{\mu_B(e_i) \wedge \mu_B(e_i)\}\right\}\right], x + \sum_{i=1}^{n} a\gamma_i f_i + z = \sum_{i=1}^{n} b\gamma_i f_i + z$$

Where $\sum_{i=1}^{n}[\gamma_i, f_i]$ is right unity of S.

$$= \vee\left\{\left\{[\mu_A(a) \wedge \mu_A(b)] \underset{i}{\wedge} [\mu_B(e_i)]\right\}\right\}$$

$= \mu_A(a) \wedge \mu_A(b)\{$Since $\mu_B(f_i) \geq \mu_B(t)$ for all t ∈ S and for all i (like semirings)$\}$

$$\nu_{Ao_hB}(x) = \wedge\left[\{\nu_A(a_i) \vee \nu_A(c_i)\} \underset{i}{\vee} \{\nu_B(b_i) \vee \nu_B(b_i)\}\right]$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma$

$$\leq \wedge\left[\{\nu_A(a) \vee \nu_A(b)\} \underset{i}{\vee} \{\nu_B(e_i) \vee \nu_B(e_i)\}\right], x + \sum_{i=1}^{n} a\gamma_i f_i + z = \sum_{i=1}^{n} b\gamma_i f_i + z$$

Where$\sum_{i=1}^{n}[\gamma_i, f_i]$ is right unity of S.

$$= \wedge\left\{\left\{[\nu_A(a) \vee \nu_A(b)] \underset{i}{\vee} [\nu_B(e_i)]\right\}\right\}$$

$= \nu_A(a) \vee \nu_A(b)\{$Since $\gamma_B(f_i) \geq \gamma_B(t)$ for all t ∈ S and for all i (like semirings) Similarly, if we take $\sum_{i=1}^{n}[e_i, \delta_i]$ be the left unity of S, we get

$$\mu_{Ao_hB}(x) \geq \mu_B(a) \wedge \mu_B(b) \text{ and } \nu_{Ao_hB}(x) \leq \nu_B(a) \vee \nu_B(b)$$

Therefore we have

$$\mu_{Ao_hB}(x) \geq [\mu_A(a) \wedge \mu_A(b)] \wedge [\mu_B(a) \wedge \mu_B(b)] = \mu_{A\cap B}(a) \wedge \mu_{A\cap B}(b)$$
$$\geq \mu_{Ao_hB}(a) \wedge \mu_{Ao_hB}(b)$$

Since the lemma 3.4, we have $\mu_{Ao_hB} \subseteq A \cap B$

$$\nu_{Ao_hB}(x) \leq [\nu_A(a) \vee \nu_A(b)] \vee [\nu_B(a) \vee \nu_B(b)] = \nu_{A\cap B}(a) \vee \nu_{A\cap B}(b)$$
$$\leq \nu_{Ao_hB}(a) \vee \nu_{Ao_hB}(b)$$

Since the lemma 3.4, we have $\nu_{Ao_hB} \subseteq A \cap B$.

If we take the representation of x as $x + \sum_{i=1}^{n} a\gamma_i f_i + z = \sum_{i=1}^{n} b\gamma_i f_i + z$ or $x + \sum_{i=1}^{n} e_i \delta_i a + z = \sum_{i=1}^{n} b\gamma_i f_i + z$ then the calculations changes but the result becomes true. Therefore, in any case μ_{Ao_hB} and ν_{Ao_hB} is a intuitionistic fuzzy left h-ideal of S.If x cannot be expressed as$x + \sum_{i=1}^{n} a\gamma_i f_i + z = \sum_{i=1}^{n} a_i\delta_i b_i + z$ where $a_i, b_i, c_i, x, y, z \in S$ and $\gamma_i, \delta_i \in \Gamma$ for $i = 1,2,3, \ldots, n$ then the proof is obvious. Therefore in any case μ_{Ao_hB} and γ_{Ao_hB} is an intuitionistic fuzzy left h-ideal of S.

Remark 3.6
For any two intuitionistic fuzzy left h-ideal A and B of S.Then $\mu_{A\Gamma_h B}$ and $\nu_{A\Gamma_h B}$ is not an intuitionistic fuzzy h-ideal of S.

Proposition 3.7
Let $A = \langle \mu_A, \nu_A \rangle$, $B = \langle \mu_B, \nu_B \rangle$, $C = \langle \mu_C, \nu_C \rangle$ be three intuitionistic fuzzy left h-ideal of S then $A\Gamma_h B \subseteq C$ if and only if $A o_h B \subseteq C$.

Proof
Since $\mu_{A\Gamma_h B} \subseteq \mu_{Ao_h B}$ (see lemma 3.4) and $\mu_{Ao_h B} \subseteq C$ it follows that $\mu_{A\Gamma_h B} \subseteq \mu_C$ & $\nu_{A\Gamma_h B} \supseteq \nu_{Ao_h B}$and $\nu_{A\Gamma_h B} \supseteq \nu_C$.
Let $x \in S$ be such that $x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$ ……. (1)
Where$a_i, b_i, c_i, d_i, z \in S$ and $\gamma_i, \delta_i \in \Gamma$, for $i = 1,2,3, \dots, n$.Then

$$\mu_C(x) \geq \left\{\mu_C\left(\sum a_i\gamma_i b_i\right) \wedge \mu_C\left(\sum c_i\delta_i d_i\right)\right\}$$

$$\geq \left\{\left\{\left(\mu_c(a_1\gamma_1 b_1)\right) \wedge \dots \dots \wedge \left(\mu_c(a_n\gamma_n b_n)\right)\right\} \wedge \left\{\left(\mu_c(c_1\delta_1 d_1)\right) \wedge \dots \dots \wedge \left(\mu_c(c_n\delta_n d_n)\right)\right\}\right\}$$

$$\geq \left\{\begin{array}{l}\left\{\left(\mu_{A\Gamma_h B}\right)(a_1\gamma_1 b_1) \wedge \left(\mu_{A\Gamma_h B}\right)(a_2\gamma_2 b_2) \wedge \dots \dots \wedge \left(\mu_{A\Gamma_h B}\right)(a_n\gamma_n b_n)\right\} \\ \wedge \left\{\left(\mu_{A\Gamma_h B}\right)(c_1\delta_1 d_1) \wedge \left(\mu_{A\Gamma_h B}\right)(c_2\delta_2 d_2) \wedge \dots \dots \wedge \left(\mu_{A\Gamma_h B}\right)(c_n\delta_n d_n)\right\}\end{array}\right\}$$

$= \{\mu_A(a_i) \wedge \mu_A(c_i)\} \underset{i}{\wedge} \{\mu_B(b_i) \wedge \mu_B(d_i)\}$Since this true for every representation of x,

We have

$$\mu_C(x) \geq \vee\left\{\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_B(b_i) \wedge \mu_B(d_i)]\right\}\right\},$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} a_i\delta_i d_i + z$$

$$= \mu_{Ao_h B}(x)$$

$$\nu_C(x) \leq \left\{\nu_C\left(\sum a_i\Upsilon_i b_i\right) \vee \nu_C\left(\sum c_i\delta_i d_i\right)\right\}$$

$$\leq \left\{\left\{\left(\nu_c(a_1\gamma_1 b_1)\right) \vee \dots \dots \vee \left(\nu_c(a_n\gamma_n b_n)\right)\right\} \vee \left\{\left(\nu_c(c_1\delta_1 d_1)\right) \vee \dots \dots \vee \left(\nu_c(c_n\delta_n d_n)\right)\right\}\right\}$$

$$\leq \left\{\begin{array}{l}\left\{\left(\nu_{A\Gamma_h B}\right)(a_1\gamma_1 b_1) \vee \left(\nu_{A\Gamma_h B}\right)(a_2\gamma_2 b_2) \vee \dots \dots \vee \left(\nu_{A\Gamma_h B}\right)(a_n\gamma_n b_n)\right\} \vee \\ \left\{\left(\nu_{A\Gamma_h B}\right)(c_1\delta_1 d_1) \vee \left(\nu_{A\Gamma_h B}\right)(c_2\delta_2 d_2) \vee \dots \dots \vee \left(\nu_{A\Gamma_h B}\right)(c_n\delta_n d_n)\right\}\end{array}\right\}$$

$= \left\{\{\nu_A(a_i)\ \nu_A(c_i)\} \underset{i}{\vee} \{\nu_B(b_i)\ \nu_B(d_i)\}\right\}$Since this true for every representation of x,

We have

$$\nu_C(x) \leq \wedge\left\{\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} [\nu_B(b_i) \vee \nu_B(d_i)]\right\}\right\}$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

$= \nu_{A o_h B}(x)$

Since $x \in S$ is arbitrary $\mu_{A o_h B} \subseteq C$ and $\nu_{A o_h B} \supseteq C$

If x cannot be expressed as (1) then the proof is trivial.

Proposition 3.8

Let A, B, C be three intuitionistic fuzzy h-ideals of S.Then $\mu_A \subseteq \mu_B$ implies that $\mu_{A o_h C} \subseteq \mu_{B o_h C}$

Proof

Let A, B, C be three intuitionistic fuzzy h-ideals of S, Such that $\mu_A \subseteq \mu_B$ and $x \in S$ be arbitrary

$$\mu_{A o_h C}(x) = \vee\left\{\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_C(b_i) \wedge \mu_C(d_i)]\right\}\right\}$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $a_i, b_i, c_i, d_i, x, z \in S$ and $\gamma_i, \delta_i \in \Gamma$, for $i = 1,2,3, \ldots, n$.

$$\leq \vee\left\{\left\{[\mu_B(a_i) \wedge \mu_B(c_i)] \underset{i}{\wedge} [\mu_C(b_i) \wedge \mu_C(d_i)]\right\}\right\} = \mu_{B o_h C}(x)$$

Thus $\mu_{A o_h C} \subseteq \mu_{B o_h C}$ and

$$\nu_{A o_h C}(x) = \wedge\left\{\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} [\nu_C(b_i) \vee \nu_C(d_i)]\right\}\right\}$$

$$x + \sum_{i=1}^{n} a_i\gamma_i b_i + z = \sum_{i=1}^{n} c_i\delta_i d_i + z$$

Where $a_i, b_i, c_i, d_i, x, z \in S$ and and $\gamma_i, \delta_i \in \Gamma$, for $i = 1,2,3, \ldots, n$.

$$\geq \wedge\left\{\left\{[\nu_B(a_i) \vee \nu_B(c_i)] \underset{i}{\vee} [\nu_C(b_i) \vee \nu_C(d_i)]\right\}\right\} = \nu_{B o_h C}(x)$$

Thus $\nu_{A o_h C} \subseteq \nu_{B o_h C}$.

Proposition 3.9

Let A, B be two intuitionistic fuzzy left (resp.right) h-ideal of S. Then $\mu_{A\oplus B}$ and $\nu_{A\oplus B}$ are intuitionistic fuzzy left (resp.right) h-ideal of S.

Proof

Suppose $x, y \in S$ and $\gamma \in \Gamma$.Then

$$\begin{aligned}
\mu_{A\oplus B}(x\gamma y) &\geq \vee\{[\mu_A(p) \wedge \mu_B(q)] \text{ for } p, q \in S\} \text{ if } x + y = p + q. \\
&\geq \vee\{[\mu_A(u+s) \wedge \mu_B(v+t)] \text{ where } u, v, s, t \in S\} \text{ if } x = u + v, y = s + t \\
&\geq \vee\{\{[\mu_A(u) \wedge \mu_A(v)]\ [\mu_B(s)\ \mu_B(t)]\}\}, \text{ if } x = u + v, y = s + t \\
&= \{\{\vee[\mu_A(u) \wedge \mu_B(v)]\}\{\vee[\mu_A(s)\mu_B(t)]\}\}, \text{ if } x = u + v, y = s + t \\
&= \mu_{A\oplus B}(x) \wedge \mu_{A\oplus B}(y) \\
\nu_{A\oplus B}(x\gamma y) &= \wedge\{[\nu_A(p) \vee \nu_B(q)] \text{ for } p, q \in S\}, \text{ if } x + y = p + q \\
&\leq \wedge\{[\nu_A(u+s) \vee \nu_B(v+t)] \text{ where } u, v, s, t \in S\}, \text{ if } x = u + v, y = s + t \\
&\leq \wedge\{\{[\nu_A(u) \vee \nu_A(v)] \vee [\nu_B(s) \vee \nu_B(t)]\}\}, \text{ if } x = u + v, y = s + t
\end{aligned}$$

$= \wedge\{\{[\nu_A(u) \vee \nu_B(v)] \vee [\nu_A(s) \vee \nu_B(t)]\}\}$, if $x = u + v, y = s + t$

$= \{\{\wedge[\nu_A(u) \vee \nu_B(v)]\} \vee \{\wedge[\nu_A(s) \vee \nu_B(t)]\}\}$, if $x = u + v, y = s + t$

$= \nu_{A\oplus B}(x) \vee \nu_{A\oplus B}(y)$.

Again

$\mu_{A\oplus B}(x\gamma y) = \vee\{[\mu_A(p) \wedge \mu_B(q)]\}$, if $x\gamma y = p + q$

$= \vee_{y=u+v}\{[\mu_A(x\gamma u) \wedge \mu_B(x\gamma v)]\}$, [since $x\gamma y = x\gamma(u + v) = x\gamma u + x\gamma v$]

$\geq \mu_A(u) \wedge \mu_B(v) = \mu_{A\oplus B}(y)$ and

$\nu_{A\oplus B}(x\gamma y) = \wedge\{[\nu_A(p) \vee \nu_B(q)]\}$, if $x\gamma y = p + q$

$= \wedge_{y=u+v}\{[\nu_A(x\gamma u) \vee \nu_B(x\gamma v)]\}$, [since $x\gamma y = x\gamma(u + v) = x\gamma u + x\gamma v$] ,

$\leq \nu_A(u) \vee \nu_B(v)$

$= \nu_{A\oplus B}(y)$

Now for h-ideal suppose $w + p + z = q + z \ldots\ldots\ldots (1)$ where $w, p, q, z \in S$. Then

$\mu_{A\oplus B}(w) = \vee\{[\mu_A(x) \wedge \mu_B(y)]\}$ if $w = x + y$, and

$\nu_{A\oplus B}(w) = \wedge\{[\nu_A(x) \vee \nu_B(y)]\}$, if $w = x + y$.

Now (1) can be written as $(x + y) + p = q + z$ *i.e* x and y have an expression of the form $x + p_1 + z = q_1 + z_1$ *and* $y + p_2 + z = q_2 + z_2$ respectively, for some $p_1, p_2, q_1, q_2, z_1, z_2 \in S$. Hence equation (1) reduced to an expression

$$(x + y) + (p_1 + p_2) + (z_1 + z_2) = (q_1 + q_2) + (z_1 + z_2) \ldots\ldots\ldots\ldots \quad (2)$$

Therefore comparing (1) and (2) we have

$\mu_{A\oplus B}(w) \geq \vee\{\{[\mu_A(p_1) \wedge \mu_A(q_1)] \wedge [\mu_B(p_2) \wedge \mu_B(q_2)]\}\}$if $w = x + y, p = p_1 + p_2,$ $q = q_1 + q_2,$

$= \vee\{\{[\mu_A(p_1) \wedge \mu_B(p_2)] \wedge [\mu_A(q_1) \wedge \mu_B(q_2)]\}\}$, if $p = p_1 + p_2, q = q_1 + q_2$

$= \{\{\vee[\mu_A(p_1) \wedge \mu_B(p_2)]\} \wedge \{\vee[\mu_A(q_1) \wedge \mu_B(q_2)]\}\}$

$= \mu_{A\oplus B}(p) \wedge \mu_{A\oplus B}(q)$

$\nu_{A\oplus B}(w) \leq \wedge\{\{[\nu_A(p_1) \vee \nu_A(q_1)] \vee [\nu_B(p_2) \vee \nu_B(q_2)]\}\}$if $w = x + y,$ $p = p_1 + p_2, q = q_1 + q_2$

$= \wedge\{\{[\nu_A(p_1) \vee \nu_B(p_2)] \vee [\nu_A(q_1) \vee \nu_B(q_2)]\}\}$, if $p = p_1 + p_2, q = q_1 + q_2$

$= \{\{\wedge[\nu_A(p_1) \vee \nu_B(p_2)]\} \vee \{\wedge[\nu_A(q_1) \vee \nu_B(q_2)]\}\}$

$= \nu_{A\oplus B}(p) \vee \nu_{A\oplus B}(q)$

Therefore $A \oplus B$ is an intuitionistic fuzzy left h-ideal of S.
Similarly we can prove the result for intuitionistic fuzzy right h-ideal.

Proposition 3.10

Let A, B, C be three intuitionistic fuzzy left h-ideals of S.Then

(i) $\mu_{A\circ_h(B\oplus C)} = (\mu_{A\circ_h B}) \oplus (\mu_{A\circ_h C})$& $\nu_{A\circ_h(B\oplus C)} = (\nu_{A\circ_h B}) \oplus (\nu_{A\circ_h C})$

(ii)$\mu_{(B\oplus C)\circ_h A} = (\mu_{B\circ_h A}) \oplus (\mu_{A\circ_h C})$&$\nu_{(B\oplus C)\circ_h A} = (\nu_{B\circ_h A}) \oplus (\nu_{A\circ_h C})$

Proof

We know that$B \subseteq B \oplus C \Rightarrow \mu_{A\circ_h B} \subseteq \mu_{A\circ_h(B\oplus C)}$

Similarly, $\mu_{A\circ_h C} \subseteq \mu_{A\circ_h(B\oplus C)}$.

Thus $(\mu_{A\circ_h B}) \oplus (\mu_{A\circ_h C}) \subseteq [(\mu_{A\circ_h(B\oplus C)}) \oplus (\mu_{A\circ_h(B\oplus C)})] = \mu_{A\circ_h(B\oplus C)}$

To prove the reverse inclusion let $x, u, v, z, a_i, b_i, c_i, d_i, b_i', b_i'', d_i' d_i'', z_1, z_2 \in S, \gamma_i, \delta_i \in \Gamma$

$$[\mu_{A\circ_h(B\oplus C)}](x) = \vee\left\{\left\{\{\mu_A(a_i) \wedge \mu_A(c_i)\} \underset{i}{\wedge} \{\mu_{(B\oplus C)}(b_i) \wedge \mu_{(B\oplus C)}(d_i)\}\right\}\right\}$$

$$x + \sum a_i\gamma_i b_i + z = \sum c_i\,\delta_i d_i + z$$

$$= \vee\left\{\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} \left[\vee\left(\mu_B(b_i') \wedge \left(\mu_C(b_i'')\right)\right)\right] \underset{i}{\wedge} \left[\vee\left(\mu_B(d_i') \wedge \left(\mu_C(d_i'')\right)\right)\right]\right\}\right\}$$

$$b = b_i' + b_i'' d = d_i' + d_i''$$

$$x + \sum a_i\gamma_i(b_i' + b_i'') + z = \sum c_i\,\delta_i(d_i' + d_i'') + z$$

$$\Rightarrow x + \left(\sum a_i\gamma_i b_i' + \sum a_i\gamma_i b_i''\right) + (z_1 + z_2) = \left(\sum c_i\,\delta_i d_i' + \sum c_i\,\delta_i d_i''\right) + z_1 + z_2$$

$$\geq \vee\left\{\left\{\vee\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_B(b_i') \wedge \mu_B(d_i')]\right\}\right\}\right.$$

$$\left.\wedge\left\{\vee\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_C(b_i'') \wedge \mu_C(d_i'')]\right\}\right\}\right\}$$

$$\geq \vee\left\{\left\{\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_B(b_i') \wedge \mu_B(d_i')]\right\}\right\}\right.$$

$$\left.\wedge\left\{\vee\left\{[\mu_A(a_i) \wedge \mu_A(c_i)] \underset{i}{\wedge} [\mu_C(b_i'') \wedge \mu_C(d_i'')]\right\}\right\}\right\}$$

$$= \vee\{\mu_{A\circ_h B}(u) \wedge \mu_{A\circ_h C}(v)\}, u + \sum a_i\gamma_i b_i' + z_1 = \sum c_i\,\delta_i d_i' + z_1$$

$$v + \sum a_i\gamma_i b_i'' + z_2 = \sum c_i\,\delta_i d_i'' + z_2 \text{ if } x = u + v$$

$$= \left((\mu_{A\circ_h B}) \oplus (\mu_{A\circ_h C})\right)(u + v)$$

$$= \left((\mu_{A\circ_h B}) \oplus (\mu_{A\circ_h C})\right)(x)$$

Therefore $\mu_{A\circ_h(B\oplus C)} \subseteq (\mu_{A\circ_h B}) \oplus (\mu_{A\circ_h C})$

$$B \supseteq B \oplus C \Rightarrow \nu_{A\circ_h B} \supseteq \nu_{A\circ_h(B\oplus C)}$$

Similarly $\nu_{A\circ_h C} \supseteq \nu_{A\circ_h(B\oplus C)}$. Thus

$(\nu_{A\circ_h B}) \oplus (\nu_{A\circ_h C}) \supseteq [(\nu_{A\circ_h(B\oplus C)}) \oplus (\nu_{A\circ_h(B\oplus C)})] = \nu_{A\circ_h(B\oplus C)}$

To prove the reverse inclusion let $x, u, v, z, a_i, b_i, c_i, d_i, b_i', b_i'', d_i' d_i'', z_1, z_2 \in S, \gamma_i, \delta_i \in \Gamma$, then

$$[\nu_{A\circ_h(B\oplus C)}](x) = \wedge\left\{\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} [\nu_{(B\oplus C)}(b_i) \vee \nu_{(B\oplus C)}(d_i)]\right\}\right\}$$

$$x + \sum a_i\gamma_i b_i + z = \sum c_i\,\delta_i d_i + z$$

$$= \wedge\left\{\left\{[\wedge(\nu_A(a_i) \vee \nu_A(c_i))] \underset{i}{\vee} \left[\wedge\left(\nu_B(b_i') \vee \left(\nu_C(b_i'')\right)\right)\right] \underset{i}{\vee} \left[\wedge\left(\nu_B(d_i') \vee \left(\nu_C(d_i'')\right)\right)\right]\right\}\right\}$$

$$b = b_i' + b_i'' d = d_i' + d_i''$$

$$x + \sum a_i \gamma_i (b_i' + b_i'') + z = \sum c_i\, \delta_i (d_i' + d_i'') + z$$

$$\Rightarrow x + \left(\sum a_i \gamma_i b_i' + \sum a_i \gamma_i b_i''\right) + (z_1 + z_2) = \left(\sum c_i\, \delta_i d_i' + \sum c_i\, \delta_i d_i''\right) + (z_1 + z_2)$$

$$\leq \wedge\left\{\left\{\wedge\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} [\nu_B(b_i') \vee \nu_B(d_i')]\right\}\right\}\right.$$

$$\left.\vee\left\{\wedge\left\{[\nu_A(a_i) \vee \nu_A(c_i)] \underset{i}{\vee} \left[\nu_C\left(b_i''\right) \vee \nu_C\left(d_i''\right)\right]\right\}\right\}\right\}$$

$$= \wedge\left\{\nu_{Ao_hB}(u) \vee \nu_{Ao_hC}(v)\right\}, u + \sum a_i \gamma_i b_i' + z_1 = \sum c_i\, \delta_i d_i' + z_1$$

$$v + \sum a_i \gamma_i b_i'' + z_2 = \sum c_i\, \delta_i d_i'' \text{ if } x = u + v$$

$$= \left((\nu_{Ao_hB}) \oplus (\nu_{Ao_hC})\right)(u + v)$$

$$= \left((\nu_{Ao_hB}) \oplus (\nu_{Ao_hC})\right)(x)$$

Therefore $\nu_{Ao_h(B \oplus C)} \supseteq (\nu_{Ao_hB}) \oplus (\nu_{Ao_hC})$

Hence the proof

Similarly, we can prove (ii).

References

[1] K. T. Atanassov, Intuitionistic Fuzzy Sets and Systems, **20**(1986), 87-96.

[2] T. K. Dutta,S.K.Sardar, On Matrix Γ-Semirings, *FarEast Journal of Mathematics and Science,***1** (2002), 17-31.

[3] T. K Dutta, S. K. Sardar, On the Operator Semiring of a Γ-Semiring, *Southeast Asian bulletin of Mathematics*, Springer-Verlag, **26** (2002), 203-213.

[4] D. Ezhilmaran. V, Krishnamoorthy, Characterization Of Intuitionistic Fuzzy h-ideal in Γ-hemirings, *Global journal of pure and applied mathematics,***10** (2014),101-112

[5] M. Henriksen , Ideals in semirings with commutative addition, *American. Mathematical Society. Notices*,**6** (1958).

[6] K. Lizuka, On the Jacobson radical of Semiring, *Tohoku Mathematical journal*, **2** (1959), 409-421**.**

[7] Y. B. Jun, M.A, Ozt¨urk and S..Z.Song, On Fuzzy h-ideals in hemiring, *Information Sciences*,**162** (2004),211-226.

[8] Y. B. Jun, C. Y. Lee, Fuzzy Γ-rings,*Pusan Kyongnam Mathematcal Journal*, **2** (1992) 163-170.

[9] D. R. LaTorre, On h-ideals and k-ideals in hemirings, *Pubicationses Mathematicea*.**12** (1965) 219-226.

[10] Ma. J. Zhan, Fuzzy h- ideals in h-hemiregular and h-simple Γ-hemirings, *Neural Computing and Application*,**19** (2010), 477-485.

[11] M. M. K. Rao, Γ-Semirings–1,*Southeast Asian Bulletin Of Mathematics*, **19**(1995), 49-54.

[12] A. Rosenfeld, Fuzzy groups, *Journal of mathematics analysis and application*, **35**(1971), 512-517.

[13] S. K. Sardar, D. Mandal, Fuzzy h-ideal in Γ-hemiring, *International journal of Pure and applied mathematics*, **3**(2009), 439-450.

[14] Xing-Yun Xie,Fuzzy ideal Extensions of semigroups, *Soochow Journal of Mathematics*,**27**(2001), 125-138.

[15] L. A. Zadeh, Fuzzy Sets, *Information and Control*,**8**(1965),338-353.

[16] J. Zhan, W. A. Dudek, Fuzzy h-ideals of hemirings, *Informationsciences*, **177** (2007), 876-886.