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# Combined Effects Of Acid And Metal On The Survival Of Resource Based Population Incorporating Nutrient Recycling: A Mathematical Model

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**ABSTRACT**

In this paper, a mathematical model is studied to discuss the Combined Effects of Acid and Metal on the Survival of Resource Based population Incorporating Nutrient Recycling. The model is formulated using the systems of non-linear ordinary differential equations. In the model there are five state variables, viz, concentration of acid in water, concentration of metal in water, density of favorable resource (Phytoplanktonic Species), density of fish population and nutrient concentration. A simple predator-prey population growth model is considered along with the Holling's Type-1 functional response which is considered as a significant component of predator-prey relationship. Conditions for local stability and feasible equilibrium points (non-living, fish extinct and interior) have been determined. Non-linear stability analysis of the non-trivial equilibrium points has been discussed. it is found that when the equilibrium level of nutrients increases than the equilibrium of acid and metal go down, further showing the synergistic effect of both the stresses. It was also observed that the amount of nutrient is increasing if the population densities of resource and fish populations are going up. It is noted that both the populations are sensitive to the specific rate of predation of fish on resource population Conditions for the existence of the equilibrium points have been drawn and the criteria for the survival or extinction of the species has been obtained using numerical simulation. Stability of the systemis explained analytically as well as graphically for the various possible cases in the form of time series and phase space and phase plan graphs.

**KeyWords:** Nutrients, LiapunovFunction, Stability, Routh-HurwitzCriteria, Sylvester's Criteria.

## 1. Introduction

In the present scenario, the most challenging problem to the society is the change in the environment caused by the pollution affecting the long term survival of the aquatic species, human life style and bio-diversity of the habitat. A great quantity of the toxicants and contaminants enter into the ecosystems continuously which threaten the survival of the exposed population including human beings. Aquatic environment is getting polluted by many different types of toxic metal which are discharged from the industries and agricultural fields. The chief pollutants which are produced by the industries are heavy metals and radioactive substances. In the agricultural fields fertilizer, pesticides and insecticides are used for more production and control of plant diseases. The fertilizers, pesticides and insecticides containing toxic metals reach the water bodies through surface runoff, causing harms to aquatic life. Sulphur and Nitrogen Oxides are important pollutants of air which are produced mainly by combustion of fossil fuels, smelters, power plants, automobile exhaust, domestic fire etc. After a long time process these oxides are oxidised into Sulphuric and Nitric acids in the presence of moisture and consequently, fall in the aquatic bodies during rains. The sequence of changes from the emitted gas to acid is:

$2SO_2 + O_2 \rightarrow 2SO_3$ then $SO_2 + H_2O \rightarrow H_2SO_4$

$2NO + O_2 \rightarrow 2NO_2$ then $4NO_2 + 2H_2O + O_2 \rightarrow 4HNO_3$

The acid rain changes the pH value of water which is harmful to the aquatic population. It has been found that the toxicity of metals in aquatic environment increases due to the acidity of water. For example many fish species die when the pH is less than five. In addition to direct effects of acid on fish, the acid mobilizes such metals as aluminum from the surrounding soil from which it enters lakes by runoff. When combined with high acidity, aluminum is toxic to fish. Considering these aspects therefore in this chapter, a mathematical model has been proposed to study the combined effects of acidity and toxicity of metals on the growth and existence of aquatic populations incorporating nutrient effects on the system. Although few mathematical models [Hallam, Miller, 1982; Patin, 1982; Hallam & Lunna, 1984; Thomann, 1989;Emmerich Woolhiser & shiriey, 1989;Lassiter & Kooijman, 1989; Shukla & Dubey, 1996; Misra & Meitei, 2005; Shukla et.al] for the effect of toxicants and pollutants on the aquatic populations have been developed, but almost no studies have been conducted to see the effects of acid rains and metal toxicity on a resource-based aquatic interacting species system with direct recycling of the nutrients. Ecologists are now actively seeking ways to integrate inter specific interaction in food waves with the functional processes of energy flow and material (nutrients) recycling. Study of the interaction between nutrients, resource waste population is therefore very much essential to assess the risk of population exposed to the toxicants ( Sarkar A.K. & Roy A.B.1989; Ghosh D.& SarkarA.K., 1998; Swati, Misra & Dhar, 2009).

In this paper a mathematical model has been proposed to discuss the combined effects of acid and metal on resource based population incorporating nutrient cycling in the system.The model is formulated using the system of non-linear ordinary differential equations. In the model there are five state variables, viz, concentration of acid in water, concentration of metal in water, density of favorable resource(phytoplanktonic species), density of fish population and nutrient concentration. Conditions for local stability and feasible equilibrium points have been determined. Non-linear stability analysis of the non-trivial equilibrium points has been discussed and criteria for the survival or extinction of the species have been obtained using numerical simulation.

## 2. Materials and Methods

### 2.1 Mathematical Model

In the model, we considered $T_1(t)$ is the concentration of acid in water, $C_w(t)$ is the concentration of metal in water, B $(t)$ is the density of favorable resource (phytoplatonic species), $N(t)$ is the density of fish population, $S(t)$ = is the amount of nutrient present in the water. The model is formulated using the system of non-linear ordinary differential equations as follows-

$$\frac{dT_1}{dt} = H_0 - \alpha_1 T_1 - \beta_1 T_1 S \quad (1)$$

$$\frac{dC_w}{dt} = Q_0 - \alpha_2 C_w - \beta_2 C_w S \quad (2)$$

$$\frac{dB}{dt} = gSB - cB - fBN \quad (3)$$

$$\frac{dN}{dt} = fBN - bN \quad (4)$$

$$\frac{dS}{dt} = S_0 - aS - gSB - \beta_1 T_1 S - \beta_2 S C_w + KcB + KbN \quad (5)$$

With the initial conditions

$$T_1(0) = T_{10} > 0, C_w(0) = C_{10} > 0, B(0) = B_{10} > 0, N(0) = N_{10} > 0, S(0) = S_{10} > 0,$$

Where, $T_1$= Concentration of acid in water, $C_w$ = Concentration of metal in water, $B$= Density of favorable resource, $N$= Density of fish population, $S$= Concentration of nutrients, $H_0$ = Total input rate of acid, $Q_0$ = Total input rate of metal,.$\alpha_1$ and $\alpha_2$ = natural washout rates of acid and metal respectively. $\beta_1, \beta_2$ = Depletion rate of nutrients due to acid and metal respectively in water. $g$ = Rate of consumption of nutrient by the resource population. $f$ =Specific rate of predation of fish on resource population. $b, c$ =Natural death rates of resource and fish populations. $S_0$ =costant nutrient input in the water. $a$ =Nutrient leaching rate. $K(0 < K < 1)$ =Proportionate amount of resource and fish populations that is being recycled back to the nutrient poolafter death.

$H_0, Q_0, S_0, a, b, c, f, g, \alpha_2, \beta_1, \beta_2, a_1, Q_0$

Are positive constants.

## 2.2 Boundness and Dynamical Behaviour

The boundness of solutions of the model (1)-(5) is given by the following lemma

**Lemma2.2.1**:- All the solutions of the model (1)-(5) will lie in the region:

$\Omega$={( $T_1, C_w, B, N, S$ )$\epsilon R_+^5$, where 0≤ $T_1 \leq \frac{H_0}{\alpha_1}$, 0≤ $C_w \leq \frac{Q_0}{\alpha_2}$, 0≤ $\frac{H_0+Q_0+S_0}{\theta_1} \leq T_1 + C_w + B + N + S$ ; $0 \leq B + N + S \leq \frac{S_0}{\theta}$ ; }as $t \to \infty$ for all positive initial values { $T_{10}, C_{10}, B_{10}, N_{10}, S_{10}$ }$\epsilon R_+^5$ where $\theta$ = min [ $(1 - K)c$, ( $1 - K$ )$b$ ] and

$\theta_1$ = max [( $\alpha_1 + \alpha_2 + \frac{2\beta_1 H_0}{\alpha_1} + \frac{2\beta_2 Q_0}{\alpha_2}, c, b, \alpha_1, \beta_2$) ]

**Proof**:-Let us consider the following function

$$W(t) = S(t) + B(t) + N(t)$$

$$\frac{d(B+N+S)}{dt} = S_0 - aS - \beta_1 T_1 S - \beta_2 S C_w - (1 - K)c - (1 - K)b$$

And if Θ = min [ $(1 - K)c$, ( $1 - K$ )$b$ ] then we obtain the following expression

$$\frac{d(W)}{dt} \leq S_0 - \theta W, S(t) + B(t) + N(t) \leq \frac{S_0}{\theta}$$

$$T_1(t) \leq \frac{H_0}{\alpha_1}, C_w(t) \leq \frac{Q_0}{\alpha_2}$$

$$W_1 = W(t) + T_1(t) + C_w(t)$$

$$\frac{dW_1}{dt} = (H_0 + Q_0 + S_0) - \alpha_1 T_1 - \alpha_2 C_w - 2\beta_1 T_1 S - 2\beta_2 S C_w - \theta W$$

If $\theta_1$ = max[ ( $\alpha_1 + \alpha_2 + \frac{2\beta_1 H_0}{\alpha_1} + \frac{2\beta_2 Q_0}{\alpha_2}, c, b, \alpha_1, \beta_2$) ]

Then we obtain the following expression

$$W_1(t) \geq S_0 + H_0 + Q_0 - \theta_1 W_1$$

$$W_1(t) \geq \frac{S_0 + H_0 + Q_0}{\theta_1}$$

Hence $B(t) + N(t) + S(t) + T_1(t) + C_w(t) \geq \frac{S_0+H_0+Q_0}{\theta_1}$ OR

$$W(t) + T_1(t) + C_w(t) \geq \frac{S_0 + H_0 + Q_0}{\theta_1}$$

From equation (1) $T_1(t) + \alpha_1 T_1 \leq \frac{H_0}{\alpha_1}$

$$T_1(t) \leq \frac{H_0}{\alpha_1}$$

This completes the proof of lemma

We will discuss the local behavior of the dynamical system (1) - (5) around the equilibrium points $E_i (i = 1, 2, 3)$

Now, we will discuss the uniform equilibrium points and interior equilibrium point. The model has the following set of non-negative equilibrium points.

(i)The first equilibrium point(non-living) $E_1$:$(\widetilde{T_1} \neq 0, \widetilde{C_w} \neq 0, \tilde{B} = 0, \tilde{N} = 0, \tilde{S} \neq 0)$

$\widetilde{T_1} = \frac{H_0}{\alpha_1 + \beta_1 \tilde{S}}$, $\tilde{C}_w = \frac{Q_0}{\alpha_2 + \beta_2 \tilde{S}}$, $\tilde{S} = \frac{S_0}{a + \beta_1 \widetilde{T_1} + \beta_2 \widetilde{C_w}}$ where

[$\tilde{S} = \frac{-g_2 + \sqrt{g_2^2 - 4 g_1 g_3}}{2 g_1} > 0$

Where $g_1 = \beta_1$

$g_2 = \alpha_1 - S_0 a \beta_1$

$g_3 = -(S_0 a \alpha_1 + \beta_1 H_0 S_0 + \beta_2 Q_0 S_0)$

Characteristic equation of the system (1) - (5) is given by$|A - \lambda\, I| = 0$.Corresponding to the equilibrium point $E_1$characteristic equationgives one root $\lambda = g\tilde{S} - c$ (out of the five roots), so by Routh-Hurwitz criteria $E_1$is Locally asymptotically stable if $\tilde{S} < \frac{c}{g}$ which implies that the equilibrium level of concentration of nutrient in the water is less than fraction of natural death rate of resource biomass to the rate of consumption of nutrient by the resource population. Further it is observed that $E_3$exist only when $E_1$is unstable i.e. $\tilde{S} > \frac{c}{g}$and in this case both the populations i.e. resource as well as fish populations will die out.

(ii) The second equilibrium(fish extinct) $E_2$: $(\overline{T_1} \neq 0, \overline{C_w} \neq 0, \bar{B} \neq 0, \bar{N} = 0, \bar{S} \neq 0)$

$\bar{T}_1 = \frac{H_0}{\alpha_1+\beta_1\bar{S}}$, $\bar{C}_W = \frac{Q_0}{\alpha_2+\beta_2\bar{S}}$, $\bar{S} = \frac{c}{g}$,

$\bar{B} = \frac{S_0-(a\bar{S}+\beta_1\overline{T_1}\bar{S}+\beta_2\bar{S}\overline{C_w})}{c(1-K)} > 0 \; if \; S_0 > (a\bar{S}+\beta_1\overline{T_1}\bar{S}+\beta_2\overline{SC_w})$ if

Characteristic equation of the system (1) - (5) is given by $|A - \lambda \, \mathrm{I}| = 0$ . Corresponding to the equilibrium point $E_2$ characteristic equationgives one root $\lambda = f\bar{B} - b$ (out of the five roots), so by Routh-Hurwitz criteria $E_2$ is Locally asymptotically stable if $\bar{B} < \frac{b}{f}$ which implies that the equilibrium level of resource population in the water is less than fraction of natural death rate of fish population to the specific rate of predation of fish on resource population. Further it is observed that $E_3$exist only when $E_2$is unstable i.e. $\bar{B} > \frac{b}{f}$ and in this case fish population will go to extinction.

(iii) Interior(co-existence) equilibrium $E_3$: $(T_1^* \neq 0, C_w^* \neq 0, B^* \neq 0, N^* \neq 0, S^* \neq 0)$

$T_1^* = \frac{H_0}{\alpha_1+\beta_1 S^*}$, $N^* = \frac{gS^*-c}{f} > 0 \; if \; gS^* > c$, $B^* = \frac{b}{f}$, $C_W^* = \frac{Q_0}{\alpha_2+\beta_2 S^*}$

$S^* = \frac{S_0+KcB^*+KbN^*}{a+gB^*+\beta_1 T_1^*+\beta_2 C_W^*}$ where $S^* = \frac{-h_2 \pm \sqrt{h_2^2-4h_1h_3}}{2h_1} > 0$

Where $h_1 = af\beta_1 + (1-K)bg\beta_1$

$h_2 = af\alpha_1 + \beta_1 H_0 f + \beta_2 Q_0 f - S_0\beta_1 + (1-K)bg\alpha_1$

$h_3 = -S_0\alpha_1 f$

Characteristic equation of the system (1)-(5) is given by$|A - \lambda \, \mathrm{I}| = 0$.Corresponding to the equilibrium point $E_3$ shows (as a result of the numerical and theoretical simplifications) that the system is stable under the condition $S^* > \frac{Kc}{g}$.

Now, in the following theorem we discuss the nonlinear stability of equilibrium $E_3$ which has been studied by Liapunov's direct method.

**Theorem 2.2.2:** The interior equilibrium $E_3$ is nonlinearly asymptotically stable if:

(i) $4[\alpha_1 B_2 a + \beta_1 S^* B_2 a + gB^* B_2 \alpha_1 + gB^* B_2 \beta_1 S^* + \beta_1 B_2 T_1^* \alpha_1 + \beta_2 B_2 \alpha_1 C_w^*] > 3[\beta_1^2 T_1^{*^2} + \beta_1^2 B_2^2 S^{*^2}] + 2[\beta_1^2 B_2 T_1^* S^*]$

(ii) $4[\alpha_2 B_2 a + \beta_2 S^* B_2 a + gB^* B_2 \alpha_2 + gB^* B_2 \beta_2 S^* + \beta_1 B_2 T_1^* \alpha_2 + \beta_1 \beta_2 B_2 S^* T_1^*] > 3[\beta_2^2 C_w^{*^2} + \beta_2^2 B_2^2 S^{*^2}] + 2[\beta_2^2 B_2 C_w^* S^*]$

(iii) $4[B_2B_1ab + gbB^*B_1B_2 + bB_1B_2\beta_1T_1^* + bB_1B_2\beta_2C_w^*] > 3[K^2b^2B_5^2] + 4\left( faB^*B_1B_2 + fgB^{*^2}B_1B_2 + fB_1B_2\beta_1T_1^*B^* + fB_1B_2\beta_2C_w^*B^*\right)$

**Proof:** Let we consider the following positive definite function:

$$X(T_1, C_w, B, N, S) = \frac{1}{2}(T_1 - T_1^*)^2 + \frac{1}{2}(C_w - C_w^*)^2 + \left(B - B^* - B^*log\frac{B}{B^*}\right) + B_1\frac{1}{2}(N - N^*)^2 + B_2\frac{1}{2}(S - S^*)^2$$

Let $Z_1 = (T_1 - T_1^*), Z_2 = (C_w - C_w^*)$, $Z_3 = (B - B^*)$, $Z_4 = (N - N^*), Z_5 = (S - S^*)$

$$\frac{dX}{dt} = Z_1\frac{dZ_1}{dt} + Z_2\frac{dZ_2}{dt} + \frac{Z_3}{B}\frac{dZ_3}{dt} + Z_4B_1\frac{dZ_4}{dt} + Z_5B_2\frac{dZ_5}{dt}$$

Where,

$$\frac{dZ_1}{dt} = -\alpha_1Z_1 - \beta_1S^*Z_1 - \beta_1T^*Z_5$$

$$\frac{dZ_2}{dt} = -\alpha_2Z_2 - \beta_2S^*Z_2 - \beta_2C_w^*Z_5$$

$$\frac{1}{B}\frac{dZ_3}{dt} = gZ_5 - fZ_4$$

$$\frac{dZ_4}{dt} = fN^*Z_3 + fB^*Z_4 - bZ_4$$

$$\frac{dZ_5}{dt} = -aZ_5 - gS^*Z_3 - gB^*Z_5 - \beta_1T_1^*Z_5 - \beta_1S^*Z_1 - \beta_2C_w^*Z_5 - \beta_2S^*Z_2 + KcZ_3 + KbZ_4$$

Now, choosing $B_1 = \frac{g}{gS^* - Kc} > 0$ and $B_2 = \frac{1}{N^*}$ and use these values in equation we get:

$$\frac{dX}{dt} = -[(\alpha_1 + \beta_1S^*)Z_1^2 + (\beta_1T_1^* + \beta_1B_2S^*)Z_1Z_5 + (B_2a + gB^*B_2 + \beta_1B_2T_1^* + \beta_2B_2C_w^*)Z_5^2 +$$

$$(\alpha_2 + \beta_2S^*)Z_2^2 + (\beta_2C_w^* + \beta_2B_2S^*)Z_2Z_5 + (B_2a + gB^*B_2 + \beta_1B_2T_1^* + \beta_2B_2C_w^*)Z_5^2 +$$

$$( -g + gS^*B_2 - KcB_2)Z_3Z_5 + (f - fN^*)Z_4Z_3 + (-fB^*B_1 + bB_1)Z_4^2 + (-KbB_2)Z_5Z_4 + (B_2a + gB^*B_2 + \beta_1B_2T_1^* + \beta_2B_2C_w^*)Z_5^2$$

$$\frac{dX}{dt} = -[(a_{11}Z_1^2 + a_{15}Z_1Z_5 + a_{55}Z_5^2) + (a_{22}Z_2^2 + a_{25}Z_2Z_5 + a_{55}Z_5^2) + (a_{44}Z_4^2 + a_{45}Z_5Z_4 + a_{55}Z_5^2)$$

Where,

$$a_{11} = \alpha_1 + \beta_1S^* a_{15} = \beta_1T_1^* + \beta_1B_2S^* a_{55} = B_2a + gB^*B_2 + \beta_1B_2T_1^* + \beta_2B_2C_w^*$$

$$a_{22} = \alpha_2 + \beta_2 S^* a_{25} = \beta_2 C_w^* + \beta_2 B_2 S^* a_{55} = B_2 a + gB^* B_2 + \beta_1 B_2 T_1^* + \beta_2 B_2 C_w^*$$

$$a_{44} = -fB^* B_1 + bB_1 a_{54} = (-KbB_2) a_{55} = B_2 a + gB^* B_2 + \beta_1 B_2 T_1^* + \beta_2 B_2 C_w^*$$

$X(t)$ is negative definite by Sylvester's criteria under the following condition:

(i) $a_{15}^2 < 2a_{11}a_{55}$

(ii) $a_{25}^2 < 2a_{22}a_{55}$

(iii) $a_{45}^2 < 2a_{44}a_{55}$

Hence, by Lyapunov's direct method it is proved that $E_3$ is nonlinearly asymptotically stable under the conditions given below-

(i) $4[\alpha_1 B_2 a + \beta_1 S^* B_2 a + gB^* B_2 \alpha_1 + gB^* B_2 \beta_1 S^* + \beta_1 B_2 T_1^* \alpha_1 + \beta_2 B_2 \alpha_1 C_w^*] > 3[\beta_1^2 {T_1^*}^2 + \beta_1^2 B_2^2 {S^*}^2] + 2[\beta_1^2 B_2 T_1^* S^*]$

(ii) $4[\alpha_2 B_2 a + \beta_2 S^* B_2 a + gB^* B_2 \alpha_2 + gB^* B_2 \beta_2 S^* + \beta_1 B_2 T_1^* \alpha_2 + \beta_1 \beta_2 B_2 S^* T_1^*] > 3[\beta_2^2 {C_w^*}^2 + \beta_2^2 B_2^2 {S^*}^2] + 2[\beta_2^2 B_2 C_w^* S^*]$

(iii) $4[B_2 B_1 ab + gbB^* B_1 B_2 + bB_1 B_2 \beta_1 T_1^* + bB_1 B_2 \beta_2 C_w^*] > 3(K^2 b^2 B_5^2) + 4\big(faB^* B_1 B_2 + fg{B^*}^2 B_1 B_2 + fB_1 B_2 \beta_1 T_1^* B^* + fB_1 B_2 \beta_2 C_w^* B^*\big)$

## 3. Results and Discussions

Consider the following set of parameters:

$S_0$= 1, $Q_0$= 2.2, $H_0$= 2.2, $a$= 0.1, $b$= 0.2, $c$= 0.3, $K$= 0.1, $f$= 0.08, $g$ = 0.51, $\alpha_1$ = 0.35, $\alpha_2$ = 0.36, $\beta_1$= 0.0001, $\beta_2$ = 0.0001, $\theta$ = 0.18, $\Theta_1$= 0.7124793

The region of attraction is:

$(T_1, C_w, B, N, S) \epsilon R_+^5$ $0 \leq T_1 + C_w + B + N + S \leq$ 7.579167563; $0 \leq S + B + N \leq 5.556$ ; $0 \leq T_1 \leq 12.2222$ ; $0 \leq C_w \leq 12.2222$ and the interior equilibrium of the model is $E_3$ =(6.28427649, 6.109752081, 2.5, 1.3549106, 0.800770296) the Figure:1shows time series graph and Figure:2 shows phase space graph. From the Figures it is observed that the Interiorequilibrium $E_3$ is asymptotically stable.

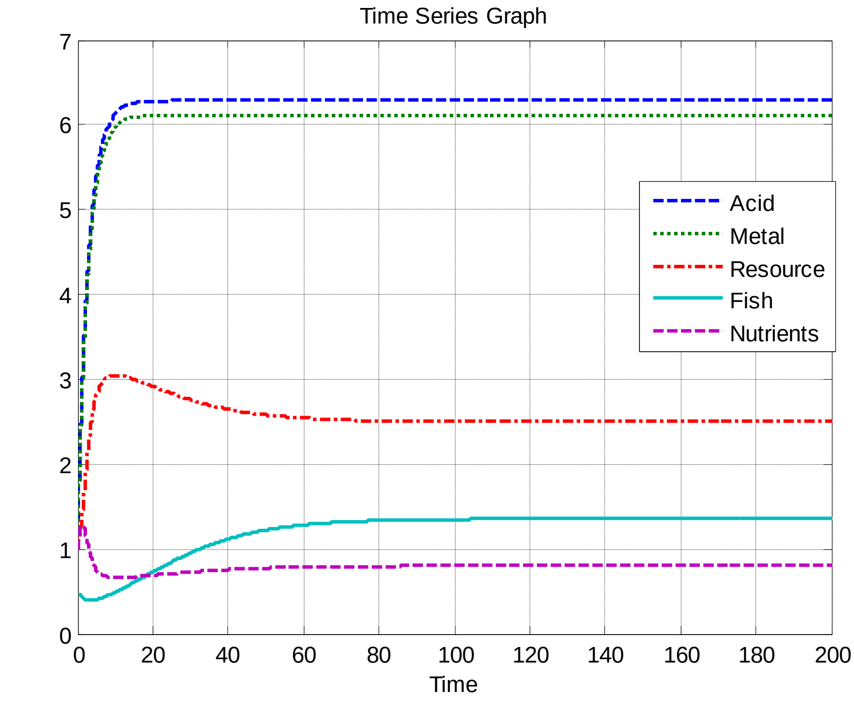


**Fig1.Time series graph of five variables of the system**

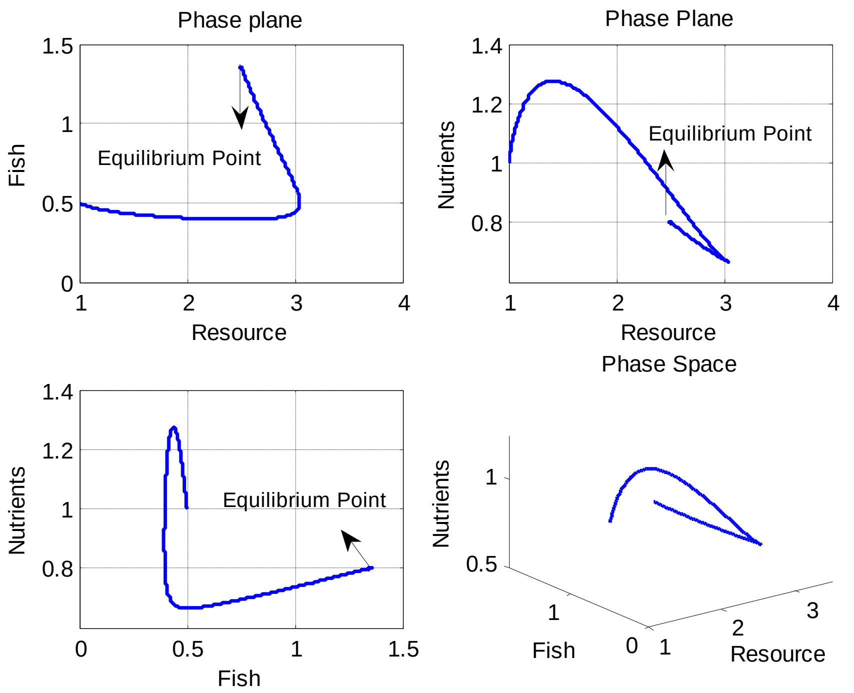


**Fig 2. Phase plane and phase space graphs**

## 4. Conclusion

In this paper, a mathematical model is studied to discuss the Combined Effects of Acid and Metal on the Survival of Resource Based population Incorporating Nutrient Recycling. The model is formulated using the systems of non-linear ordinary differential equations. In the model there are five state variables, viz, concentration of acid in water, concentration of metal in water, density of favorable resource (Phytoplanktonic Species), density of fish population and nutrient concentration. A simple predator-prey population growth model is considered along with the Holling's Type-1 functional response which is considered as a significant component of predator-prey relationship. Conditions for local stability and feasible equilibrium points (non-living, fish extinct and interior) have been determined. Non-linear stability analysis of the non-trivial equilibrium points has been discussed. It has been observed that due to toxicity in the nutrient pool, equilibrium level of resource population and fish population decreases. In the local behavior of all the feasible equilibrium it has been found that the interior equilibrium exists only when the non-living equilibrium and fish extinct equilibrium are unstable. Further, the non-living equilibrium is stable, if the equilibrium level of concentration of nutrient in the water is less than fraction of natural death rate of resource biomass to the rate of consumption of nutrient by the resource population. The fish extinct equilibrium is stable, if the equilibrium level of resource population in the water is less than fraction of natural death rate of fish population to the specific rate of predation of fish on resource population.We have also established that the interior equilibrium is locally as well as non-linearly asymptotically stable under some conditions. It has been observed that interior equilibrium is sensitive to Specific rate of predation of fish on resource population. It is found that when the equilibrium level of nutrients increases than the equilibrium of acid and metal go down, further it is showing the synergistic effect of both the stresses. It was also observed that the amount of nutrient is increasing if the population densities of resource and fish populations are going up. Conditions for the existence of the equilibrium points have been drawn and the criteria for the survival or extinction of the species has been obtained using numerical simulation. Stability of the system is explained analytically as well as graphically for the various possible cases in the form of time series and phase space and phase plan graphs.

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