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Intuitionistic Fuzzy Totally Regular Weakly Generalized Continuous Mappings

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy totally regular weakly generalized continuous mappings and intuitionistic fuzzy totally regular weakly generalized open mappings in intuitionistic fuzzy topological space. Some of their properties are explored.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy regular weakly generalized closed set, Intuitionistic fuzzy regular weakly generalized open set, Intuitionistic fuzzy totally regular weakly generalized continuous mappings, Intuitionistic fuzzy totally regular weakly generalized open mappings.

AMS Classification: 54A40, 03E72.

Introduction

Fuzzy set (FS) as proposed by Zadeh [19] in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology.

By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space.

In this paper, we introduce the notion of intuitionistic fuzzy totally regular weakly generalized continuous mappings and intuitionistic fuzzy totally regular weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy totally regular weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

Preliminaries

Definition 2.1

[1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2

[1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then,

a. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
b. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
c. $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X\}$,
d. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X\}$,
e. $A \cup B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$ shall be used instead of the longer $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3:

[3] An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

a) $0_\sim, 1_\sim \in \tau$,
b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4:

[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

int(A) = ∪ { G / G is an IFOS in X and G ⊆ A }, cl(A) = ∩ { K / K is an IFCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5:
An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ | x ∈ X} in an IFTS (X, τ) is said to be

a) *intuitionistic fuzzy semi closed set* [6] (IFSCS in short) if int(cl(A)) ⊆ A,
b) *intuitionistic fuzzy α-closed set* [6] (IFαCS in short) if cl(int(cl(A))) ⊆ A,
c) *intuitionistic fuzzy pre-closed set* [6] (IFPCS in short) if cl(int(A)) ⊆ A,
d) *intuitionistic fuzzy regular closed set* [6] (IFRCS in short) if cl(int(A)) = A,
e) *intuitionistic fuzzy generalized closed set* [16] (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS,
f) *intuitionistic fuzzy generalized semi closed set* [15] (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS,
g) *intuitionistic fuzzy α generalized closed set* [13] (IFαGCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS,
h) *intuitionistic fuzzy γ closed set* [5] (IF*γ* CS in short) if int(cl(A)) ∩ cl(int(A)) ⊆ A.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy γ open set* (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS, IFαGOS and IF*γ O*S) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS, IFαGCS and IF*γ* CS respectively.

Definition 2.6:
[8] An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ | x ∈ X} in an IFTS (X, τ) is said to be an *intuitionistic fuzzy regular weakly generalized closed set* (IFRWGCS in short) if cl(int(A)) ⊆ U whenever A ⊆ U and U is an IFROS in X .

The family of all IFRWGCSs of an IFTS (X, τ) is denoted by IFRWGC(X).

Definition 2.7:
[8] An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ | x ∈ X} is said to be an *intuitionistic fuzzy regular weakly generalized open set* (IFRWGOS in short) in (X, τ) if the complement A^c is an IFRWGCS in X.

The family of all IFRWGOSs of an IFTS (X, τ) is denoted by IFRWGO(X).

Result 2.8:
[8] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFRWGCS but the converses need not be true in general.

Definition 2.9
[9] Let (X, τ) be an IFTS and A = ⟨ x, μ_A, ν_A ⟩ be an IFS in X. Then the *intuitionistic fuzzy regular weakly generalized interior* and an *intuitionistic fuzzy regular weakly*

generalized closure are defined by wgint(A) = ∪ {G | G is an IFRWGOS in X and G ⊆A}, wgcl(A) = ∩ {K | K is an IFRWGCS in X and A ⊆ K}.

Definition 2.10

[3] Let f be a mapping from an IFTS (X, τ) into an IFTS(Y, σ). If B = {⟨y, $\mu_B(y)$, $\nu_B(y)$⟩ | y ∈ Y} is an IFS in Y, then the *pre-image* of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B)$ = {⟨x, $f^{-1}(\mu_B(x))$, $f^{-1}(\nu_B(x))$⟩ | x ∈ X}, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ | x ∈ X} is an IFS in X, then the *image* of A under f denoted by f(A) is the IFS in Y defined by f (A) = {⟨y, f ($\mu_A(y)$), f_($\nu_A(y)$)⟩ | y ∈ Y} where f_(ν_A) = 1 – f(1 – ν_A).

Definition 2.11

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then f is said to be

a) *intuitionistic fuzzy continuous* [4] (IF continuous in short) if $f^{-1}(B)$ ∈ IFO(X) for every B∈ σ,
b) *intuitionistic fuzzy α continuous* [6] (IFα continuous in short) if $f^{-1}(B)$ ∈ IFαO(X) for every B∈ σ,
c) *intuitionistic fuzzy pre continuous* [6] (IFP continuous in short) if $f^{-1}(B)$ ∈ IFPO(X) for every B∈ σ,
d) *intuitionistic fuzzy generalized continuous* [16] (IFG continuous in short) if $f^{-1}(B)$ ∈ IFGO(X) for every B∈σ,
e) *intuitionistic fuzzy* α *generalized continuous* [14] (IFαG continuous in short) if $f^{-1}(B)$ ∈ IFαGO(X) for every B∈ σ,
f) *intuitionistic fuzzy regular weakly generalized continuous* [10] (IFRWG continuous in short) if $f^{-1}(B)$ ∈ IFRWGO(X) for every B∈ σ,
g) *intuitionistic fuzzy almost continuous* [17] (IFA continuous in short) if $f^{-1}(B)$ ∈ IFO(X) for every IFROS B∈ σ,
h) *intuitionistic fuzzy almost regular weakly generalized continuous* [11] (IFARWG continuous in short) if $f^{-1}(B)$ ∈ IFRWGO(X) for every IFROS B∈ σ,
i) *intuitionistic fuzzy regular weakly generalized irresolute* [9] (IFRWG irresolute in short) if $f^{-1}(B)$ ∈ IFRWGO(X) for every IFRWGOS B∈ σ,
j) *intuitionistic fuzzy totally continuous mapping* [7] if the inverse image of every IFCS in Y is an intuitionistic fuzzy clopen subset in X,

Definition 2.12

[8] An IFTS (X, τ) is said to be an intuitionistic fuzzy rwT1/2 ((IF rwT1/2 in short) space if every IFRWGCS in X is an IFCS in X.

Definition 2.13

[8**]** An IFTS (X, τ) is said to be an intuitionistic fuzzy rwgT1/2 ((IF rwgT1/2 in short) space if every IFRWGCS in X is an IFPCS in X.

Intuitionistic Fuzzy Totally Regular Weakly Generalized Continuous Mappings

In this section, we introduce intuitionistic fuzzy totally regular weakly generalized continuous mappings and study some of their properties.

Definition 3.1

An IFS A is called intuitionistic fuzzy regular weakly generalized clopen set in (X, τ) if it is both intuitionistic fuzzy regular weakly generalized open and intuitionistic fuzzy regular weakly generalized closed in (X, τ).

Definition 3.2

A mapping f : (X, τ) → (Y, σ) from an IFTS (X, τ) into an IFTS (Y, σ) is said to be an intuitionistic fuzzy totally regular weakly generalized continuous mapping if the inverse image of every IFOS in Y is an intuitionistic fuzzy regular weakly generalized clopen set in X.

Theorem 3.3

Let f : (X, τ) → (Y, σ) be a mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.

a) f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping,
b) $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X for each IFCS B in Y.

Proof:

(a) ⇒ (b): Let B be an IFCS in Y. Then B^c is an IFOS in Y. Since f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, $f^{-1}(B^c) = (f^{-1}(B))^c$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X.

(b) ⇒ (a): Let B be an IFOS in Y. Then B^c is an IFCS in Y. By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is an intuitionistic fuzzy regular weakly generalized clopen set in X, which implies $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Theorem 3.4

Every intuitionistic fuzzy totally regular weakly generalized continuous mapping is an intuitionistic fuzzy regular weakly generalized continuous mapping but not conversely.

Proof:

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy totally regular weakly generalized continuous mapping and B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. That is, $f^{-1}(B)$ is an IFRWGOS and

IFRWGCS in X. Hence f is an intuitionistic fuzzy regular weakly generalized continuous mapping.

Example 3.5
Let X = {a, b}, Y = {u, v} and T_1= <x, (0.2, 0.6) , (0.8, 0.3)>, T_2= <y, (0.2, 0.6), (0.8, 0.3)>. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy regular weakly generalized continuous mapping but not an intuitionistic fuzzy totally regular weakly generalized continuous mapping, since the IFS T_2^c = <y, (0.8, 0.3), (0.2, 0.6)> is an IFCS in Y but $f^{-1}(T_2^c)$ = <x, (0.8, 0.3) , (0.2, 0.6)> is not an intuitionistic fuzzy regular weakly generalized clopen set in X.

Theorem 3.6
Every intuitionistic fuzzy totally continuous mapping is an intuitionistic fuzzy totally regular weakly generalized continuous mapping but not conversely.

Proof
Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy totally continuous mapping and B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Since every IFOS and IFCS is an IFRWGOS and IFRWGCS, $f^{-1}(B)$ is an IFRWGOS and IFRWGCS in X. Thus $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Example 3.7
Let X = {a, b}, Y = {u, v} and T_1= <x, (0.3, 0.5) , (0.1, 0.4)>, T_2= <y, (0.2, 0.5), (0.2, 0.5)>. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping but not an intuitionistic fuzzy totally continuous mapping, since the IFS T_2^c = <y, (0.2, 0.5) , (0.2, 0.5)> is an IFCS in Y but $f^{-1}(T_2^c)$ = <x, (0.2, 0.5) , (0.2, 0.5>> is not an intuitionistic fuzzy clopen set in X.

Theorem 3.8
Let f : (X, τ) → (Y, σ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping from an from an IFTS (X, τ) into an IFTS (Y, σ) and (X, τ) an $IF_{rw}T_{1/2}$ space. Then f is an intuitionistic fuzzy totally continuous mapping.

Proof
Let B be an IFOS in Y. Since f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Since (X, τ) is an $IF_{rw}T_{1/2}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Hence f is an intuitionistic fuzzy totally continuous mapping.

Theorem 3.9
Every intuitionistic fuzzy perfectly regular weakly generalized continuous mapping is an intuitionistic fuzzy totally regular weakly generalized continuous mapping but not conversely.

Proof
Let f : (X, τ) → (Y, σ) is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping and B be an IFOS in Y. Since every IFOS is an IFRWGOS in Y, B is an IFRWGOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Since every IFCS and IFOS are IFRWGCS and IFRWGOS, $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Example 3.10
Let X = {a, b}, Y = {u, v} and T_1 = <x, (0.6, 0.6) , (0.3, 0.3)>, T_2 = <y, (0.5, 0.5) , (0.5, 0.5)>. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f : (X, τ) → (Y, σ) defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy totally weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly regular weakly genralized continuous mapping, since the IFS T_2^c = <y, (0.5, 0.5) , (0.5, 0.5)> is an IFRWGCS in Y but $f^{-1}(T_2^c)$ = <x, (0.5, 0.5) , (0.5, 0.5)> is not an intuitionistic fuzzy clopen set in X.

Theorem 3.11
Let f : (X, τ) → (Y, σ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping from an from an IFTS (X, τ) into an IFTS (Y, σ) and (X, τ), (Y, σ) be $IF_{rw}T_{1/2}$ spaces. Then f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Proof
Let B be an IFRWGOS in Y. Since (Y, σ) is an $IF_{rw}T_{1/2}$ space, B is an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Since (X, τ) is an $IF_{rw}T_{1/2}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Hence f is an intuitionistic fuzzy perfectly regular weakly generalized continuous mapping.

Theorem 3.12
Let f : (X, τ) → (Y, σ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f(rwgcl(A)) ⊆ cl(f(A)) for every IFS A in X.

Proof
Let A be an IFS in X. Then cl(f(A)) is an IFCS in Y. Since f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, $f^{-1}(cl(f(A)))$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Thus $f^{-1}(cl(f(A)))$ is an

IFRWGCS in X. Clearly A ⊆ f^{-1}(cl(f(A))). Therefore rwgcl(A) ⊆ rwgcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))). Hence f(rwgcl(A)) ⊆cl(f(A)) for every IFS A in X.

Theorem 3.13

Let f : (X, τ) → (Y, σ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then rwgcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for every IFS B in Y.

Proof

Let B be an IFS in Y. Then cl(B) is an IFCS in Y. By hypothesis, f^{-1}(cl(B)) is an intuitionistic fuzzy regular weakly generalized clopen set in X. Thus f^{-1}(cl(B)) is an IFRWGCS in X. Clearly B ⊆ cl(B) implies f^{-1}(B) ⊆ f^{-1}(cl(B)). Therefore rwgcl(f^{-1}(B)) ⊆ rwgcl(f^{-1}(cl(B))) = f^{-1}(cl(B)). Hencer rwgcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for every IFS B in Y.

Theorem 3.14

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy totally regular weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f^{-1}(int(B)) ⊆ rwgint(f^{-1}(B)) for every IFS B in Y.

Proof

Let B be an IFS in Y. Then int(B) is an IFOS in Y. By hypothesis, f^{-1}(int(B)) is an intuitionistic fuzzy regular weakly generalized clopen set in X. Thus f^{-1}(int(B)) is an IFRWGOS in X. Clearly int(B) ⊆ B implies f^{-1}(int(B)) ⊆ f^{-1}(B). Therefore f^{-1}(int(B)) = rwgint(f^{-1}(int(B))) ⊆rwgint(f^{-1}(B)). Hence f^{-1}(int(B)) ⊆ rwgint(f^{-1}(B)) for every IFS B in Y.

Theorem 3.15

Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be two intuitionistic fuzzy totally regular weakly generalized continuous mappings. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping if (Y, σ) an $IF_{rw}T_{1/2}$ space.

Proof

Let A be an IFCS in Z. Then g^{-1}(A) is an intuitionistic fuzzy regular weakly generalized clopen set in Y, by hypothesis. Since (Y, σ) is an $IF_{rw}T_{1/2}$ space, g^{-1}(A) is an intuitionistic fuzzy clopen set in Y and hence an IFCS in Y. Further, since f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, f^{-1}(g^{-1}(A)) = $(gof)^{-1}$(A) is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Theorem 3.16

Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be any two mappings, then the following statements hold.

1. Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy totally regular weakly generalized continuous mapping and g : (Y, σ) → (Z, δ) an intuitionistic fuzzy continuous mapping. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.
2. Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy totally regular weakly generalized continuous mapping and g : (Y, σ) → (Z, δ)an intuitionistic fuzzy regular weakly generalized continuous mapping. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping if (Y, σ) is an $IF_{rw}T_{1/2}$ space.
3. Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy regular weakly generalized irresolute mapping and g : (Y, σ) → (Z, δ) an intuitionistic fuzzy totally regular weakly generalized continuous mapping. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Proof

a. Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y. Since f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.
b. Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFRWGCS in Y. Thus $g^{-1}(A)$ is an IFCS in Y, as (Y, σ) is an $IF_{rw}T_{1/2}$ space. Since f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.
c. Let A be an IFCS in Z. Since g is an intuitionistic fuzzy totally regular weakly generalized continuous mapping, $g^{-1}(A)$ is an intuitionistic fuzzy regular weakly generalized clopen set in Y. Further, since f is an intuitionistic fuzzy regular weakly generalized irresolute mapping, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is an intuitionistic fuzzy regular weakly generalized clopen set in X. Hence gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Intuitionistic Fuzzy Totally Regular Weakly Generalized Open Mappings

In this section, we introduce intuitionistic fuzzy totally regular weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties.

Definition 4.1
A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy totally regular weakly generalized open mapping if the image of every IFOS in X is an intuitionistic fuzzy regular weakly generalized clopen set in Y.

Theorem 4.2
Let f : (X, τ) → (Y, σ) is a mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.

a) f is an intuitionistic fuzzy totally regular weakly generalized open mapping,
b) f(B) is an intuitionistic fuzzy regular weakly generalized clopen set in Y for each IFCS B in X.

Proof:

a) ⇒ (b): Let B be an IFCS in X. Then B^c is an IFOS in X. Since f is an intuitionistic fuzzy totally regular weakly generalized open mapping, $f(B^c) = (f(B))^c$ is an intuitionistic fuzzy regular weakly generalized clopen set in Y. Hence f(B) is an intuitionistic fuzzy regular weakly generalized clopen set in Y.
b) ⇒ (a): Let B be an IFOS in X. Then B^c is an IFCS in X. By assumption, $f(B^c) = (f(B))^c$ is an intuitionistic fuzzy regular weakly generalized clopen set in Y, which implies f(B) is an intuitionistic fuzzy regular weakly generalized clopen set in Y. Hence f is an intuitionistic fuzzy totally regular weakly generalized open mapping.

Theorem 4.3
Let f : (X, τ) → (Y, σ) is a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.

(a) Inverse of f is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.
(b) f is an intuitionistic fuzzy totally regular weakly generalized open mapping.

Proof:

a) ⇒ (b): Let A be an IFOS in X. By assumption, $(f^{-1})^{-1}(A) = f(A)$ is an intuitionistic fuzzy regular weakly generalized clopen set in Y. Hence f is an intuitionistic fuzzy totally regular weakly generalized open mapping.
b) ⇒ (a): Let B be an IFOS in X. Then f(B) is an intuitionistic fuzzy regular weakly generalized clopen set in Y. That is, $(f^{-1})^{-1}(B) = f(B)$ is an intuitionistic fuzzy regular weakly generalized clopen set in Y. Hence f^{-1} is an intuitionistic fuzzy totally regular weakly generalized continuous mapping.

Theorem 4.4
Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be two intuitionistic fuzzy totally regular weakly generalized open mappings. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized open mapping if (Y, σ) an $IF_{rw}T_{1/2}$ space.

Proof
Let A be an IFOS in X. Then f(A) is an intuitionistic fuzzy regular weakly generalized clopen set in Y, by hypothesis. Since (Y, σ) is an $IF_{rw}T_{1/2}$ space, f(A) is an intuitionistic fuzzy clopen set in Y and hence an IFOS in Y. Further, since g is an intuitionistic fuzzy totally regular weakly generalized open mapping, g(f(A)) = (gof)(A) is an intuitionistic fuzzy regular weakly generalized clopen set in Z. Hence gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized open mapping.

Theorem 4.5
Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy open mapping and g : (Y, σ) → (Z, δ) an intuitionistic fuzzy totally regular weakly generalized open mapping. Then their composition gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized open mapping.

Proof
Let A be an IFOS in X. By hypothesis, f(A) is an IFOS in Y. Since g is an intuitionistic fuzzy totally regular weakly generalized open mapping, g(f(A)) = (gof)(A) is an intuitionistic fuzzy regular weakly generalized clopen set in Z. Hence gof : (X, τ) → (Z, δ) is an intuitionistic fuzzy totally regular weakly generalized open mapping.

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