International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 6 (2015) pp. 13841-13853

http://www.ripublication.com

# Optimal Placement of PMU Based on Vertex Colouring and AVL Tree Technique

**Gomathi Venugopal[1] and Uma Velayutham[2]**
*[1]College of Engineering, Department of Electrical & Electronics, Anna University, Chennai, India-600025*
*[2]St.Joseph's College of Engineering, Department of Electrical & Electronics, Chennai, India-600 119*
*gomuma16@gmail.com[1],umafuji2007@gmail.com[2]*

## Abstract

Phasor measurement units (PMUs) are devices which are used to measure the phasors of voltage and current in real time system. A suitable methodology is needed to determine the optimal location of PMUs in the power system. This paper proposes two different concepts namely vertex colouring and AVL tree technique for the optimal placement of Phasor Measurement Units (PMUs). The proposed methods are used to solve the optimal PMUs placement solutions for the IEEE 6 bus, IEEE 9 bus, IEEE 14 bus and IEEE 30 bus system.

**Index Terms:** Phasor Measurement Units, Optimal PMU Placement, Observability, Vertex Colouring, AVL Tree.

## Introduction

Phasor Measurement Unit (PMU) is a device that allows synchronized real-time measurements of multiple remote measurement points on the grid by providing the phasor information in real time. The advantage of referring phase angle to global reference time using Global Positioning System (GPS) is helpful in capturing the wide area snapshot of the power system. GPS is the recommended time source and IRIG-B is the basic format used for time communication [1]. It is useful in mitigating blackouts in learning the real time behavior of the power system. Therefore analysis over the network has to be done to find optimal points to place the Phasor Measurement Units.

PMUs are used to measure the voltage and current phasors at its associated bus and all incident buses. In order to avoid redundant use of PMUs the optimal locations for the PMUs must be determined. Installation of PMUs will be a gradual process, requiring decisions on the best possible locations for a limited number of PMUs in the beginning. Hence a systematic method is needed for finding the best locations for the new PMUs in the presence of other already placed PMUs and/or conventional measurements. More recently synchronized phasor measurements have started to become available at selected substations in the system. In recent years, there has been a significant research activity on the problem of finding the minimum number of PMUs and the optimal location. A power system is completely observable only when all of its states can be uniquely determined [2]. The present work also focuses to find the minimum number of 14 bus systems are given in Section V and are compared with existing methods. Finally conclusions are given in Section VII.

## Different Topologies

A suitable methodology is needed to determine the optimal locations of PMUs in a power system. The PMU placement methodology presented here ensures that the system is topologically observable during normal operating conditions. In the past few years, there has been a significant research activity on the problem of finding the minimum number of PMUs and their optimal locations. Phadke used the phasor measurement to improve the efficiency of the conventional state estimation method [3,4]. Baldwin used modified bisecting search simulated annealing to find the optimal points to place PMUs [5]. Xu.Bei proposed graph theory to find the optimal location of PMU in [6]. Nuqui presented techniques for identifying placement sites for PMUs in a power system based on incomplete observability [7]. Saikat chakrabarti proposed a method for optimal placement of PMU using binary search algorithm [9]. Nuqui used spanning tree technique for the constraints on the PMU placement algorithm found the optimal location using simulated annealing technique [8].

## Problem Formulation

The optimal placement of PMU becomes an important problem to be solved in power system state estimation. The PMU placement problem is formulated as a binary integer linear programming, in which the binary decision variables (0,1) determine whether to install a PMU at each bus, while preserving the system observability and lowest system metering economy. It is neither economical nor necessary to install a PMU at every node of a wide-area interconnected network.

The cost of a PMU depends on a number of factors, including the number of measuring terminals (channels), CT and PT connection, power connection, station ground connection, and GPS antenna connection. The main purpose of performing

PMU placement problem is to minimize the number of installed PMUs. Thus for an n-bus system the optimization problem is given as:

$$minimize \sum_{i}^{n} w_i x_i$$

PMUs to make the system topologically observable, as well as the optimal locations of these PMUs.

The remaining parts of the paper are organized as follows. Section II briefly gives idea about the existing topologies for

Subject to f(x) ≥ î

the optimal placement of PMUs. Section III formulates the optimal PMU placement problem. Section IV presents the Where the installation is cost of the PMU at bus i

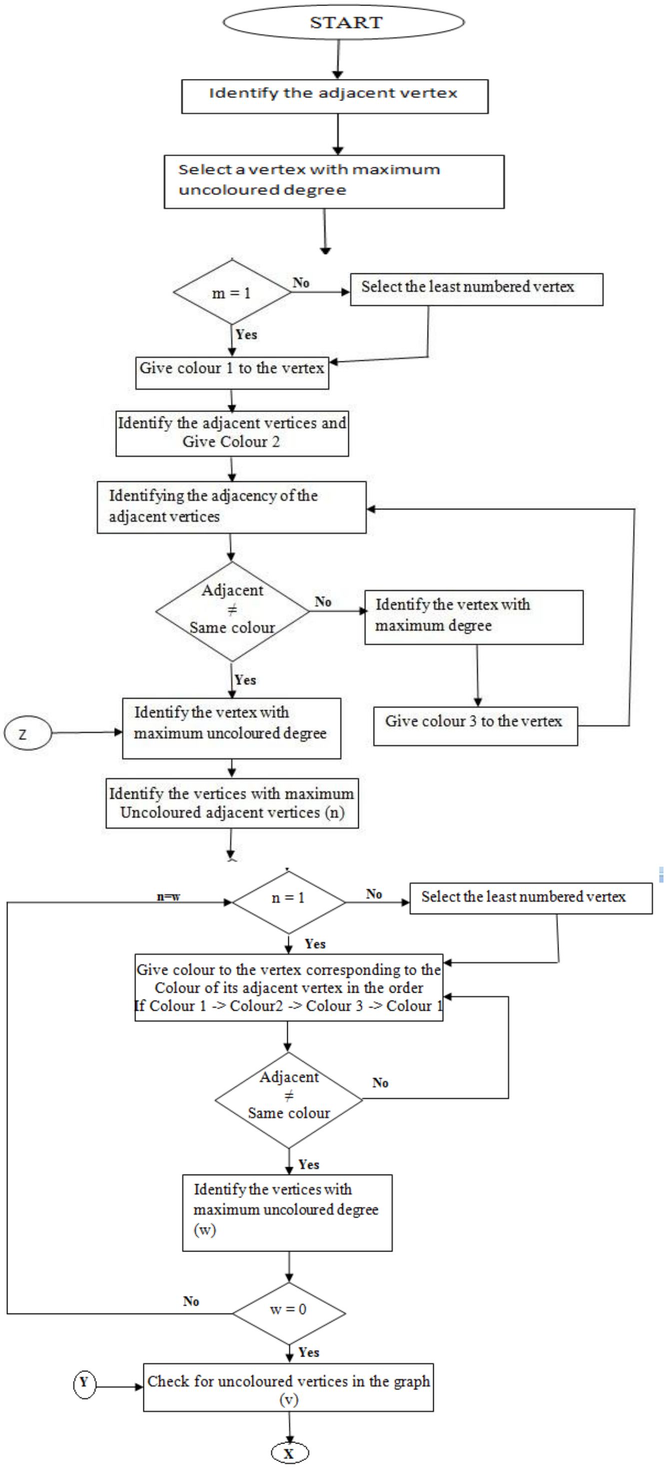
Assume =1

implementation of the proposed methods for different test systems. Test Results on IEEE 6 bus, IEEE 9 bus and IEEE f (X) is a vector function representing the constraints. î is a vector whose entries are all equal to 1.

The above OPP problem formulation is solved here using new proposed methodologies namely Vertex Colouring and AVL tree technique.

### A. Vertex Colouring

This topic deals with the implementation of vertex colouring for finding the locations for the optimal PMU placement in a bus system.


START
Identify the adjacent vertex
Select a vertex with maximum uncoloured degree
m = 1
No
Select the least numbered vertex
Yes
Give colour 1 to the vertex
Identify the adjacent vertices and Give Colour 2
Identifying the adjacency of the adjacent vertices
Adjacent ≠ Same colour
No
Identify the vertex with maximum degree
Give colour 3 to the vertex
Yes
Z
Identify the vertex with maximum uncoloured degree
Identify the vertices with maximum Uncoloured adjacent vertices (n)
n=w
n = 1
No
Select the least numbered vertex
Yes
Give colour to the vertex corresponding to the Colour of its adjacent vertex in the order If Colour 1 -> Colour2 -> Colour 3 -> Colour 1
Adjacent ≠ Same colour
No
Yes
Identify the vertices with maximum uncoloured degree (w)
w = 0
No
Yes
Y
Check for uncoloured vertices in the graph (v)
X

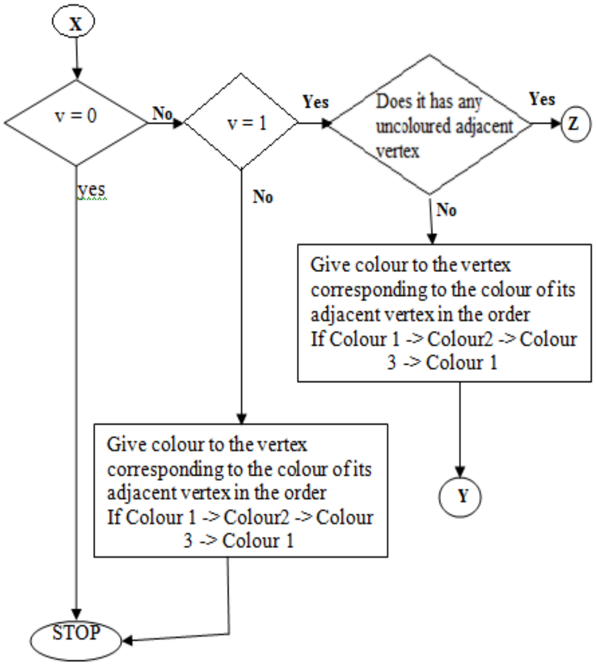


**Figure 1:** Flow Chart For Colouring The Vertices of The Graph.

Initially assume the buses to be the nodes or vertices of a graph and the transmission lines to be the edges connecting those nodes and convert it into a graph G with a non-empty vertex set V(G) and a non empty edge set E(G). The degree of a vertex in a graph G is the number of edges of G incident to it [10]. Adjacency is the property in which the vertex is connected to another vertex through an edge.

A k-vertex colouring of G is an assignment of k colours say 1, 2 . . . , k, to the vertices of G. The colouring is proper if no two distinct adjacent vertices have the same colour. The chromatic number $\chi(G)$, of G is the minimum k for which G is k-colourable. If $\chi(G) = k$, G is said to be k-chromatic. The uncoloured degree of a vertex in G is said to be the number of edges connecting uncoloured vertex with that vertex.

The flow chart in Fig. 1 describes the steps followed to optimally place PMUs in a network.

A k-chromatic graph is obtained and the colour that is used minimum is considered and its corresponding vertices are found to be the optimum places for the placement of PMU. If more than one colour has minimum chromatic number, then based on the satisfaction of observability the optimal solution is found. Even if there is only one colour with minimum chromatic number the topological observability has to be verified.

### B. AVL Tree Technique

The other proposed method for the optimal PMU placement problem is the AVL tree technique. AVL trees were invented by Adelson-Velskii and Landis in 1962. AVL tree is a binary search tree where the height of the left subtree differs from the height of the right subtree by at most 1 level, if it exceeds 1 level then rebalancing occurs. Insertions and deletions may require the tree to be rebalanced by one or more tree rotations.

A balanced binary tree is a binary tree where every node satisfies the property that the left and right subtrees of node differ in height by one.

BalanceFactor = height (right-subtree) – height (left-subtree)An AVL tree is a binary search tree in which the balance- factor of each node is in the range -1 to 1.

-1: height of the left subtree is one greater than the right subtree.

0: height of the left and right subtrees is equal.

+1: height of the right subtree is one greater than the left subtree.

Buses are considered as a nodes and based on the connectivity of the nodes an adjacency matrix is formed. An adjacency matrix is a matrix which describes a graph by representing which vertices are adjacent to which other vertices. If *G* is a graph of order *n*, then its adjacency matrix is an *n*×*n* square matrix, which is called vertex matrix or adjacency matrix. The adjacency matrix is defined by

$$M_{ij} = \begin{cases} 1 & P_i \rightarrow P_j \\ 0, Otherwise \end{cases}$$

From the adjacency matrix form the order of the matrix and create a graph.

For making the system topologically observable using PMUs, following simple rules are used,

1. If voltage phasor and current phasor at one end of a branch are known, voltage phasor at the other end of the branch can be calculated using Ohm’s law.
2. If voltage phasors at both the ends of a branch are known, branch current can be calculated.
3. If there is a zero injection bus without a PMU, whose outgoing currents are known expect for one, then the unknown outgoing current can be calculated using Kirchhoff’s Current Law (KCL).

## Case Studies

### A. Vertex Colouring

The vertex colouring method for the optimal placement of PMUs is tested on the IEEE 6 bus, IEEE 9 bus and IEEE 14 bus systems. Standard IEEE 14 bus system shown in Fig. 2 is modeled as a graph in Fig 3. Here the buses are considered as nodes and the transmission lines are considered as edges.

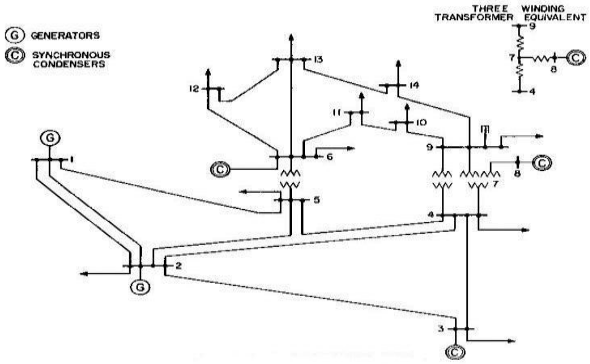


**Figure 2:** IEEE 14 bus test system

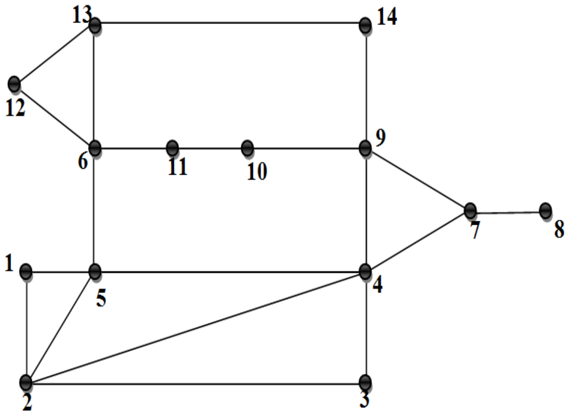


**Figure 3:** Graphical Representation of IEEE 14 Bus System

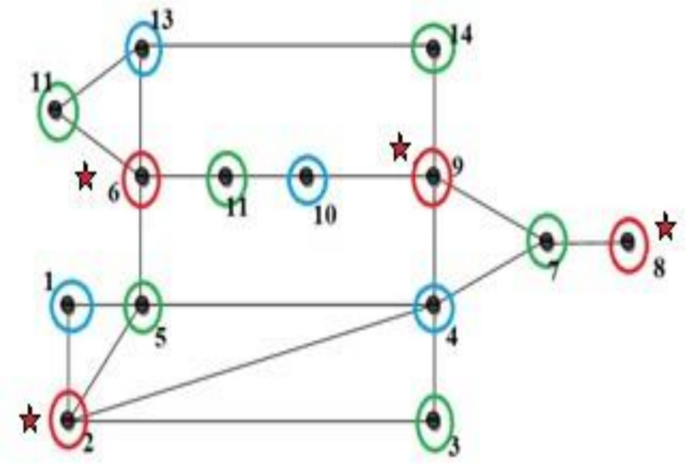


**Figure 4:** Network Coloured Using Vertex Colouring

The graph for which the vertex colouring algorithm is applied is shown in Fig. 4.

The vertices corresponding to the least chromatic number, satisfying the observability are found to be the optimal places for locating the PMUs. In IEEE 14 bus system, PMUs are placed in 2,6,8,9. is used to indicate the location of placement of the PMU in Fig. 4.

## B. AVL Tree Technique

In the proposed strategy the set of nodes where the PMUs are to be optimally located is found using a non-linear tree structure. For finding the solution, each node in the power system network is assigned a weight based on the connectivity strength using which the nodes are ordered for constructing the AVL Tree. An adjacency matrix is to be constructed to know the connectivity strength of each node to other node. The AVL tree technique for the optimal placement of PMUs is tested on Anderson Fouad 9 bus, IEEE 14 bus, IEEE 30 bus systems.

The adjacency matrix of the Anderson and Fouad 9 Bus system is as follows:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 7 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 9 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

The set of nodes in the 9 Bus system with connectivity strength is given below:
{ (1, 1), (2, 1), (3, 1), (4, 3), (5, 2), (6, 2), (7, 3), (8, 2), (9, 3) }

The nodes are to be arranged in such a way that the next node considered from the adjacency matrix should not have connectivity to the next node to be visited. For instance, in the 9-Bus system, consider the node number 2 as the starting node. The starting node is to be selected from the list of nodes, which is having the lowest connectivity strength. The next node to be visited is 1, 3, 4, 5, 6, 8 or 9. The complete list should be constructed in the way such that no two consecutive nodes have connectivity. Since the node 2 is connected with node 7, it can be assumed that the next node to be considered is 6. Since node 6 is connected to 4, the next node to be considered is 3. Subsequently, the next nodes to be considered for the construction of AVL tree are 8, 5, 1, 7, 9 and 4. Hence the order of the nodes to be used to construct the AVL tree is:{2, 6, 3, 8, 5, 1, 7, 9, 4}

The AVL tree constructed for the 9 Bus system in the above order is shown in Fig. 5. The nodes 4, 7 and 9 are the optimal locations for the placement of PMUs, which are leaf nodes in the AVL Tree.

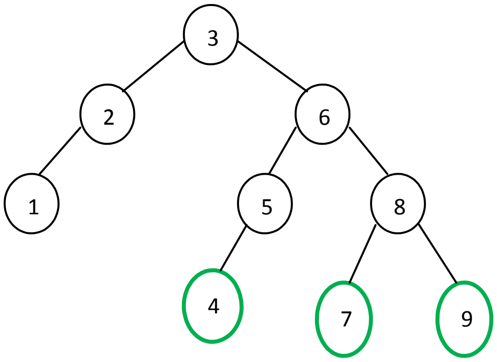


**Figure 5:** AVL structure for 9 bus

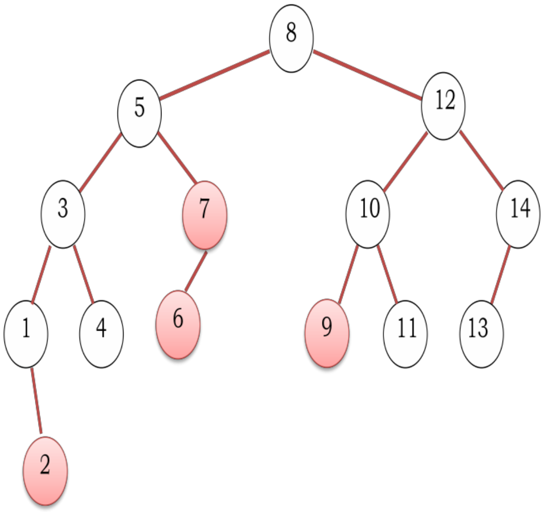


**Figure 6:** AVL Tree structure for IEEE 14 Bus system

It is interesting to note that the nodes having high value of importance factor with respect to the retained nodes always presented in the bottom level, most probably as leaves in the right sub-tree, sometimes the parent of one child in the bottom level. The same procedure is followed for IEEE 14 bus system and the result is shown in Fig. 6. The observability condition is checked in leaf nodes and its parent nodes, if the condition is satisfied PMUs are placed in that buses.

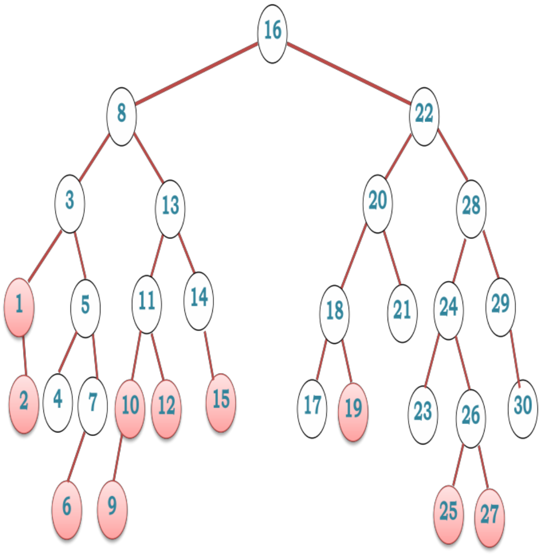


**Figure 7:** AVL Tree Structure For IEEE 30-Bus System

From the AVL tree obtained for the IEEE 30 bus system shown in Fig. 7, it is again found that the majority of the nodes where PMUs are to be installed form the leaves in the right sub-tree, sometimes the parent of one child in the bottom level. In the IEEE 30 bus system PMUs are placed in the following places 1,2,6,9,10,12,15,19,26,27 using AVL tree technique for the network to be completely observable.

## Results

In this paper, the optimal PMU placement is formulated to minimize the number of PMU installation subjecting to full network observability. The minimum number of PMUs needed to make a system observable is found by using different methodologies namely Vertex Colouring and AVL tree technique. Table I shows the Optimal placement of PMUs using vertex colouring for IEEE 6 bus, IEEE 9 bus and IEEE 14 bus test system.

**Table 1:** Optimal Placement of Pmus For Different Bus Test Systems Using Vertex Colouring

| **System** | **Number of PMUs** | **Placement** |
|---|---|---|
| IEEE 6 bus | 2 | 3,6 |
| IEEE 9 bus | 3 | 4,6,8 |
| IEEE 14 bus | 4 | 2,6,8,9 |

**Table 2:** Optimal Placement Of Pmus For Different Bus Test Systems Using AVL Tree Technique

| **System** | **umber of PMUs** | **Placement** |
|---|---|---|
| Anderson and Fouad 9 Bus | 3 | 4,7,9 |
| IEEE 14 bus | 4 | 2,6,7,9 |
| IEEE 30 bus | 10 | 1,2,6,9,10,12,15,19,25,27 |

**Table 3:** Comparison Of The Results Of Proposed Methods With The Already Existing Methods

| **Method** | **umber of PMUs** | **Placement** |
|---|---|---|
| **Vertex Colouring** | **4** | **2,6,8,9** |
| **AVL Tree** | **4** | **2,6,7,9** |
| Binary particle swarm optimization | 4 | 2,6,7,9 |
| Simulated Annealing | 4 | 1,4,6,9 |
| Binary search algorithm | 4 | 2,6,7,9 |
| Modified invasive weed optimization | 4 | 2,6,7,9 |

Table II shows the optimal placement of PMUs using AVL tree technique for Anderson and Fouad 9 bus and IEEE 14 bus test system. Table III shows the comparison of different methods used for the optimal placement of PMUs in IEEE 14 bus system. From Table III, it is concluded that the number of PMUs required for maintaining the observability is the same.

## Conclusion

In this paper, two different approaches have been proposed for determining the optimal number and location of PMUs such that the entire power system becomes completely observable. Compared to the exiting methods for Optimal PMU placement, the proposed methods have faster convergence and can be provide effective solution for real time networks also. The proposed vertex colouring method has been applied to different test systems namely IEEE 6 bus, IEEE 9 bus and IEEE 14 bus systems. The AVL tree technique has been tested with Anderson Fouad 9 bus, IEEE 14 bus and IEEE 30 bus systems. The test results obtained from the proposed methods have been compared with the results obtained by several other techniques available in the literature and the solution obtained has been validated.

## References

[1]. K.E.Martin, G.Benmouyal, and M.G.Adamiak, 1998. IEEE Standard for Synchrophasors for Power Systems. IEEE Transactions on Power Delivery, 13(1):

[2]. A.Abur and A.G.Exposito, 2004. Power System State Estimation: Theory and Implementation. New York: Mercel Dekker.

[3]. A.G.Phadke, 1993. Synchronized phasor measurements in power systems. IEEE Computer Applications in Power, 6(2),10-15.

[4]. A.G.Phadke, J.S.Thorp and K.J.Karimi, 1986. State Estimation with Phasor Measurements. IEEE Transactions on power systems, 1(1), 233-241.

[5]. T. L. Baldwin, L. Mili, M. B. Boisen, Jr., and R. Adapa, 1993. Power system observability with minimal phasor measurement placement, IEEE Transaction on Power Systems, 8(2), 707-715.

[6]. Xu Bei, Yeo Jun Yoon and Abur, 2005. Optimal placement and utilization of Phasor measurements for state estimation, IEEE Transaction on Power Delivery. 20(4), 2831-2838.

[7]. R. F. Nuqui, A. G, Phadke, 2005. Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability. IEEE Transaction on Power Delivery, 20(4),2381 – 2388, October 2005

[8]. R. F. Nuqui and A. G. Phadke, 2005. Phasor Measurement Unit placement techniques for complete and incomplete observability. IEEE Transaction Power Delivery, 20(4),2381-2388.

[9]. Saikat Chakrabatri and Elias, 2008. Optimal Placement of phasor measurement units for Power system Obsevability. IEEE Transaction on Power delivery, 23(3).

[10]. J.A.Bondy and U.S.R.Murty, 1982. Graph Theory with Applications,Elsevier Science Publishing Co., Inc., 5th Edition.