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Space Vector and Space Vector Modulation In Electrical Machines

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Abstract

The concepts of Space-Vectors and Space-Vector Modulation in Electrical machines suddenly appear in post graduate texts dealing with speed control of three phase electrical machines where as they hardly make any appearance in undergraduate texts dealing with the basic subject of Electrical Machines .The situation is rather uncomfortable for students and this paper, therefore, presents a suitable step-by-step introduction to these concepts, which should make things more comfortable for the students .

Keywords: Three Phase Induction Motors, Mmf Distribution, Space Vector, Space Vector Modulation

Symbols:-

n_c = Number of turns per phase
F_a= mmf distribution of the phase winding a
F_{av} = Space-vector representation of F_a
ω = electrical frequency rad/sec

Intoduction

Text-books dealing with electrical machines and drives [1]-[4] discuss space-vector modulation at great length but do not introduce the space-vector concept explicitly or clearly.

A heteropolar rotating electrical machine is essentially cylindrical in physical structure. It is the narrow annular cylindrical air-gap between stator and rotor where electromechanics of the machine happen. This gap is the space in which the space-vectors of our interest are defined .The definition of a space-vector, the existence of the different space-vectors in a three phase machine and their addition are discussed in the subsequent sections of this paper.

Definition of MMF Space Vectors

Fig. 1 shows the developed diagram of an elementary three-phase machine .The magneto–motive force distribution of phase a is also shown in fig.1

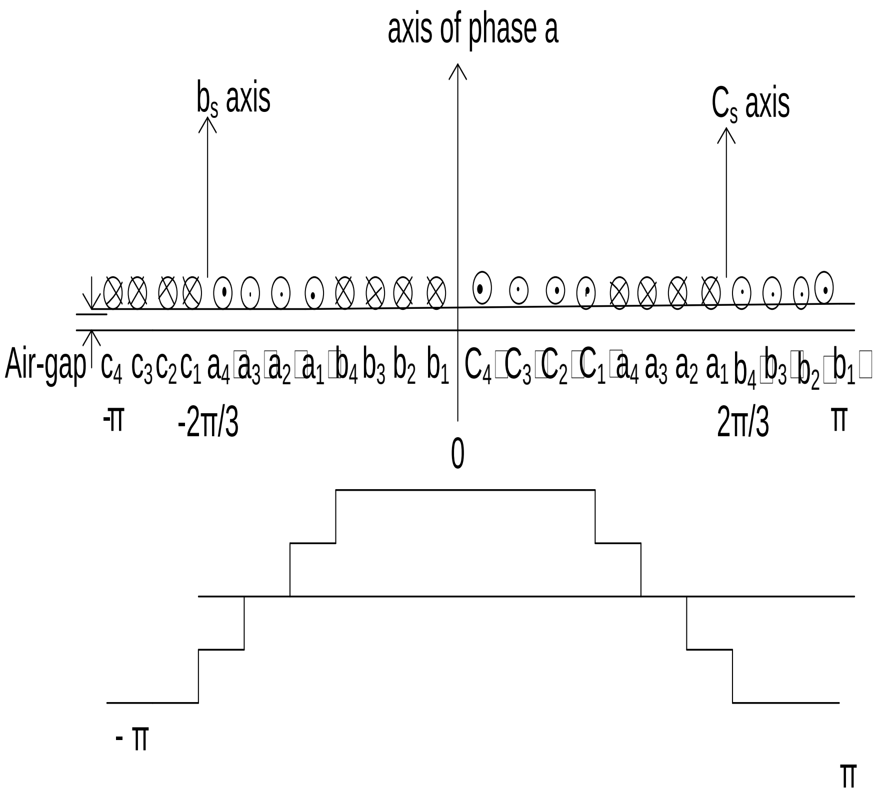

Figure 1: The magneto motive force distribution of phase ‘a’

The mmf distribution for phase b and c will be similar but with their axes shifted by $\frac{2\pi}{3}$ radians successively **.**The waveform of the illustrative winding is not sinusoidaly distributed in space**.**

Further the resultant mmf distribution in the air-gap can be obtained by summing up the individual waveforms. To take an example if *ia* =Im and *ib*=*ic*=-Im/2. (This condition corresponds to the instant when the three phase currents are balanced and the current in phase a is maximum**)** the resultant mmf distribution waveform will be as shown in Fig. 2

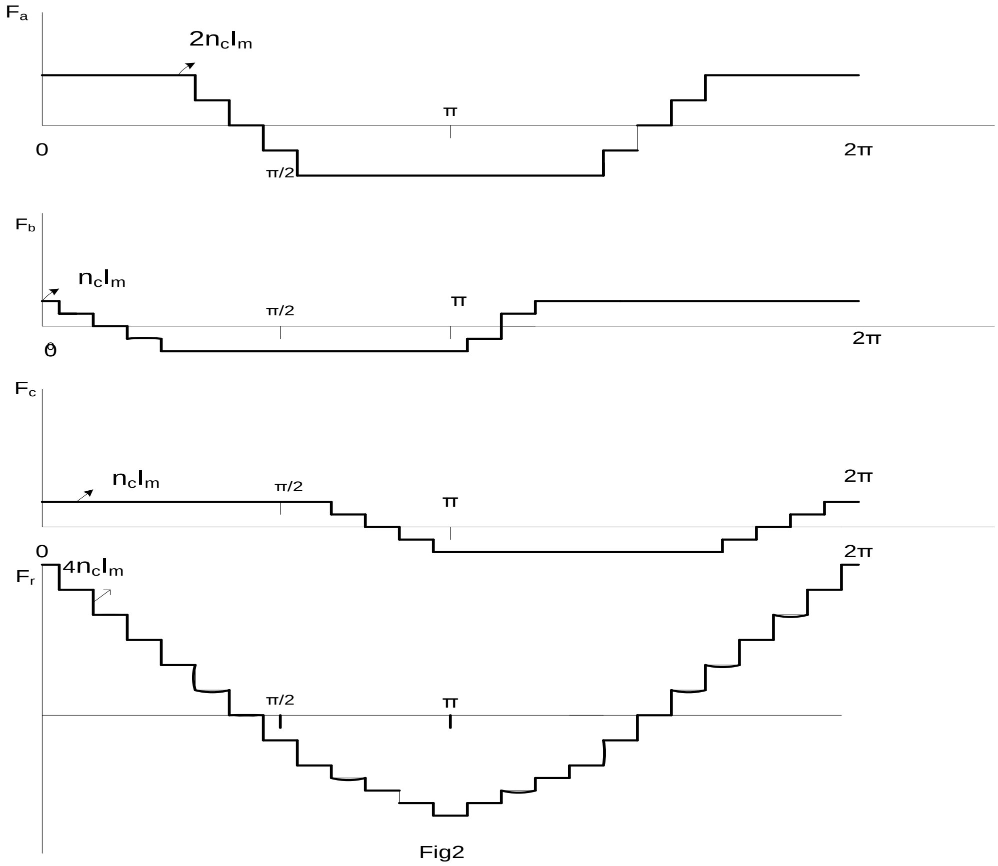

Figure 2: Mmmfs F_a **,**F_b,F_c andF_r

It is seen that though the mmf distributions of individual phases are similar in shape, but the resultant waveform is completely different. This can be verified by assuming any other set of values for the currents i_a,i_b, and i_c This situation is not satisfactory. It is not desirable that different waveforms of space distribution of mmfs are to be dealt with..

However, if the currents in the slots of a phase are sinusoidally distributed [1], then the mmf distributions of the phases turn out to be pure sine waves. Taking the axis of phase a as our benchmark for measuring angular displacement along the air–gap i.e. θ =0 at the air-gap location corresponding to the axis of phase a the mmf distribution of phase a can be written as

: $\mathbf{F}_a = k i_a \cos\theta$ (1)

Similarly the mmf distribution of phase b and c can be written as

: $\mathbf{F}_b = k i_b \cos[(\theta - \frac{2\pi}{3})]$ (2)

: $\mathbf{Fc} = k i_c \cos[(\theta + \frac{2\pi}{3})]$ (3)

The resultant $\mathbf{F}_r$ can be graphically determined as was done in earlier case, but in the present case, the summation can be done analytically with ease .Thus

: $\mathbf{F}_r = k i_a \cos\theta + k i_b \cos(\theta - \frac{2\pi}{3}) + k i_c \cos(\theta + \frac{2\pi}{3})$

$= k[(i_a - 0.5 i_b - 0.5 i_c)\cos\theta + \sqrt{3}/2 (i_b - i_c)\sin\theta]$

: $= k i_r \cos(\theta - \Phi)$: (4)

Where $i_r = \{(i_a - 0.5 i_b - 0.5 i_c)^2 + 0.75 (i_b - i_c)^2\}^{1/2}$

: And $\tan \Phi = \frac{\sqrt{3}(ib - ic)}{2ia - \text{ib} - ic}$: (5)

Eqn(4) says that the individual sinusoidaly space distributed mmf waves add up to give a resultant distribution of mmf which is also sinusoidal with the same periodicity in space .Thus with sinusoidal space distribution secured for each phase ,their addition is also maintained as sinusoidal.

In the annular air-gap space where θ is varying, a space –vector mmf can now be defined .The angle of this vector corresponds to the value of the angular position where the maximum mmf is obtained and this maximum. mmf itself corresponds to the magnitude of this vector. Following **definition**, the space-vectors for the three phase mmfs of Eqns.(1) –(3) can be written as

: $\mathbf{F_{av}} = k i_a \angle 0^o$: (6)

: $\mathbf{F_{bv}} = k i_b \angle \frac{2\pi}{3}^o$: (7)

: $\mathbf{F_{cv}} = k i_c \angle - \frac{2\pi}{3}^o$: (8)

The motivation of using space-vector notation is the convenient way in which they can be added . Thus

$\mathbf{F_r} = k i_a \angle 0^{o+} k i_b \angle \frac{2\pi}{3}^o + k i_c \angle - \frac{2\pi}{3}^o$

: $= k[i_a - 0.5 i_b - 0.5 ic]\mathbf{a_x} + k[\frac{\sqrt{3}}{2}(i_b - i_c)]\mathbf{a_y}$ (9)

: Hence $|Fr| = k\{[i_a - 0.5 i_b - 0.5 i_c]^2 + 0.75 [i_b - i_c]^2\}^{1/2}$ (10)

: And $\angle F_r = \tan^{-} \frac{\sqrt{3}(ib - \text{Ic})}{2ia - 2ib - 2ic}$ (11)

This agrees with the results of Eqns.(4) and (5) as it should ,but the addition in terms of space vectors is more easy and elegant.

The Process of Space-Vector Modulation

Eqn. (9) shows that the resultant space-vector is resolved into two components in the x and y directions .The magnitude of each component is dependent on the values of the currents i_a, i_b, and i_c .This means that by adjusting these values suitably, it is possible to control the magnitude and angle of the resultant space-vector.

The deliberate process of adjusting the values of these currents to achieve the desired magnitude and angle of the resultant space vector is space vector modulation intended in the control of induction motor drives.

Though the resultant mmf space -vector discussed so far is made up of three constituent space –vectors having their individual directions $\frac{2\pi}{3}$ radians apart from each other .Eqn. (9) shows that two space vectors at 90^o apart from each other are the minimum necessary and sufficient ones for implementing space-vector modulation . This will correspond to the operation of a two- phase machine.

On the other hand ,the expression of space-vector as a magnitude at some angle θ indicates that if phase windings are available for every angle Φ possible ,the space-vector modulation can be obtained just by switching currents continuously from one phase winding at an angle Φ to the next phase winding available at an angle Φ +d Φ .

However having a large number of phases for the induction motor is neither practical nor desirable .Hence ‘ three as the number of phases just seems to be fine.

A Look At The Conventional Three Phase Induction Motor

It is well-known that in a conventional three-phase induction motor running from a constant voltage constant frequency bus bars, the phase currents can be written as –

$$: i_a= I_m\cos \omega t \quad (12)$$

$$: i_b=I_m\cos(\omega t-\frac{2\pi}{3}) \quad (13)$$

$$: i_c=I_m\cos(\omega t+\frac{2\pi}{3}) \quad (14)$$

Corresponding mmf space vector are as

$$\mathbf{F_{av}} =kIm\cos\omega t\angle 0^o$$

$$: =kI_m\cos\omega t a_x \quad (15)$$

$$\mathbf{F_{bv}} = kI_m\cos(\omega t-\frac{2\pi}{3}) \angle\frac{2\pi}{3}$$

$$=kI_m\cos(\omega t-\frac{2\pi}{3})\cos\frac{2\pi}{3}\mathbf{a_x}+kI_m\cos(\omega t-\frac{2\pi}{3})\sin\frac{2\pi}{3}\mathbf{a_y}$$

$$: =-0.5kI_m\cos(\omega t-\frac{2\pi}{3})\mathbf{a_x}\frac{+\sqrt{3}}{2}kI_m\cos(\omega t-\frac{2\pi}{3})\mathbf{a_y} \quad (16)$$

$$\mathbf{F_{cv=}} kI_m\cos(\omega t+ \frac{2\pi}{3}) \angle - \frac{2\pi}{3}$$

$$=kI_m\cos(\omega t- \frac{2\pi}{3}) \cos\frac{2\pi}{3} a_x- kI_m\cos(\omega t-\frac{2\pi}{3})\sin\frac{2\pi}{3}a_y$$

$$: =0.5Im\cos(\omega t+\frac{2\pi}{3})\mathbf{a_x}-\sqrt{3}/2kI_m\cos(\omega t+\frac{2\pi}{3})\mathbf{a_y} \quad (17)$$

Hence the x –component **F** rx of the resultant mmf **F** r is

$\mathbf{F}_{rx} = kI_m[\cos\omega t - 0.5\cos(\omega t - \frac{2\pi}{3}) - 0.5\cos(\omega t + \frac{2\pi}{3})]$

$= kI_m[\cos\omega t + \cos\omega t \cos\frac{2\pi}{3})$

$$: = 1.5kI_m\cos\omega t \qquad (18)$$

And the y- component $\mathbf{F}_{ry}$ of F_r is

$\mathbf{F}_{ry} = \sqrt{3}/2kI_m[\cos(\omega t - \frac{2\pi}{3}) - \cos(\omega t + \frac{2\pi}{3})]$

$= 1.5\ kI_m(2\sin\omega t \sin\frac{2\pi}{3})$

$$: = 1.5\ kI_m\sin\omega t \qquad (19)$$

Hence $\mathbf{F}_r = \mathbf{F}_{rx} a_x + \mathbf{F}_{ry} a_y$

$= 1.5kIm[\cos\omega t\ \mathbf{a_x} + \sin\omega t\ \mathbf{a_y}]$

$$: = 1.5kI_m \angle \omega t \qquad (20)$$

Equation (19) implies that the system of three phase currents modulate the angle of the resultant space-vector in a continuous analog fashion .The magnitude is decided by the voltage of the system. Thus the millions of three phase induction motors have been implementing space-vector modulation for about hundred years now, though the terms space-vector and space–vector modulation are coined only recently.(since1975)

However, there are very severe restrictions to this space vector modulation. Both the magnitude and frequency 'ω of the supply voltage being constant ,the space-vector magnitude and the rotational speed of the vector is constant .The three-phase induction motor fed from the utility mains is confined essentially to a constant speed operation However ,it has been appreciated for a long time that a variable –voltage ,variable frequency supply will free the induction motor from the shackles of a constant –speed operational environment

Voltages As Space Vectors

If a voltage V_a is applied to the phase winding a of an induction motor, a current Ia will be established in it. This **Ia** is really the space-vector, but since it is V_a which is causing the current Ia, the voltage V_a applied to the phase winding'a can also be considered as a space vector. Similarly the voltages V_b and V_c can also be considered as space vectors. Thus if the angles corresponding to the axes of phases a , b and c respectively are 0, $\frac{2\pi}{3}$ and $-\frac{2\pi}{3}$radians respectively. the corresponding space vectors are

$$: \mathbf{V_a} = V_a \angle 0 \qquad (21)$$

$$: \mathbf{V_b} = Vb \angle \frac{2\pi}{3} \qquad (22)$$

$$: \mathbf{V_c} = Vc \angle -\frac{2\pi}{3} \qquad (23)$$

Voltage Space Vector Modulation Using Three Leg Inverter

Fig.3 shows the familiar three –leg inverter configuration. The upper and lower switches are complementary.

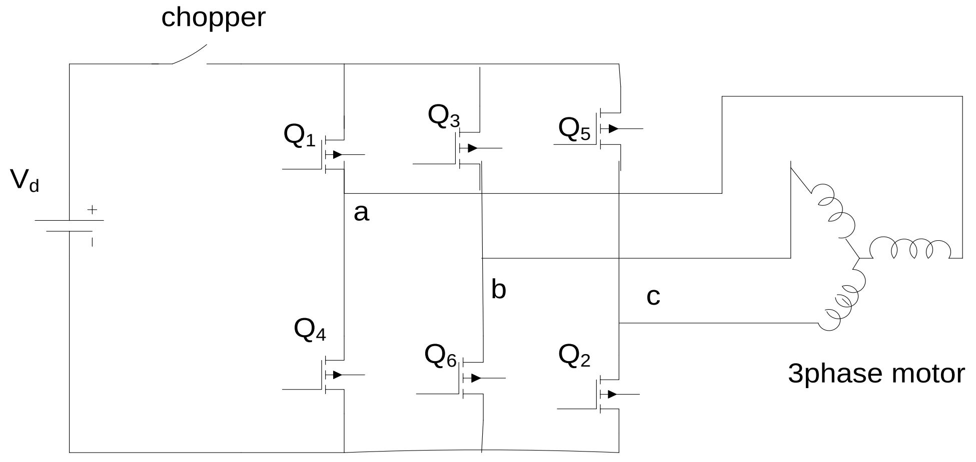

Figure 3: Three leg inverter driving three phase motor

Hence there are eight possible switch combinations 000,001,010,011,100,101,110, and111 (0 corresponds to open switch and 1 corresponds to closed position for the upper switches.) Table 1 gives a summary of the switching states and the corresponding phase to neutral voltage of an isolated neutral three–phase machine. Consider for example state 2 where switches Q_1,Q_3, and Q_2 are closed. In this case phase □ a □□ and phase □ b □ are connected to the positive bus and the phase □c □ is connected to the negative bus .Obviously the three voltage space vectors are

: $\mathbf{V_{an}}=\frac{1}{3}V_d\angle 0$ (24)

: $\mathbf{V_{bn}}=\frac{1}{3}V_d\angle +\frac{2\pi}{3}$ (25)

: $\mathbf{V_{cn}}=\frac{2}{3}V_d\angle -\frac{2\pi}{3}$ (26)

Hence resultant space vector is

$\mathbf{V_R} = \frac{2}{3}(\mathbf{V_{an}}+\mathbf{V_{bn}}+\mathbf{V_{cn}})$

$=\frac{2}{9}V_d[1\angle 0+ 1\angle\frac{2\pi}{3}-2\angle\frac{4\pi}{3}]$

: $= -\frac{2}{3}V_d\angle -2\pi/3 = \frac{2}{3} V_d\angle\frac{\pi}{3}$ (27)

Similar calculations can be done for the active-states 1 to 6 .For switch states 0 and 7, the resultant space vector is zero .The six –active space-vectors are shown in fig.4

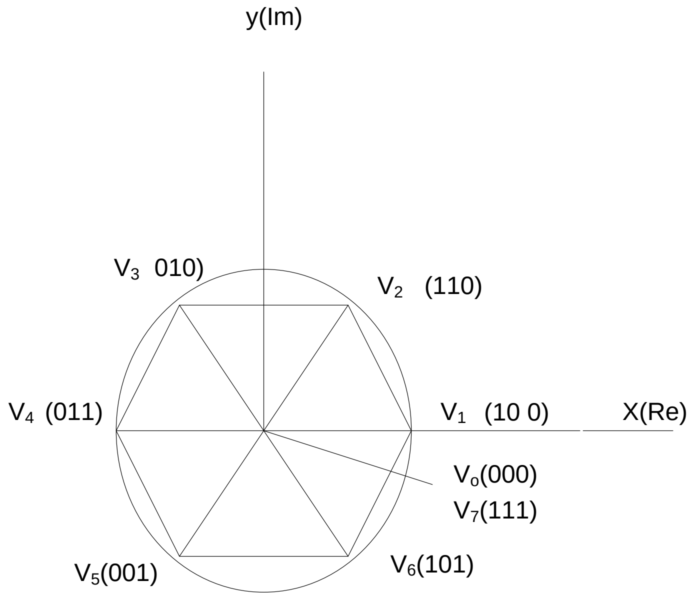

Fig.4
Space vectors and the corresponding switching states

Table 1:

State	A B C	**Va**	**V_b**	**V_c**	**V_R**
0	000	0	0	0	0
1	100	$\frac{2}{3}V_d$	$-\frac{1}{3}V_d$	$\frac{1}{3}V_d$	$\frac{2}{3}V_d\angle 0$
2	110	$\frac{1}{3}V_d$	$\frac{1}{3}V_d$	$-\frac{2}{3}V_d$	$\frac{2}{3}V_d\angle 60$
3	010	$-\frac{1}{3}V_d$	$\frac{2}{3}V_d$	$-\frac{1}{3}V_d$	$\frac{2}{3}V_d\angle +120$
4	011	$-\frac{2}{3}V_d$	$\frac{1}{3}V_d$	$\frac{1}{3}V_d$	$\frac{2}{3}V_d\angle 180$
5	001	$-\frac{1}{3}V_d$	$-\frac{1}{3}V_d$	$\frac{1}{3}V_d$	$\frac{2}{3}V_d$
6	101	$\frac{1}{3}V_d$	$-\frac{2}{3}V_d$	$\frac{1}{3}V_d$	0
7	111	0	0	0	0

The active state vectors are $\frac{\pi}{3}$ radians apart and their end points are vertices of a regular hexagon boundary. The two zero-vectors **V_0**(000) and **V7**(111) are at the origin. If the switching states V_1-V_2-V3-V4-V5-V6 are implemented with an interval of T_s seconds between them, the phase –voltages are square waves and the resultant space vector jumps by $\frac{\pi}{3}$ radians every T_s seconds ,thus completing one revolution in 6 T_s seconds. Obviously if T_s is reduced the rotational speed of space vectors will increase .Additionally if the input voltage to the inverter is controlled by chopping the voltage V_d .The magnitude of the space vector is also changed .

This in essence is the concept of voltage space vector modulation to obtain a variable voltage variable frequency three phase supply for three phase induction motors. The space vector modulation explained here is rather coarse .To obtain a space vector which rotates more smoothly ,refined methods are used .These methods are discussed in great detail in the text books on electrical drives mentioned earlier [2]-[4].Hence further discussions about them is not considered necessary here.

Conclusions

The space in which the space vectors are defined is the annular cylindrical air gap between the stator and rotor cores of an electrical machine .The mmf and the flux density distribution are primarily periodic functions of angle θ along the periphery of the air gap. When these periodic distributions are sinusoidal in space they are defined as space vectors and can be added like the ordinary vectors in a two dimensional space. The voltage applied to a phase can be looked upon as space vector oriented along its axis. Through the use of semiconductor switches the magnitudes of the three voltage space vectors can be suitably chosen to obtain any specific magnitude and angle for the resultant space vector. This is the mechanism involved in space vector modulation.

The three phase balanced voltages in a three phase induction motors are sinusoidaly varying functions of time with a successive phase difference of $\frac{2\pi}{3}$ radians .The three space vectors add up to give resultant space vector of constant amplitude rotating in θ space at a constant angular speed .this is an analogue space vector modulation.

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