

Denoising Methods for Ultrasound Renal Images – A Comparative Study

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Abstract

This paper addresses problem of denoising the speckle in ultrasound (US) renal images. The speckle noise mimics calculi and affects the diagnosis process; hence it should be eliminated for effective segmentation and detection of calculi. A suitable denoising method is necessary to denoise and enhance the image. There are various types of spatial domain, frequency domain, adaptive, non-adaptive and multi scale filters available for speckle reduction. Evaluation have been done on various existing despeckle filters in terms of the performance metrics such as RMSE, PSNR and SSIM. The edge preservation property of various filters also analyzed. Significant remarks have been found out for this analysis providing valuable information on best despeckle filter.

Keywords: Ultrasound B-scan images, Speckle noise, spatial domain filters, Frequency domain filters, Wavelet domain filters, Diffusion filter, Edge Preservation.

Introduction

The early detection of kidney stone is required for two reasons. First, it can be treated promptly without huge medication and second to identify whether the pain is due to presence of calculi or other problems such as appendicitis, cyst, diverticulitis etc. Ultrasonography is widely used for imaging organs due to its non-invasive nature, absence of radiation, low cost and the portability. However the presence of speckle noise components caused by the image acquisition system due to coherent waves degrades the quality of US images. The speckle noise mainly occurs while imaging of soft organs such as liver and kidney whose underlying structures are too small to be resolved by the large ultrasound wavelength. This is due to the improper contact between the transducer probe and the human body, beam forming process and the

signal processing stage. The speckle noise in ultrasound images are assumed to have multiplicative error model and it must be eliminated before diagnosis otherwise it degrades the image quality and leads to misdiagnosis. There are major approaches used in speckle noise reduction, they are spatial filtering and transform domain filtering.

Peter C. Tay et al. [1] proposed a contrast enhancement technique by decreasing pixel variation in homogeneous region while maintaining differences in mean values of distinct region for cardiac images. Mario Mastriani [2] presented a wavelet based novel algorithm for speckle reduction in SAR images. W.M. Hafizah et al. [3] compared various image enhancement techniques for US images. C. P. Loizou et al. [4] performed comparative evaluation of filters for US carotid artery images. Motwani M.C. et al. [5] reviewed various image denoising approaches to facilitate proper selection of filters. Y. S. Kim et al. [6] proposed a wavelet based US image improvement approach for 2D B-mode images. The proposed approach was compared with anisotropic approach and various shrinkage schemes for speckle reduction and edge enhancement. S. Jiang et al. [7] proposed a transform domain denoising method for image.

They developed a hybrid Fourier-wavelet denoising method which improves denoising performance. L. Ganon and A Jouan [8] compared the complex wavelet based shrinkage with several standard filters for SAR image processing. D. L. Donoho [9] proposed mathematical model for denoising by using soft thresholding. Pizurica A. et al. [10] and A.K. Talukdar et al. [11] proposed a noise filtering for medical images in robust wavelet domain for contrast enhancement of medical US images. A. Achim et al. [12] presents a novel speckle reduction scheme for the logarithmic transformed data in multiscale wavelet domain. J.S. Lee [13] proposed a non-recursive contrast enhancement and noise filtering technique based on their local mean and variance. This method does not require any kind of transform. T. Loupas et al. [14] presented an adaptive weighted median filter for US images and it preserves the edges and important details.

H. Cheng et al. [15] presented a robust noise suppression method for medical US image by fusing wavelet denoising technique support vector algorithm. Y. Yang [16] proposed an image enhancement algorithm in wavelet domain. They used both wavelet transform and Haar transform to decompose the sub-band of the image and soft threshold is applied. T. Y. [17] Sun presented an wavelet based noise reduction in which the universal threshold is determined by genetic algorithm for levels of wavelets. S. G. Chang et al. [18] developed an adaptive data drive wavelet soft thresholding called Bayes shrink for denoising and compression and in [19] a spatially adaptive wavelet thresholding based on contest modeling. E.J. Balster used wavelet shrinkage algorithm with two threshold validation process. These two thresholds are used in coefficient selection process. Rabbani H et al. [21] proposed a novel spatially adaptive noise reduction algorithm using discrete complex wavelet transform.

Speckle Noise

The speckle noise is common phenomena occur in all coherent imaging system such

as laser, medical ultrasound and SAR. The speckle noise is considered as a data drop out noise caused by the data transmission error. This speckle noise is multiplicative in nature occurs in all coherent imaging systems and follows the gamma distribution given as

$$F(g) = \left[\frac{g^{\alpha-1}}{(\alpha-1)! a^\alpha} e^{-\frac{g}{a}} \right] \tag{1}$$

Where a^2 is the variance, α and g are the grey levels. In [1],[2],[12],[13] the speckle noise can be modeled as

$$s(x, y) = f(x, y)\beta m(x, y) + \beta a(x, y) \tag{2}$$

In this $f(x, y)$ is the original noise free image, $S(x, y)$ is the noisy image and $\beta m(x, y)$, $\beta a(x, y)$ are multiplicative and additive noises respectively. The additive component in ultrasound images are less significant than the effect of multiplicative component hence it can be ignored. Now (2) can be written as

$$s(x, y) = f(x, y)\beta m(x, y) \tag{3}$$

To convert the multiplicative model into additive model, logarithmic transform [12] is applied on both sides

$$\log s(x, y) = \log f(x, y) + \log \beta m(x, y) \tag{4}$$

Expression (4) can be rewritten as

$$S(x, y) = F(x, y) + M(x, y) \tag{5}$$

Where $S(.)$, $F(.)$ and $M(.)$ are the logarithms of $s(.)$, $f(.)$ and $\beta m(.)$ respectively.

The images corrupted by multiplicative noise have the characteristics of brighter area where it is noisier [13]. Smoothing of speckle and preservation of edges are necessary for an efficient segmentation of region of interest. The various methods were proposed to address the speckle removal for variety of applications. When addressing speckle as an unwanted noise, adaptive filters like Lee, Frost, Kaun, adaptive weighted median filters, anisotropic diffusion filter and other methods have been proposed. In this paper evaluation done on various filters applied to ultrasound kidney images.

Spatial Domain Filters

The Frost filter is an adaptive Wiener filter which convolves the pixel values within the fixed size window with an exponential impulse response. This filter is based on the coefficient variation, which is defined as the ratio between local standard deviation to the local mean of the noisy image. Due to this the Frost filter is also

called exponentially weighted average filter and is given as

$$w(x, y) = e^{-kC_l^2(x', y')|(x, y)|} \quad (6)$$

Where, K is the constant controlling the damping rate of the impulse function, (x', y') is the pixel to be filtered $C_l(x', y')$ is the variation coefficient. When $C_l(x', y')$ is small, it smoothens the speckle in the noisy image. When $C_l(x', y')$ is large it preserves the original observed image. Frost filter is primarily used to filter speckled SAR image.

Frost filter smooths the image without affecting edges and sharp features. The Frost filter is an adaptive filtering technique based on pixel distance to reduce the multiplicative noise. In uniform regions it acts as a median filter and in contrast region it acts as a high pass filter and preserves edges. It produces the output which is approximately similar to Lee and Kaun filter output. Mean Square Error (MSE) has to be regressively reduced in order to obtain better results.

The Lee and Kaun filters are earliest spatial domain filter based on local statistics, which works directly on image intensities and improves smoothing in homogeneous regions of ultrasound images [12], [14]. These filters are based on minimum mean square error and is given by

$$U(x, y) = S(x, y)W(x, y) + S'(x, y)(1 - W(x, y)) \quad (7)$$

Where S' is the mean value of the intensity within the selected window and $W(x, y)$ is the adaptive filter coefficient. The Lee filter is primarily used to despeckle the radar data. Smoothing of image is performed, if variance of the region is low. If variance is high, the smoothing is not performed. The main advantage of Lee filter is the effective control of amount of smoothing hence avoids blurring effect and it can directly work on multiplicative noise. The disadvantage of this filter is it tends to ignore speckle noise in the areas closest to line and edges. The Kaun filter also smooths the image without removing edges and sharp features. It is only applicable for radar intensity images. It first converts the multiplicative noise model to signal dependent additive noise model. The advantage this filter is it preserve edges and sharp feature better as compared to Lee filter.

The median filter is a nonlinear spatial filter. It has been widely used in image processing, including medical imaging because of its edge preserving properties and simplicity of implementation [14]. The basic idea behind is to replace the center pixel value within the selected window by its median or weighted median value of the neighbor pixel within that window. The median filter is also called the order specific filter because it is based on statistics derived from ordering the elements of a set rather than taking the means. This filter is popular for reducing noise without blurring edges [15] of the image. The noise-reducing effect of the median filter depends on two factors: the spatial extent of the neighborhood and the number of pixels involved in the median calculation. The above discussed filters remove the speckle noise also enhance the image quality. However these filters remove the useful information along with noise which is small and less contrast lesion of interest.

Frequency domain filters

In many cases, filtering in frequency domain is more straightforward than in spatial domain when reducing noises because noises can be easily identified in frequency domain. When an image is transformed into the Fourier or wavelet domain, the low frequency components usually correspond to smooth regions or blurred structures of the image, whereas high-frequency components represent image details, edges, and noises. In Fourier transform the textures of images are effectively represented but it would poorly represent the image edges whereas the wavelet transform sparsely represents the signal which contains singularities and sharp edges[7].

The frequency domain filters such as low pass filter, high pass filter, band pass filter etc. can be used for despeckling of ultrasound images. When an image is transformed using Fourier transform (FT), Fast Fourier Transform (FFT), Discrete Cosine Transform (DCT), etc. into frequency domain, the smooth regions and blurred structures are represented by low frequency components whereas the image details, edges and noises are represented using high frequency components. Thereby one can design filters according to the image frequency components to either smooth regions or remove noise or to enhance the edges. However in frequency domain, the commonly used filter is low pass filter based on Gaussian function. The function of Gaussian low pass is given by

$$H(u, v) = e^{-D^2 (u,v)/2\sigma^2} \tag{8}$$

Where σ^2 the variance and $D^2 (u, v)$ is the distance from the origin of the Fourier transform. These filters also suffer the same problem like spatial domain filters and causes the useful information to disappear. In this study the Gaussian band pass filter and Butterworth filter [16] has been employed and is given by equation (9) and (10)

$$H_{u,v} = e^{-D^2(u,v)/2D_1^2} - e^{-D^2(u,v)/2D_2^2} \tag{9}$$

$$H_{u,v} = \delta_L + \frac{\delta_H}{1 + \left[\frac{D_0}{D_{u,v}}\right]^2} \tag{10}$$

$$D_{u,v} = \sqrt{(u - N/2)^2 + (v - N/2)^2} \tag{11}$$

Where D_0 is the cutoff frequency of the filter, δ_L is the lower frequency gain, δ_H is the higher frequency gain, u and v are the spatial co-ordinates of the frequency transformed image and N is the dimension in u and v space. The wiener filter is an adaptive least mean square filter. It performs smoothing on speckle corrupted image by calculating the local variance of the noisy image. If the calculated variance is high then it perform little smoothing and if the local variance is small then it performs better smoothing. Hence this filter preserves edges and high frequency information. The wiener filter is given by

$$f(u, v) = \left[\frac{H(u, v)^*}{H(u, v) + [S_n(u, v)/S_f(u, v)]} \right] G(u, v) \quad (12)$$

Where $H(u, v)^*$ is complex conjugate of degraded function, $G(u, v)$ is the degraded image and $S_n(u, v), S_f(u, v)$ are the power spectrum of the noisy and original image. The Wiener filter is an optimal filter derived under a minimum of mean-squared error criteria.

Wavelet filtering

Wavelets are developed in applied mathematics for the analysis of multiscale image structures. Wavelet functions are distinguished from other transformations such as Fourier transform because they not only dissect signals into their component frequencies but also vary the scale at which the component frequencies are analyzed. As a result, wavelets are exceptionally suited for applications such as data compression, noise reduction, and singularity detection in signals.

Now days the wavelet transform is a widely used as a denoising tool for speckle reduction in ultrasound images. There are varieties of wavelets used such as Haar, daubechies, coiflet, symlet, etc. in wavelet transform. The most commonly used wavelet transform is the DWT. The DWT is applied to noisy image, which will decompose the image into LL, LH, HL and HH components. The HH sub-band gives the diagonal information, the HL sub-band gives the horizontal information, LH sub-band gives the vertical information and the LL sub-band is low resolution residual which contains the low frequency components. This LL sub-band is considered for the higher level decomposition [2],[12],[16]. There are number of wavelet denoising techniques are available based on the method of selection of threshold.

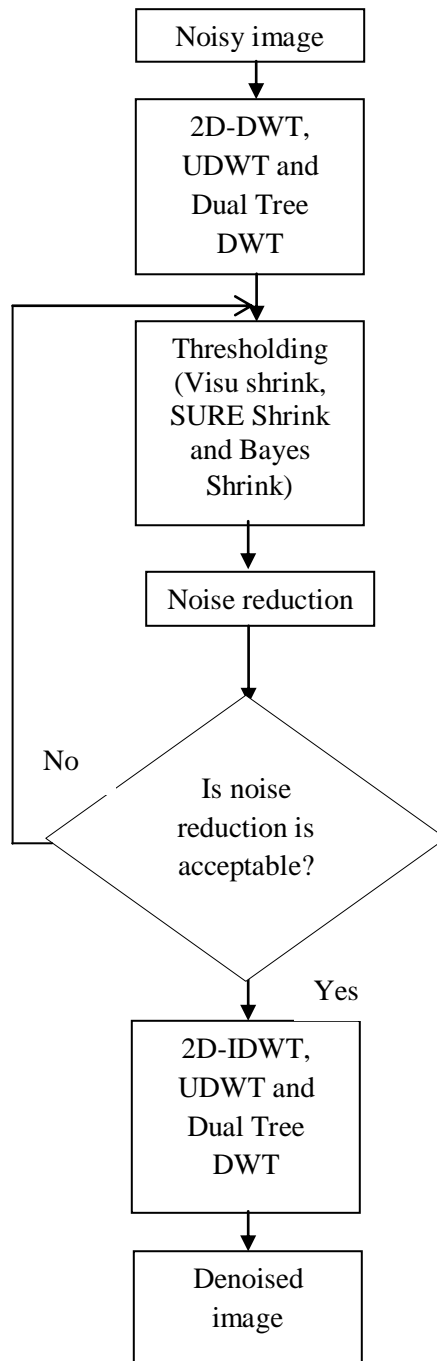


Figure 1. WaveletDenoising Algorithm

There are two types of thresholding algorithms mainly used:

Hard Thresholding

The hard thresholding operator is defined as

$$\begin{aligned}
 T(N, \theta) &= N \text{ for all } |N| > 0 \\
 &= 0 \text{ otherwise}
 \end{aligned} \tag{13}$$

Soft Thresholding

The soft thresholding operator is defined as

$$\begin{aligned}
 T(N, \theta) &= 0 \text{ for all } |N| \leq \theta \\
 &= \text{sgn}(N)(|N| - \theta) \text{ otherwise}
 \end{aligned} \tag{14}$$

The hard threshold is a keep or kills procedure whereas the soft threshold shrinks the magnitude of the coefficient above the threshold in absolute value. The equation and calculation of its parameter for threshold [9] is given as:

$$T_n = \frac{\gamma \sigma^2}{\sigma_x} \tag{15}$$

Where γ is the scale parameter, σ^2 is the variance of the sub-band and σ_x is the standard deviation of the sub-band. The scaling parameter is given by

$$\gamma = \sqrt{\log \left[\frac{L_i}{D_i} \right]} \tag{16}$$

Where the L_i is the length of the sub-band at i^{th} level of decomposition and D_i is the number of decomposition level. The noise variance is given by [3],[8]

$$\sigma^2 = \left[\frac{\text{median}(|Y_{ij}|)}{0.6745} \right] Y_{ij} \in \text{subband HH} \tag{17}$$

The most of the existing wavelet denoising techniques modifies the wavelet coefficient based on thresholding of Gaussian based statistical estimation method. Most of the existing wavelet denoising algorithms were developed for the images corrupted by additive noise. The ultrasound images are corrupted by multiplicative noise, hence requires log-transform of the image when the application of wavelet. An exponential operation is performed to convert the image back into non logarithmic format [15],[17],[18],[19].

In [11], the adaptive Bayesian shrinkage based on context modeling for ultrasound images was discussed. The proposed method with wiener filter and AWMF works better in terms of noise removal and edge preservation. The summarization complete wavelet based thresholding algorithm is given below:

1. Decompose the Noisy image into sub-band using 2D-DWT
2. Compute the σ^2 in the noisy image using equation (17).
3. For each level of decomposition calculate the scale parameter using the equation (16).

4. For HL, LH,HHsub-band compute standard deviation and threshold T using equation (15).
5. Apply soft thresholding to the sub-bands HH_i , LH_i , HL_i .
6. Apply the required filtering algorithm to the LL_i sub-band for removal of noisy coefficient.
7. Compute 2D-IDWT.

Generally, speckle noise is modeled as a multiplicative noise. The speckle reduction is done by multiplying wavelet coefficients by a speckle reduction ratio. It should be mentioned that the speckle reduction aims to improve the subjective image quality and the resulting images should look natural. In[20], the author proposed a wavelet based denoising method which reduces the computational complexity. In this two threshold values are used to perform denosing and the result is compared with various algorithms.

Diffusion Filtering

The diffusion filtering concept utilizes the nonlinear partial differential equation for smoothing the image. This diffusion is given by

$$\frac{\partial X}{\partial t} = \text{div}[C \|\nabla X\|. \nabla X] \quad (18)$$

$$X(t = 0) = X_0$$

Where div is the divergence operator, $\|\nabla X\|$ is the gradient magnitude of the image X, C is the diffusion co-efficient and X_0 is the original image. The gradient magnitude is used to detect the edge in anisotropic diffusion method. The speckle reducing anisotropic diffusion filtering (SRAD) has been developed for speckle reduction with edge preservation.

Results and discussion

The image quality evaluation methods can be classified into two as objective methods and subjective methods. The subjective methods are based on human expert's judgment whereas the objective methods are based on comparisons of various numerical criteria calculated explicitly. In this section we present a result of eight despeckle filters discussed in the above sections. These filters are applied on 120 ultrasound kidney images and some sample output images are shown in Figure 2. Furthermore two image quality in terms of the both subjective and objective methods were calculated and discussed. The two metrics RMSE, PSNR were computed and given in Table 1 using the formula [21],[22],[23]

$$RMSE = \sqrt{\frac{\sum (f(x,y) - F(x,y))^2}{MN}} \quad (19)$$

$$PSNR = 20 \log_{10} \frac{255}{RMSE} \quad (20)$$

Here, $f(x, y)$ is the original image and $F(x, y)$ is the denoised image. When RMSE is low the PSNR value is larger, then better denoising is obtained. The table 1 gives the numerical values of RMSE and PSNR. The Wavelet Filtering has highest PSNR value of followed by Lee Filter. The Lee Filter, Median filter and Gaussian Band pass filter has the same nearest values as 37.9373db, 37.9114db and 37.7309db respectively. The Frost and wiener filters also have the nearest value as 36.8792db and 36.642db. The frequency domain Butterworth Filter has the lowest value of 29.5742db. The Wavelet filter has the highest PSNR value, which means the filtered image has useful signal content than other filters.

The PSNR value approaches infinity value as the RMSE approaches zero value provides a higher image quality according to (19), (20). The PSNR and RMSE measurements are not consistent with human eye perception. The SSIM is an image quality metric which is correlated to the visual perception of human visual system. It can be used as a benchmark to measure the image quality there by it checks the performance of various image processing algorithms. The SSIM quantifies the similarity measurement of two images in three components luminance, contrast and structural. The luminance between two images is determined by mean intensity of pixels in the image, contrast is determined by standard deviation of image and the structural is determined by the correlation between two images[24]. Let $f(x, y)$ and $F(x, y)$ then

$$L(f, F) = (2\mu_f\mu_F + C_1)/(2\mu_f^2 + \mu_F^2 + C_1) \quad (21)$$

$$C(f, F) = (2\sigma_f\sigma_F + C_2)/(2\sigma_f^2 + \sigma_F^2 + C_2) \quad (22)$$

$$S(f, F) = (\sigma_{fF} + C_3)/(\sigma_f\sigma_F + C_3) \quad (23)$$

Where, μ_f is the mean over a window in the image $f(x, y)$

μ_F is the mean over a window in the image $F(x, y)$

σ_f is the standard deviation over a window in the image $f(x, y)$

σ_F is the standard deviation over a window in the image $F(x, y)$

σ_{fF} is the co-variance over a window in the image $f(x, y)$ and $F(x, y)$ C_1, C_2 and C_3 are constants.

The SSIM is the manipulation of three components.

$$\text{Set } C_3 = C_2/2 \quad (24)$$

$$SSIM(f, F) = ((2\mu_f\mu_F + C_1) * (2\sigma_{fF} + C_2)) / ((\mu_f^2 + \mu_F^2 + C_1) * (\sigma_f^2 + \sigma_F^2 + C_2)) \quad (25)$$

The mean SSIM is the average of all local windows. The window is moved across the image one pixel at a time. The SSIM value lies between -1 for a bad and 1 for a good similarity between the original and despeckled images. The SSIM index is computed for the image with respect to the reference image. The ultrasound kidney images used for the experiment are in BMP format. The stone images also experimented with various filters. There are five images were tested with all the discussed filters and the performance was ensured in terms of subjective method of image quality evaluation. The visual evaluations of these images were made by a sonologist. The Lee Filter output gives better identification of stone.

The wavelet filter has better PSNR and MSSIM, minimum RMSE. But its performance is poor in terms of visual perception of a human expert. This means that the wavelet filter over-smooths the speckle noise. The Lee filter is next to wavelet in the aspects of RMSE, PSNR but it performs well in terms of visual perception of a human expert.

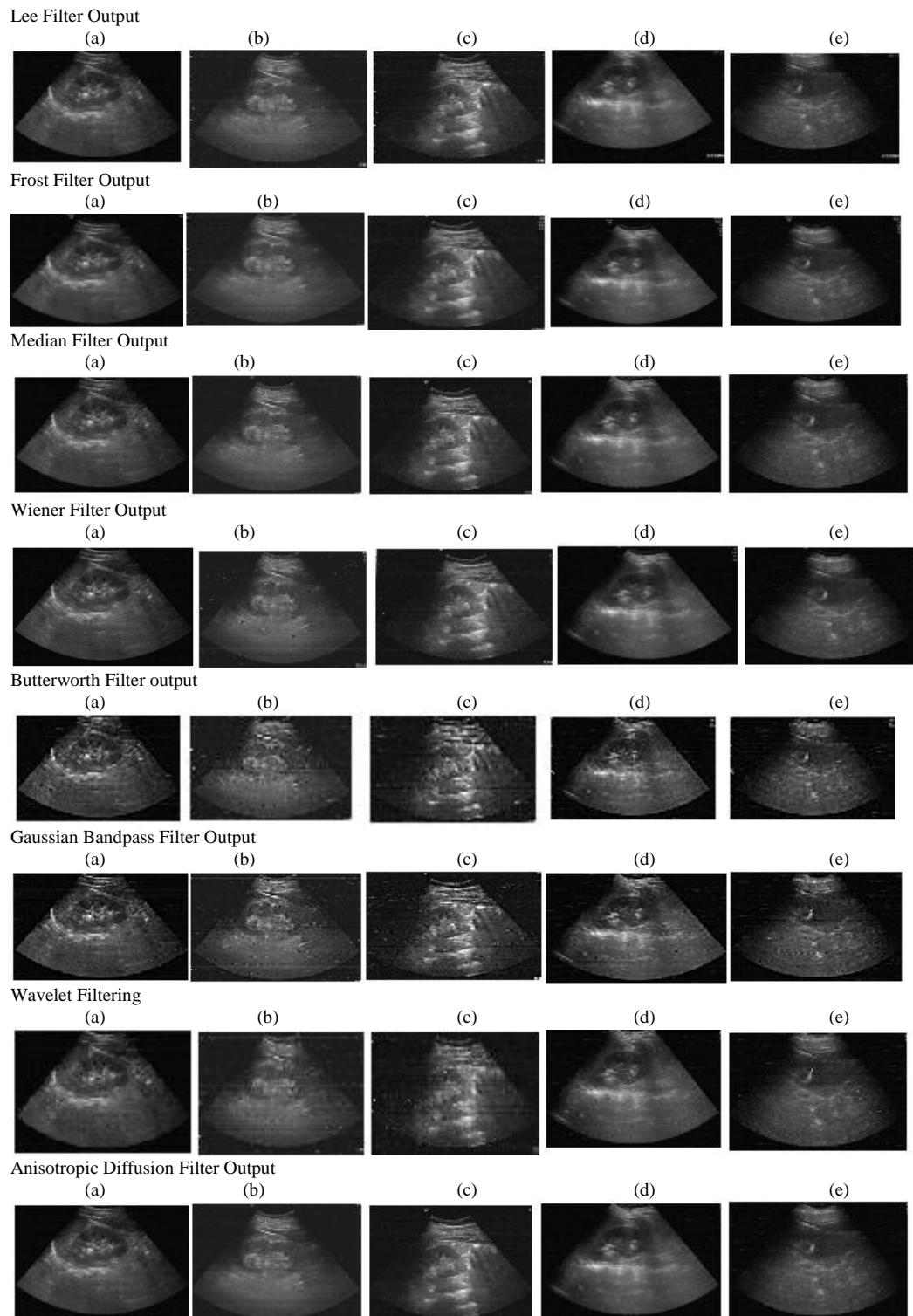


Figure 2. Filtered images with different filters

Table 1. Comparison of RMSE and PSNR Values

Performance Metrics	RMSE					PSNR				
	(a)	(b)	(c)	(d)	(e)	(a)	(b)	(c)	(d)	(e)
Kidney Images										
Lee Filter	3.1787	3.9040	4.4254	3.1107	3.3866	38.0858	36.3007	35.2118	38.2735	37.5354
Frost Filter	3.7124	4.3543	4.8135	3.5778	3.6178	36.7377	35.3524	34.4815	37.0585	36.6178
Median	3.2377	3.9818	4.4185	3.1661	3.4807	37.9261	36.1292	35.2253	38.1204	37.2976
Wiener Filter	3.6901	3.4967	4.9064	3.6030	3.9341	36.7899	37.2578	34.3156	36.9974	36.2339
Butterworth	9.1364	7.9308	10.0495	8.3843	8.1656	28.7377	30.1444	28.0879	29.6615	29.8911
Bandpass Filter	3.4980	3.2233	4.1174	3.2430	3.0909	37.2543	37.9647	35.8384	37.9119	38.3291
Wavelet Filter	1.8319	2.0889	2.5748	1.7157	1.9890	42.8726	41.7327	39.9160	43.4422	42.1581

The edge preservation properties of speckle filters are important characteristics for segmentation of ROI. The edge preservation of each filter were analyzed using edge preservation index (β) given by the equation as

$$\beta = \frac{\Gamma(\Delta f - \overline{\Delta f}, \Delta F - \overline{\Delta F})}{\sqrt{\Gamma(\Delta f - \overline{\Delta f}, \Delta f - \overline{\Delta f}) \cdot \Gamma(\Delta F - \overline{\Delta F}, \Delta F - \overline{\Delta F})}} \tag{26}$$

Table 2. Comparison of Edge Preservation Index (β)

Performance Metrics	β				
	(a)	(b)	(c)	(d)	(e)
Kidney Images					
Lee Filter	0.8789	0.6997	0.7112	0.6943	0.7026
Frost Filter	0.3557	0.3143	0.4098	0.3423	0.4718
Median	0.5617	0.4981	0.5364	0.4667	0.5103
Wiener Filter	0.1647	0.2143	0.2978	0.2323	0.2218
Butterworth	0.2557	0.3343	0.3998	0.1823	0.2988
Band pass Filter	0.3657	0.3103	0.2298	0.4973	0.3588
Wavelet Filter	0.7657	0.6103	0.6998	0.5923	0.6788

Comparing to all other filters discussed, the Lee filter is having better edge preservation index. The median and wavelet filters are also having the better edge preservation.

Conclusion

In this evaluation on various speckle removing filters for medical ultrasound B-scan

kidney images were compared and tested with the performance measurement parameters RSME, PSNR, SSIM and edge preservation index. From this the wavelet thresholding technique, median and Lee filter gives better noise removal compared to the other methods. The region of interest can also be easily detected in Lee Filter. The future work will be focused on hybrid filtering technique based on wavelet transform applied to various kidney images with renal calculus problem.

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