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A Study On The Complexity Of Visco-Elastic Fluid Flows In A Porous Medium

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Abstract:

Visco-elastic substances exhibit a dual nature of behaviour by showing signs of both viscous fluids and elastic solids. visco-elasticity can be modelled by combining Newton's law for viscous fluids (stress / rate of strain) with Hook's law for elastic solids (stress / strain), as given by the original Maxwell model and extended by the convected Maxwell models for the nonlinear visco-elastic fluids. The behaviour of visco-elastic fluids is drastically different from that of Newtonian and inelastic non-Newtonian fluids. This includes the presence of normal stresses in shear flows, sensitivity to deformation type, and memory effects such as stress relaxation and time-dependent viscosity. Visco-elastic fluids are those that show partial elastic recovery upon the removal of a deforming stress. Such materials possess properties of both viscous fluids and elastic solids. The complexity of visco-elasticity is phenomenal and the subject is notorious for being extremely difficult and challenging.

The constitutive equations for visco-elastic fluids are much too complex to be treated in a general manner.

Key words: Oldroyd-B fluids, visco-elastic flow, Porous medium.

Introduction:

A common feature of visco-elastic fluids is stress relaxation after a sudden shearing displacement where stress overshoots to a maximum then starts decreasing exponentially and eventually settles to a steady-state value. This phenomenon also takes place on cessation of steady shear flow where stress decays over a finite measurable length of time. This reveals that visco-elastic fluids are able to store and release energy in contrast to inelastic fluids which react instantaneously to the imposed deformation [1, 6 and 13]. A defining characteristic of visco-elastic materials associated with stress relaxation is the relaxation time which may be defined as the time required for the shear stress in a simple shear flow to return to zero under constant strain condition. Hence for a Hookean elastic solid the relaxation time is infinite, while for a Newtonian fluid the relaxation of the stress is immediate and the relaxation time is zero. Relaxation times which are infinite or zero are never realized in reality as they correspond to the mathematical idealization of Hookean elastic solids and Newtonian liquids. In practice, stress relaxation after the imposition of constant strain condition takes place over some finite non-zero time interval [16].

These features underlie the observed peculiar visco-elastic phenomena such as rod-climbing (Weissenberg effect), die swell and open-channel siphon [13]. Most visco-elastic fluids exhibit shear-thinning and an elongational viscosity that is both strain and extensional strain rate dependent, in contrast to Newtonian fluids where the elongational viscosity is constant and in proportion to shear viscosity [2]. The behaviour of visco-elastic fluids at any time is dependent on their recent deformation history, that is they possess a fading memory of their past. Indeed a material that has no memory cannot be elastic, since it has no way of remembering its original shape.

Consequently, an ideal visco-elastic fluid should behave as an elastic solid in sufficiently rapid deformations and as a Newtonian liquid in sufficiently slow deformations. The justification is that the larger the strain rate, the more strain is imposed on the sample within the memory span of the fluid [1, 13, 2]. Many materials are visco-elastic but at different time scales that may not be reached. Dependent on the time scale of the flow, visco-elastic materials show mainly viscous or elastic behaviour. The particular response of a sample in a given experiment depends on the time scale of the experiment in relation to a natural time of the material. Thus, if the experiment is relatively slow, the sample will appear to be viscous rather than elastic, whereas, if the experiment is relatively fast, it will appear to be elastic rather than viscous. At intermediate time scales mixed visco-elastic response is observed. Therefore the concept of a natural time of a material is important in characterizing the material as viscous or elastic. The ratio between the material time scale and the time scale of the flow is indicated by a non-dimensional number: the Deborah or the Weissenberg number [9].

Further complications arise from the confusion created by the presence of other phenomena such as wall effects and polymer-wall interactions, and these appear to be system specific [11]. Therefore, it is doubtful that a general fluid model capable of predicting all the flow responses of visco-elastic systems with enough mathematical simplicity or tractability can emerge in the foreseeable future [6, 4]. Understandably, despite the huge amount of literature composed in the last few decades on this subject, the overwhelming majority of these studies have investigated very simple cases in which substantial simplifications have been made using basic visco-elastic models.

Formulation and Solution:

Important aspects of non-Newtonian flow in general and visco-elastic flow in particular through porous media are still presenting serious challenge for modeling and quantification. There are intrinsic difficulties in characterizing non-Newtonian effects in the flow of polymer solutions and the complexities of the local geometry of the porous medium. This geometry gives rise to a complex and pore space dependent flow field in which shear and extension coexist in various proportions that cannot be quantified. Flows through porous media cannot be classified as pure shear flows as the converging-diverging passages impose a predominantly extensional flow fields especially at high flow rates. The extension viscosity of many non-Newtonian fluids also increases dramatically with the extension rate. As a consequence, the relationship between the pressure drop and flow rate very often do not follow the observed Newtonian and inelastic non-Newtonian trend. Further complications arise from the fact that for complex fluids the stress depends not only on whether the flow is a shearing, extensional, or mixed type, but also on the whole history of the velocity gradient [5, 10, 12, 14, 15 and 17].

The Oldroyd-B model is a simple form of the more elaborate and rarely used Oldroyd 8-constant model which also contains the upper convected, the lower convected, and the co-rotational Maxwell equations as special cases. Oldroyd-B is the second simplest nonlinear visco-elastic model and is apparently the most popular in visco-elastic flow modeling and simulation. It is the nonlinear equivalent of the linear Jeffreys model, as it takes account of frame invariance in the nonlinear regime. Oldroyd-B model can be obtained by replacing the partial time derivatives in the differential form of the Jeffreys model with the upper convicted time derivatives

$$\boldsymbol{\tau} + \lambda_1 \overset{\triangledown}{\boldsymbol{\tau}} = \mu_o \left(\dot{\boldsymbol{\gamma}} + \lambda_2 \overset{\triangledown}{\dot{\boldsymbol{\gamma}}} \right) \tag{1.5}$$

Where $\overset{\triangledown}{\dot{\boldsymbol{\gamma}}}$ is the upper convected time derivative of the rate of strain tensor given by

$$\overset{\triangledown}{\dot{\boldsymbol{\gamma}}} = \frac{\partial \dot{\boldsymbol{\gamma}}}{\partial t} + \mathbf{v} \cdot \nabla \dot{\boldsymbol{\gamma}} - (\nabla \mathbf{v})^T \cdot \dot{\boldsymbol{\gamma}} - \dot{\boldsymbol{\gamma}} \cdot \nabla \mathbf{v} \tag{1.6}$$

There are many possible sets of rheological equations of state, with the right invariance properties for general validity under all conditions of motion and stress, which reduce to equations Newtonian fluid, when the velocity gradients and shear stresses are sufficiently small so that their squares and products to be neglected. None of the possible general forms of the equations of state is linear in the velocity gradients dv_i/dx_k and the stresses p'_{ik}. The simplest are linear in the stresses alone, and include terms of the second degree in the stresses and velocity gradients taken and we confine our attention to these. Assuming isotropy and incompressibility, so that eii is zero, we have to consider the class of liquids whose behaviour defines the non-Newtonian fluids.

$$p'_{ik}+\lambda_1\frac{\mathscr{D}p'_{ik}}{\mathscr{D}t}+\mu_0 p'_{jj}e_{ik}-\mu_1(p'_{ij}e_{jk}+p'_{jk}e_{ij})+\nu_1 p'_{jl}e_{jl}\delta_{ik}$$
$$=2\eta_0\left(e_{ik}+\lambda_2\frac{\mathscr{D}e_{ik}}{\mathscr{D}t}-2\mu_2 e_{ij}e_{jk}+\nu_2 e_{jl}e_{jl}\delta_{ik}\right), \quad (1.7)$$

Where μ_0, μ_1, μ_2, v_1 and v_2 are five more arbitrary scalar physical constants, each with the dimensions of time. This equation has been made symmetrical in the free suffixes so that it represents six distinct differential equations for the six distinct components p'_{ik} in terms of the e_{ik}*'s*. The usual summation convention is to be understood for repeated suffixes, and the material derivative denoted by *D/Dt* is a total derivative following the typical fluid elements, taking into account the linear and angular motion of the element which are measured by the velocity vi and the vorticity tensor

$$\omega_{ik}\equiv\frac{1}{2}\left(\frac{\partial v_k}{\partial x_i}-\frac{\partial v_i}{\partial x_k}\right).$$

For any Cartesian tensor $b_{ik\ldots r}$ associated with a moving fluid, expressed as a function of position x_i and the time *t*,

$$\frac{\mathscr{D}b_{ik\ldots r}}{\mathscr{D}t} \equiv \frac{\partial b_{ik\ldots r}}{\partial t} + v_j \frac{\partial b_{ik\ldots r}}{\partial x_j} + \omega_{ij} b_{jk\ldots r} + \omega_{kj} b_{ij\ldots r} + \ldots + \omega_{rj} b_{ik\ldots j}$$

Oldroyd discussed Two particular types of liquid of this class were discussed by liquid A liquid B one corresponds to the values

$$\eta_0 > 0, \quad \lambda_1 = -\mu_1 > \lambda_2 = -\mu_2 \geqslant 0, \quad \mu_0 = \nu_1 = \nu_2 = 0, \tag{1.8}$$

Which refers to Oldroyd liquid A and next the values which refers to liquid B

$$\eta_0 > 0, \quad \lambda_1 = \mu_1 > \lambda_2 = \mu_2 \geqslant 0, \quad \mu_0 = \nu_1 = \nu_2 = 0. \tag{1.9}$$

Conclusion:

1. It was shown that Oldroyd liquid B would exhibit the Weissenberg climbing effect when sheared at a finite rate between rotating coaxial cylinders, and that liquid A would show the effect in reverse, sinking down (instead of rising up) near the inner cylinder.

2. It has also been shown that, with some exceptions, among them liquids A and B, the liquids whose behaviour is represented by equations (1.7) with $\mu_0 = v_1 = v_2 = 0$ will show a variation of apparent viscosity with rate of shear in steady flow between rotating cylinders.

3. Some of the inelastic non-Newtonian liquids considered by Rivlin are formally included in the class represented by equations (1.6) and (1.5), if we put.

$$\eta_0 > 0, \quad \mu_2 \neq 0, \quad \lambda_1 = \lambda_2 = \mu_0 = \mu_1 = \nu_1 = \nu_2 = 0.$$

Such liquids have been shown by Rivlin to be capable of exhibiting the Weisenberg effect. Newtonian liquids can also be regarded as a special case, in which all physical constants except the viscosity η_o vanish.

Bibliography:

1. Bird. R.B.; R.C. Armstrong; O. Hassager. 1987., Dynamics of Polymeric Liquids, volume 1. John Wily & Sons, second edition, p. 2, 7, 8, 9, 12, 13, 14, 15, 16, 19, 37, 45, 46, 49, 50, 52, 55, 70.
2. Boger.D.V., Visco-elastic flows through contractions. Annual Review of Fluid Mechanics, 19:157{182}, p. 16, 45, 1987.
3. Carreau. P.J.; D. De Kee; R.P. Chhabra. Rheology of Polymeric Systems. Hanser Publishers, p. 8, 9, 10, 12, 14, 15, 19, 37, 48, 49, 54, 55, 60, 61, 1997.
4. Chhabra.R.P.; J. Comiti; I. Machac., Flow of non-Newtonian fluids in fixed and fluidised beds. Chemical Engineering Science, 56(1):1{27, p. 22, 46, 48, 53, 54, 2001.
5. Dauben. D.L.; D.E. Menzie., Flow of polymer solutions through porous media. Journal of Petroleum Technology, SPE 1688-PA, pages 1065{1073}, p. 47, 54, 57, 1967.
6. Deiber. J.A., Modeling the flow of Newtonian and visco-elastic fluids through porous media. PhD thesis, Princeton University, p. 11, 28, 46, 48, 53, 58, 1978.
7. Dullien. F.A.L., Single phase flow through porous media and pore structure. The Chemical Engineering Journal, 10(1):1{34}, p. 54, 1975.
8. Gogarty. W.B.; G.L. Levy; V.G. Fox., Visco-elastic effects in polymer flow through porous media. SPE 47th Annual Fall Meeting, 8-11 October, San Antonio, Texas, SPE 4025, p. 23, 52, 53, 54, 55, 58, 65, 1972.
9. H.A. Barnes; J.E Hutton; K. Walters., An introduction to rheology. Elsevier, 2, 4, 7, 45, 48, 49, 50, 52, 55, 60, 70, 1993.

10. J.R.A. Pearson; P.M.J. Tardy., Models for flow of non-Newtonian and complex fluids through porous media. Journal of Non-Newtonian Fluid Mechanics, 102(2):447{473}, p. 23, 25, 35, 47, 61, 64, 2002.
11. Jones.W.M.; S.P. Ho., The flow of dilute aqueous solutions of macromolecules in various geometries: VII. Mechanisms of resistance in porous media. Journal of Physics D: Applied Physics, 12(3):383{393}, p. 46, 1979.
12. Larson.R.G., The Structure and Rheology of Complex Fluids. Oxford University Press, p. 12, 16, 37, 47, 48, 49, 50, 52, 55, 70, 1999.
13. Larson.R.G., Constitutive Equations for Polymer Melts and Solutions. Butterworth Publishers, p. 12, 13, 14, 15, 19, 45, 46, 49, 50, 70, 1988.
14. Marshall. R.J.; A.B. Metzner., Flow of visco-elastic fluids through porous media. Industrial & Engineering Chemistry Fundamentals, 6(3):393{400}, p. 47, 53, 55, 56, 65, 1967.
15. Mendes. P.R.S.; M.F. Naccache., A constitutive equation for extensional thickening fluids flowing through porous media. Proceedings of 2002 ASME International Mechanical Engineering Congress & Exposition, November 17-22, New Orleans, Louisiana, USA, p. 47, 49, 57, 59, 2002.
16. Owens. R.G.; T.N. Phillips., .Computational Rheology. Imperial College Press, Vol. 2., p. 2, 7, 11, 14, 16, 19, 46, 50, 52, 60, 2002.
17. Plog. J.P.; W.M. Kulicke. 2000, Flow behaviour of dilute hydrocolloid solutions through porous media. Influence of molecular weight, concentration and solvent. Institute of Technical and Macromolecular Chemistry, University of Hamburg, 2000, p. 47.
18. Sadowski. T.J., Non-Newtonian flow through porous media. PhD thesis, University of Wisconsin, p. 23, 27, 54, 58, 65, 1963.
19. Savins. J.G., Non-Newtonian flow through porous media. Industrial and Engineering Chemistry, Vol. 61(10):18{47}, p. 9, 54, 56, 57, 58, 1969.

20. Wissler. E.H., Visco-elastic effects in the flow of non-Newtonian fluids through a porous medium. Industrial & Engineering Chemistry Fundamentals, 10(3):411{417}, p. 46, 54, 58.

21. Vradis. G.C., J. Dougher, S. Kumar, "Entrance pipe flow and heat transfer for a Bingham plastics", *International Journal of Heat Mass transfer*, Vol. 96 (1993), pp. 543–550.

22. Oldroyd.J.G., Proc. Roy. Soc. Sec. A, Vol. 200 (1950), p. 523.

23. Oldroyd.J.G., Proc. Camb. Phil. Soc. Vol. 47 (1951a), p. 410.

24. Oldroyd.J.G., Quart. J. Mech. Appl. Math., Vol.4 (1951b), p. 271.

25. Oldroyd.J.G., Proc. Roy. Soc. Sec. A, Vol. 245 (1958), p. 278.