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# Nonholonomic Car Control Using Higher Order Sliding Mode Controllers

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**Abstract**

**This Higher siding mode control problem is to stabilize the auxiliary system stable to the origin in a finite time. The purpose of switching control law is to derive the non-linear plant state trajectory on to a pre specified .The Sliding mode control is used to wheel slip control for vehicle motion. It is design to trajectory move towards switching continuous. A non-linear control strategy based on sliding mode, which is a standard approach to gear the parametric & modeling uncertainties of a non-linear system is chosen for slip control. We introduce the Robust Fuller's Problem that is a robust generalization of the Fuller's problem, by solving robust fuller problem, possible to obtain feedback laws that are higher order sliding mode algorithm of generic order. The Goal of this paper is steer the car from a given position to the sliding surface. The connection between the design of high order sliding modes algorithms and the solution to some optimal control problems for a perturbed chain of integrators has been investigated in this paper. Reaching mode to state trajectory moves from the initial condition towards the switching surface. In an additional provide optimal finite time reaching of the sliding manifold.**

*Keywords* — **Chattering effect, Higher Order Sliding Mode Control, Holonomic car control, Sliding mode.**

## I. INTRODUCTION

Sliding mode controllers are powerful tools to control uncertain systems. They are able to achieve finite time reaching and exact keeping of a suitably chosen sliding manifold in the state space by means of a discontinuous control. The choice of the sliding manifold is strictly related to the control objective to be attained. In standard formulations, the controlled system state is first steered to the sliding manifold in finite time, and then maintained confined to the manifold itself, giving rise to the so-called sliding mode, so that the equilibrium point corresponding to the fulfillment of the control objective is made asymptotically stable.

In order to overcome the chattering problem in the sliding mode higher order sliding mode (HOSM) control was introduced in PhD dissertation the of Arie Levant (Levantovsky). Levant systematized the second-order sliding mode algorithms and obtained estimates of their accuracy.

The system's trajectory is steered to the sliding manifold by the designed higher order sliding mode controller in finite time. Higher Order Sliding Mode (HOSM) controllers are effective in extending the good properties of standard sliding mode controllers to systems with higher relative degree, and can be used also to reduce the chattering effect.

## II. SLIDING MODE CONTROL

Alter the dynamics of non-linear system. This is a form of variable structure control and it is a non-linear control method that alters the dynamics of a non-linear system. This implementation is high-frequency switching control laws. It is an electro mechanically system for example pendulum, antennas and robotics manipulators. Sliding mode control is robust with respect to matched internal & external disturbances. The Sliding accuracy is proportional to the square of the switching time del
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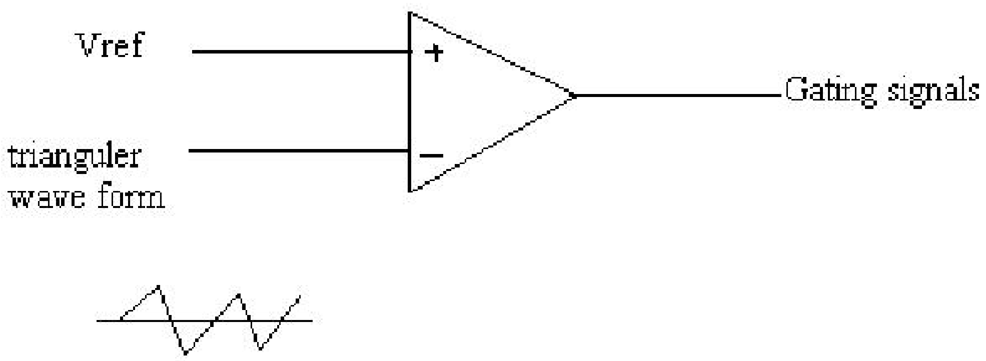


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**Fig. 1. Sliding mode controller block diagram**

## III. SLIDING SURFACES

This section investigates Variable Structure Control (VSC) as a high-speed switched feedback control resulting in sliding mode. The purpose of the switching control law is to drive the nonlinear plant's state trajectory onto a pre specified (user-chosen) surface in the state space and to maintain the state of the plant trajectory on this surface for subsequent time. The surface is called as a switching surface.

When the state of the plant trajectory is “above” the surface and a feedback path gain is one form to the different form, if the trajectory drops “below” the surface. This surface is defining the rule of proper switching control and this surface is also called as a sliding surface (sliding manifold). In a perfectly once intercept the switched control maintains the plant’s state trajectory on the surface for all subsequent time and the plant’s state trajectory slides along the surface. The most important task is to design a switched control that will drive the plant state to the switching surface and maintain it on the surface upon interception. A Lyapunov approach is used to characterize this task. Lyapunov method is usually used to determine the stability properties of an equilibrium point without solving the equation of state. Let V (x) be a continuously differentiable scalar function defined in a domain D that contains the origin. A function of V(x) is said to be positive definite if V (0) = 0 and if V(x) > 0. It is said to be negative definite if V (0) = 0 and if V(x) < 0. Lyapunov method satisfies that the function is positive definite when it is negative and the function is negative definite when it is positive. In this way the stability is established.

A generalized Lyapunov function that characterized the motion of the state trajectory to the sliding surface is defined in terms of the surface and each chosen switched control structure is the “gains", so that the derivative of this Lyapunov function is negative definite and thus guaranteeing motion of the state trajectory to the surface. After a proper design of the surface is a switched controller is constructed so that the tangent vectors of the state trajectory point towards the surface such that the state is driven to and maintained on the sliding surface and such controller result is discontinuous closed-loop systems. Let a single input nonlinear system be defined as

$$x(n) = f(x, t) + b(x, t)\, u(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (1)$$

Here x (t) is the state vector u (t) is the control input (in our case braking torque or pressure on the pedal and x is the output state). The other states in the state vector are the higher order derivatives of x up to the (n-1)th order. The superscript n and x(t) shows the order of differentiation f( x ,t) and b( x ,t) are generally nonlinear functions of time and states. The function f( x ) is not exactly known but the extent of the imprecision on f( x ) is upper bounded by a known continuous function of x ; similarly, the control gain b ( x ) is not exactly known but is of known sign and is bounded by known continuous functions of x and the control problem is to get the state x to track a specific time varying state x d in the presence of model imprecision on f(x) and b(x). A time varying surface s (t) is defined in the state space R (n) by equating the variable s(x; t) defined below to zero.

$$s(x; t) = (d/dt + \delta)n\text{-}1\ x\square(t) \quad .......................... \qquad (2)$$

Here, δ is a strict positive constant taken to be the bandwidth of the system and is the error in the output state where x d(t) is the desired state. The problem of tracking the (n-1)th dimensional vector x d (t) can be replaced by a first-order stabilization problem in s. s(x; t ) verifying [2] is referred to as a sliding surface, and the systems behavior once on the surface is called sliding mode or sliding control and

From (2) the expression of s contains we only need to differentiate s once for the input u to appear. Furthermore, bounds on scan be directly translated into bounds on the tracking error vector x and therefore the scalar s represents a true measure of tracking performance.

The set remains in the set for all future and past times. Furthermore (4) also implies that some disturbances or dynamic uncertainties can be tolerated while still keeping the surface an invariant set.

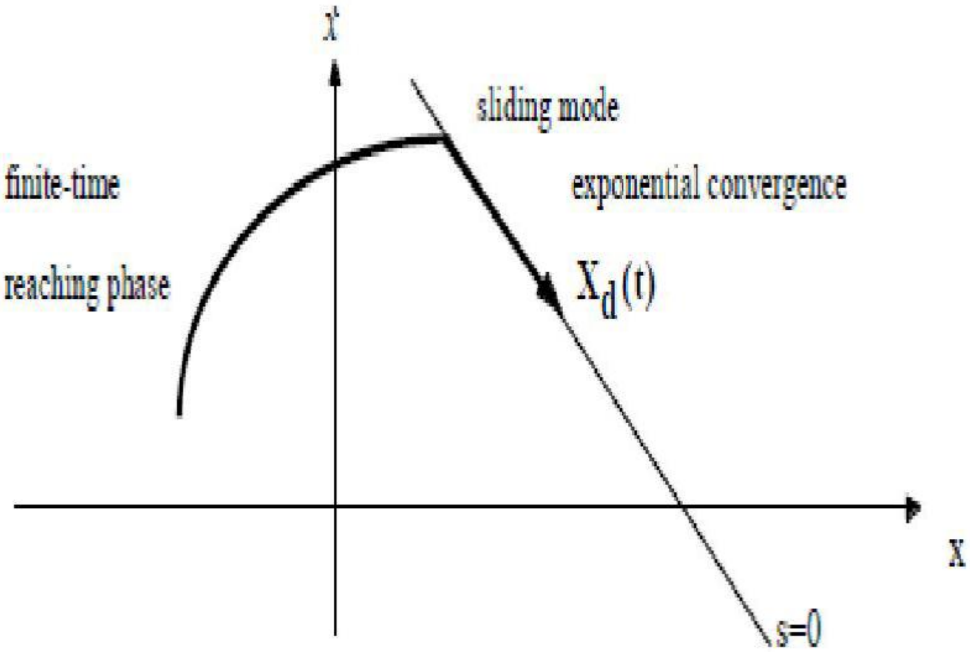


**Fig. 2. Graphical interpretations of equations (2) and (4) (n=2)**

Finally satisfying guarantees that if x(t=0) is actually off xd(t=0), the surface S(t) will be reached in a finite time smaller than the corresponding transformations of performance measures assuming x (o) =0 is

$$\forall\ t \geq 0 , |s(t)| \leq \varphi => \forall\ t \geq 0, |\ x\square(i)(t)| \leq (2\ \delta)i\ \varepsilon\ i=0,\ldots,n-1 \qquad (3)$$

Where $\tau=\varphi/\delta$ (n-1).In this way, an nth-order tracking problem can be replaced by a 1st-order stabilization problem. The simplified, 1st-order problem of keeping the scalar s at zero can now be achieved by choosing the control law u of (1) such that outside of S(t)

$$(1/2)\ (d/dt)\ s2 \leq -\ \eta|s| \ldots\ldots\ldots\ldots \qquad (4)$$

Where $\eta$ is a strictly positive constant and States that the squared "distance" to the surface, as measured by S2, decreases along all system trajectories. Thus it constrains trajectories to the point towards the surface s (t). In particular, once on the surface the system trajectories remain on the surface. In other words, satisfying the sliding condition makes the surface an invariant set (a set for which any trajectory starting from an initial condition with in $|s(t-0)| \ /\eta$ Assume for instance that s(t=0)>0, and let t reach be the time required to hit the surface s=0.

Integrating (4) between t=0 and t reach leads to

$$0-S\ (t=0)=S(t=t\ reach)-S(t=0)-\eta(t\ reach-0) \qquad (5)$$

Which implies that?

$$t_{reach} \leq S(t=0)/\eta \qquad (6)$$

The similar result starting with s (t=0)>0 can be obtained as

$$t_{reach} \leq |\ S(t=0)|/\eta \qquad (7)$$

Starting from any initial condition the state trajectory reaches the time-varying surface in a finite time smaller than $s(t = 0)\ /\eta$ and then slides along the surface towards x d ( t ) exponentially with a time-constant equal to $1 / \lambda$. In summary, the idea is to use a well-behaved function of the tracking error s and then select the feedback control law u in 1 such that s remains characteristic of a closed-loop system and presence of model imprecision

## IV. CHATTERING REDUCTION

An ideal sliding mode exists only when the state trajectory x(t) of the controlled plant agrees with the desired trajectory at every t for some t1 This may require infinitely fast switching. In the real system a switched controller has imperfections which limit switching to a finite frequency. The typical point of oscillates within a neighborhood of the switching surface. This oscillation is called as a chattering and illustrated on figure 3.

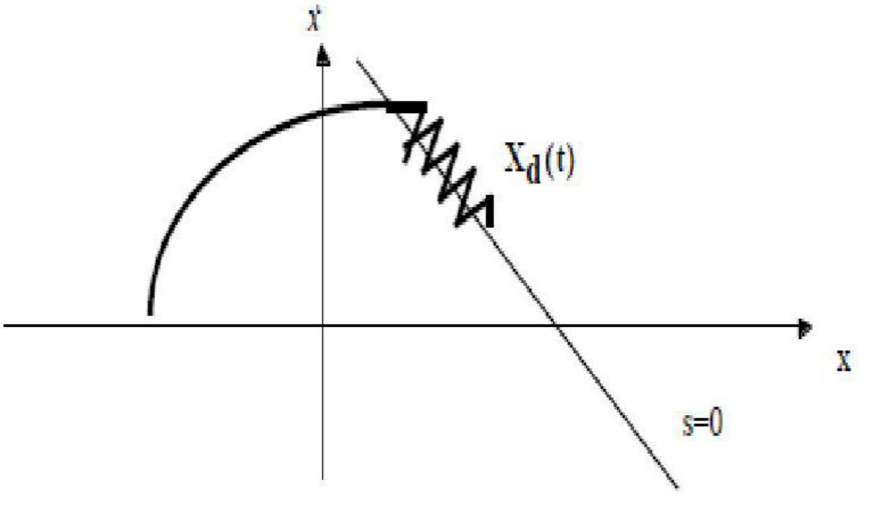


**Fig. 3. Chattering as a result of imperfect control switching**

Control laws which satisfying sliding condition and lead to “perfect” tracking in the face of model uncertainty are discontinuous across the surface S(t) thus causing control chattering. The chattering is undesirable since it involves extremely high control activity and furthermore may excite high-frequency dynamics neglected in the course of modeling. This chattering must be reduced (eliminated) for the controller to perform properly.

This approach leads to tracking within a guaranteed precision (rather than perfect tracking) and more generally guarantees that for all trajectories starting inside B(t=0)

$$\Psi\ t\geq 0, \left| x^{?(t)}(t) \right| \leq (2\lambda)^t \in i=0,\ldots,n-1 \ldots\ldots\ldots\ldots \quad (8)$$

## V. LEVANT CONTROLLER

Two other major contributions by Levant: arbitrary-order HOSM control in 2001 and arbitrary-order asymptotically optimal robust exact differentiators in allowed the design and the implementation of universal arbitrary-order HOSM output-feedback controller. However the design of new types of HOSM controllers still remained complicated. Thus the recently generalized algorithms for designing universal arbitrary-order HOSM controllers have been developed based on the homogeneous and quasi-homogeneous properties of HOSM dynamic. The aim of Special Issue is to report the current state of art of modern HOSM control. The Higher order sliding modes (HOSM) which generalize the sliding mode notion remove that restriction

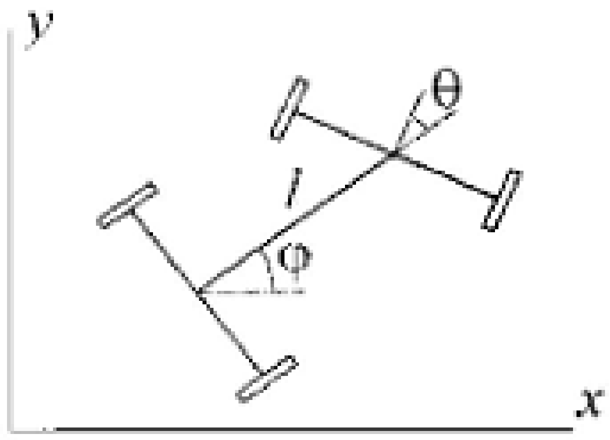


**Fig. 4. Kinematic model**

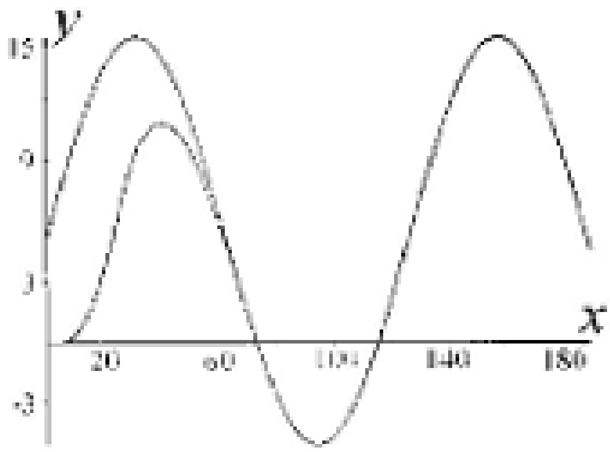


**Fig. 5. 3-Sliding trajectory**

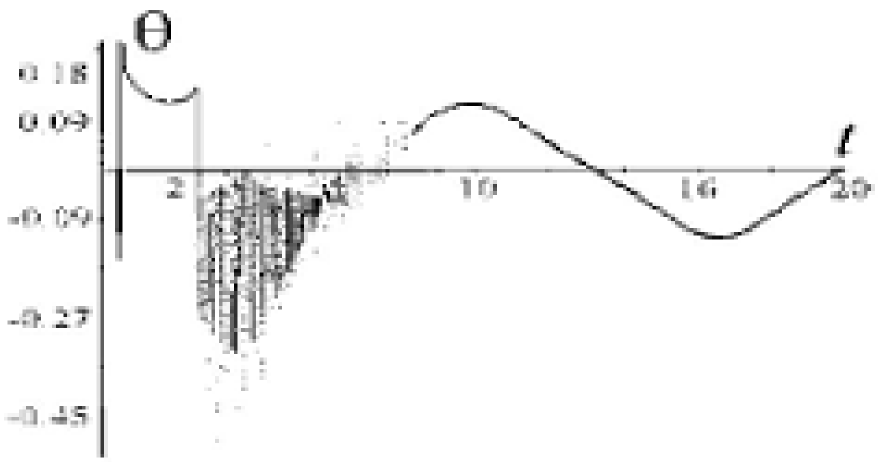


**Fig. 6. Steering angle: 3- Sliding**

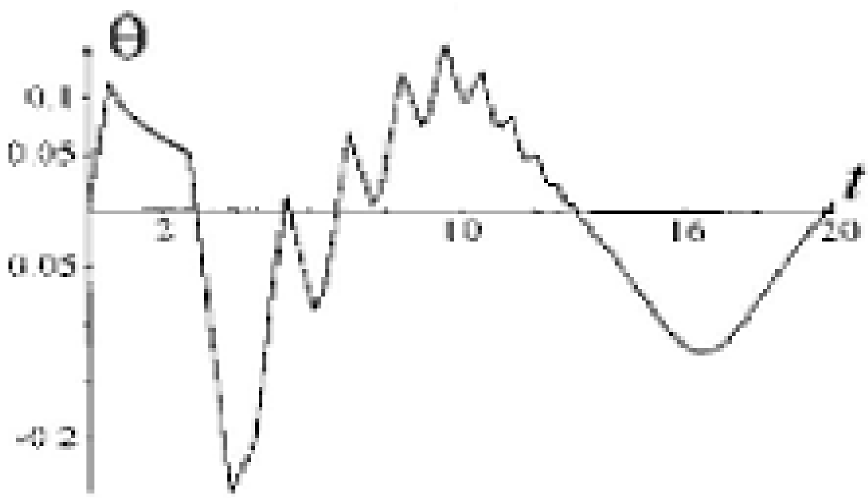


**Fig. 7. Steering angle: 4-Sliding**

## VI. NONHOLONOMIC SYSTEM

A nonholonomic system is a physics and mathematics system whose state depends on the path taken to achieve it and such a system is described by a set of parameters subject to differential constraints such that when the system evolves along a path in its parameter space (the parameters varying continuously in values) but finally returns to the original set of values at the start of the path system itself may not have returned its original state. Exactly a nonholonomic system also called an nonholonomic system is one in which there is a continuous closed circuit of the governing parameters by which the system may be transformed from any given state to any other state because the final state of the system depends on the intermediate values of its trajectory through parameter space the system cannot be represented by a conservative potential function as can for example the inverse square law of the gravitational force this is an example of a holonomic system: path integrals in the system depend only upon the initial and final states of the system (positions in the potential) completely independent of the trajectory of transition between those state. The system is therefore said to be integrable while the nonholonomic system is said to be non integrable. When a path integral is computed in a nonholonomic system the value represents a deviation within some range of admissible values and this deviation is said to be a homonymy produced by the specific path under consideration.

## VII. KINEMATIC CAR

It is known that the model of a kinematic car with two inputs, which are the longitudinal velocity and the steering angle (or its time derivative), is differentially flat Since time-scaling is involved in the sequel, we precise that the time in this system is denoted by t and x denotes the time derivative of the function x(t) such that t

= 1.The motion planning can be done off- line prior to the motion and the time-scaling does not need the redesign of the reference. It follows that more involved method scan be also applied.

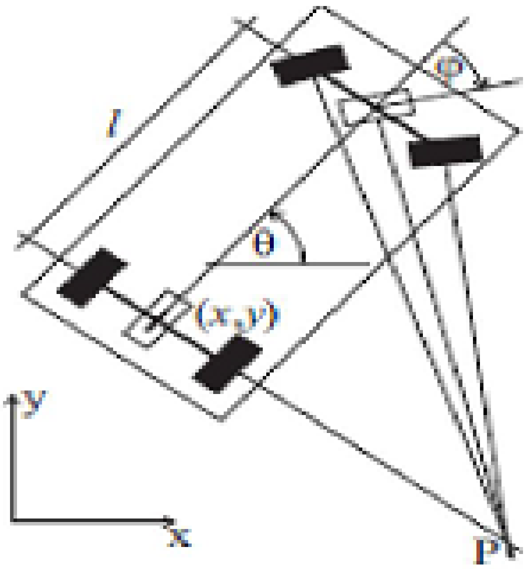


**Fig. 8. Notations of the kinematic model**

## VIII. SIMULATION RESULTS

Applied nonholonomic car motion planning for testing performance of high sliding mode system is described by

$$\dot{x}=\vartheta \cos \phi,\ \dot{y}=\vartheta \sin \phi,\ \dot{\phi}=(\vartheta/l)\tan\theta,\ \dot{\theta}=u$$

Where

(x, y) are the Cartesian coordinates of the rear-axle middle point, $\varphi$ is the orientation angle, is the steering angle and u is the control variable V=10, L=5. Goal is to steer the car from a given initial position

(x0, y0, φ0, 0) = (0, 0, 0, 0) to the trajectory

$y = 10 \sin(x/20) + 5$

Sliding Variable $\sigma = y-10\sin(x/20)-5$

Under assumption of small axis/relation degree is 3,

Lm = LM = 20, C 5

Simulations have been carried out in the time horizon 0 to 20 second and the controller have been enabled only at t = 0:5s (in the first half- second are set to zero) in order to better compare the results with. Results are summarize in Fig.1 dotted lines are relative to UL the car trajectory obtained with the three controllers is shown together with the reference trajectory y = 10sin(x=20) +5. In the auxiliary system state y = 10 sin(x/20) +5 is plotted against time for all the controllers. Finally the dynamics of the steering angle θ is reported. From F we see that $U_L$ performs very well in terms of reaching time; in this respect it consistently outperforms the other two controllers .$U_L$ produces quite effective steering angles while avoiding dangerous vibration the author obtained a reaching time similar to that of this experiment by using only β1 = 1. This also shows that our bounds on C and Km are very conservative. In our setting using β2 = 1 corresponds to C < Km (feasibility condition). We have made additional simulations by fixing t= 1 and trying several values of =1 in the interval [1:4]. UOR still obtains the best reaching times. In the fig, x-axis time varying from (0 to 20sec) and y axis velocity in time. The proposed controllers have simple rules to select the design parameters and exhibit state of the art performance. Levant controller produces quite effective steering angles while avoiding dangerous vibration and slower than the process.

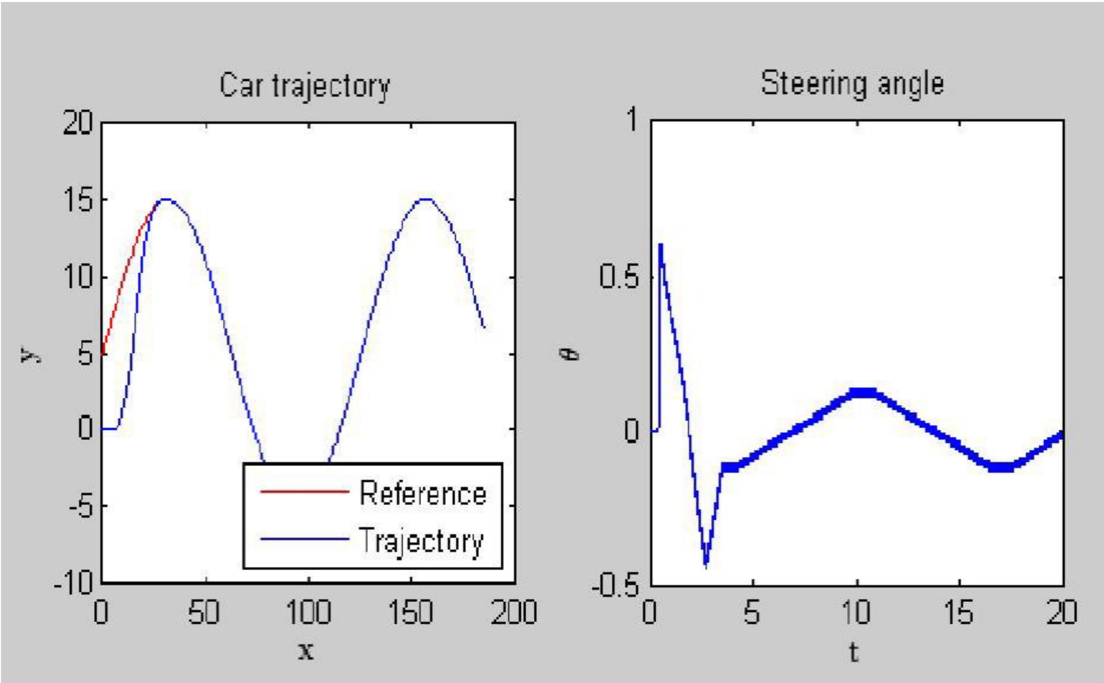


**Fig. 9. Simulated Response For HOSM**

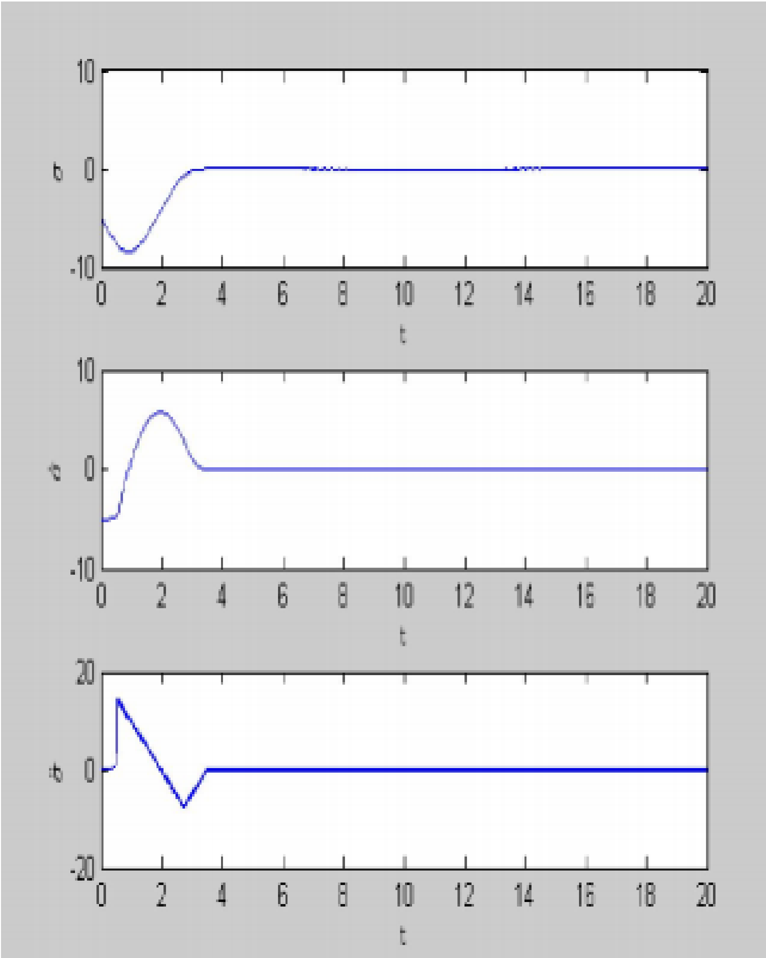


**Fig. 10. Simulated Response for Nonholonomic Car Control**

## IX. CONCLUSION

The connection between the design of high order sliding modes algorithms and the solution to some optimal control problems for a general family of HOSM controllers is obtained which guarantees optimal reaching of the sliding manifold with respect to a suitable optimality index. This family includes several already known HOSM control algorithms A control derivative of some order being treated as a new control a higher order controller can be applied to providing for the prescribed control smoothness and removing the chattering. Special cases namely second and third order algorithms are analyzed in the paper. The design is to ensure that the controller will obtain closed loop stability. The proposed controllers have simple rules to select the design parameters and exhibit state of the art performances. Levant controller produces quite effective steering angles, while avoiding dangerous vibration and slower than the process.

**REFERENCES**


[1] Fridman L. M "Singularly perturbed analysis of chattering in relay control systems,"

[2] Boiko I, and. Fridman L. M. "Analysis of chattering in continuous sliding-mode controllers,"

[3] Koshkouei A, Burnham K, and Zinober A. "Dynamic sliding mode control design," IEE Proceedings - Control Theory and Applications,

[4] "Chattering avoidance by second order sliding mode control," IEEE Transaction on Automatic Control,

[5] Bartolini G., Ferrara A., Usai E., and. Utkin V. I, "On multi-input chattering-free second order sliding mode control,"

[6] Levant A., "Universal single-input–single-output (SISO) sliding-mode controllers with finite-time convergence,"

[7] Quasi-continuous high-order sliding-mode controllers,"

[8] Liberzon D. Mors e A. S, and Sontag E. D, "Output-input stability and minimum-phase nonlinear systems,"

[9] Sussman H. J. and Yang Y. "On the stabilizability of multiple integrators by means of bounded feedback," in Proc. IEEE conference on Decision and Control

[10] Feldbaum A. A, "On synthesis of optimal systems with the help of phase space,"