

## **Determination Of Weber-Ampere Characteristic For Electrical Devices Based On The Solution Of Harmonic Balance Inverse Problem**

**Gorbatenko Nikolay Ivanovich, Lankin Anton Michailovich, Lankin Michail Vladimirovich, Shayhutdinov Danil Vadimovich**

*Federal State educational institution of higher professional education  
"South-Russian State Polytechnic University (NPI) named after MI Platov",  
Russia, 346428, Novocherkassk, Prosvesheniya street, 132*

### **Abstract**

The aim of the study is the development weber-ampere characteristic determination method for electrical devices consisting of a magnetic core and a coil. The method is based on the following principle: sinusoidal voltage is supplied on the electrical device coil with a certain amplitude and frequency. The amplitudes of flowing current harmonics are measured. The inverse problem of harmonic balance is solved according to these data and the expression approximation coefficients describing the desired weber-ampere characteristic. The approximation expression degree influence on the measurement result error is investigated. The obtained results showed the possibility of the proposed method application to determine weber - ampere characteristic of electrical devices. The error of weber-ampere characteristic determination does not exceed 3%.

**Keywords:** weber-ampere characteristic, electrical devices, methods of measurement, harmonic balance, Fourier series, inverse problem solution.

### **1. Introduction**

All electrical devices (electric magnets, electric magnet relays, motors) have a movable and a fixed part of a magnetic core and at least one working coil [1]. During the operation of an electrical device current flows in the coil. An operational magnetic flux is developed in the fixed part of the magnetic core, driving its movable part. The magnetic flux value is determined by the design and the mutual disposition of the magnetic core and operational coil working parts, as well as by the number of turns of the operational coil and the magnitude of the current within these parts. During the operating cycle of an electrical device the movable part of the magnetic core is moved

relative to a fixed part, which also leads to magnetic flux change. All this suggests the conclusion that the integral characteristic containing not only information about the operational parameters of electrical devices, but also about the quality of its parts is the weber - ampere characteristic (WAC) of the operation cycle. Let's suggest that the WAC of an operation cycle is the dependence of a magnetic flux penetrating the working coil of an electrical device from the current flowing through it. At that, the movable part of the electrical device magnetic core performs a typical working movement. In order to obtain WAC sample of an electrical device it is necessary to make an impact on a magnetic core with an external changing magnetic field and to measure a magnetic flux developed in its section with a special sensor [2]. To obtain the WAC of the operation cycle an assembled electrical device is tested which makes the use of the magnetic flux sensors impossible.

Therefore, the use of existing measurement methods is not possible. Thus it is necessary to develop a method determining the operation cycle WAC of an electrical device with an acceptable accuracy and which does not require additional sensors. The harmonic balance method (HBM) is a promising one for the current problem solution [3].

## 2 Harmonic balance method essence

The direct task of HBM [3, 4] is the determination of the current shape flowing through a non-linear element with a known volt-ampere characteristic, the amplitude and the shape of supply voltage. The method idea is based on the voltage periodic function decomposition applied to the non-linear element and the current flowing through it in a Fourier series. In general, the unknown variables in a nonlinear electrical circuit are nonsinusoidal and contain an infinite spectrum of harmonics. An expected solution may be represented as the sum of the primary and several upper harmonics. Substituting this amount in a non-linear differential equation written for an unknown value and equating the coefficients in a resulting expression before the harmonics (sinusoidal and cosine functions) of the same frequencies in its left and right sides, we obtain the system of  $n$  algebraic equations, where  $n$  is the number of recorded harmonics. Solving the system of equations, we obtain the desired unknown. This method is based on the inverse problem of harmonic balance, which is to determine the unknown volt-ampere characteristic of a nonlinear element with the known current form flowing through it, the shape and the amplitude of the supply voltage.

The solution of the direct HBM problem allows to determine the current shape  $i(t)$ , flowing through the coil of an electrical device defined as a nonlinear inductance [5], using the decomposition into Fourier series:

$$i(t) = \sum_{m=1}^n I_{(2m-1)} \sin((2m-1)\omega t), \quad (1)$$

where  $I_{(2m-1)}$  –  $(2m-1)$  current harmonic amplitude. At that the voltage shape and amplitude  $U_a$  applied to nonlinear inductance is known:

$$u(t) = U_a \sin(\omega t), \quad (2)$$

as well as the WAC of nonlinear inductance defined by approximated expression:

$$\Phi(i) = \sum_{m=1}^n k_{(2m-1)} i^{2m-1}, \tag{3}$$

where  $\Phi$  is the value of the magnetic flux through the nonlinear inductance,  $k_{(2m+1)}$  are the approximating WAC expression coefficients,  $m = \overline{(1, n)}$ ,  $n$  is the number of component in an approximated expression,  $i$  is the current flowing through the nonlinear inductance.

The expression (1) has no elements of the cosine decomposition part in a Fourier series due to the WAC symmetry an electrical device concerning coordinate origin.

The inverse HBM problem to determine the WAC of an electrical device is developed as follows. There is a nonlinear inductance with an unknown WAC. There are known laws of the voltage variation applied to the nonlinear inductance (2) and the current flowing through it (1). It is necessary to determine the coefficients  $k_{(2m-1)}$  of an approximating WAC expression (3).

Let's write the circuit equation with a nonlinear inductance an active resistance  $R$ :

$$u(t) = Ri + \frac{d\Phi}{dt}.$$

Let's rewrite it taking into account the known current (1) and voltage (2) variation laws:

$$U_a \sin \omega t = R \sum_{m=1}^n I_{(2m-1)} \sin((2m-1)\omega t) + \frac{d \sum_{m=1}^n k_{(2m-1)} \sum_{m=1}^n I_{(2m-1)} \sin((2m-1)t\omega)^{2m-1}}{dt} \tag{4}$$

Knowing the degree of  $(2n - 1)$  of an approximating WAC expression let's define  $n$  value of the sine function argument in the equation (4). The argument values are taken from the interval  $]0; \pi/2[$ .

Thus, we obtain the system from the linear equations  $n$ . The resulting system of equations provides a voltage amplitude  $U_a$ , current harmonic amplitudes  $I_{(2m-1)}$ , the value of active resistance  $R$  and the angular frequency value of the flowing current  $\omega$ , since these circuit parameters with a nonlinear inductance may be measured during an electrical device test. Solving this system of equations, we obtain the coefficients  $k_{(2m-1)}$  of an approximating WAC expression (3).

### 3. Results

#### 3.1. Model experiment

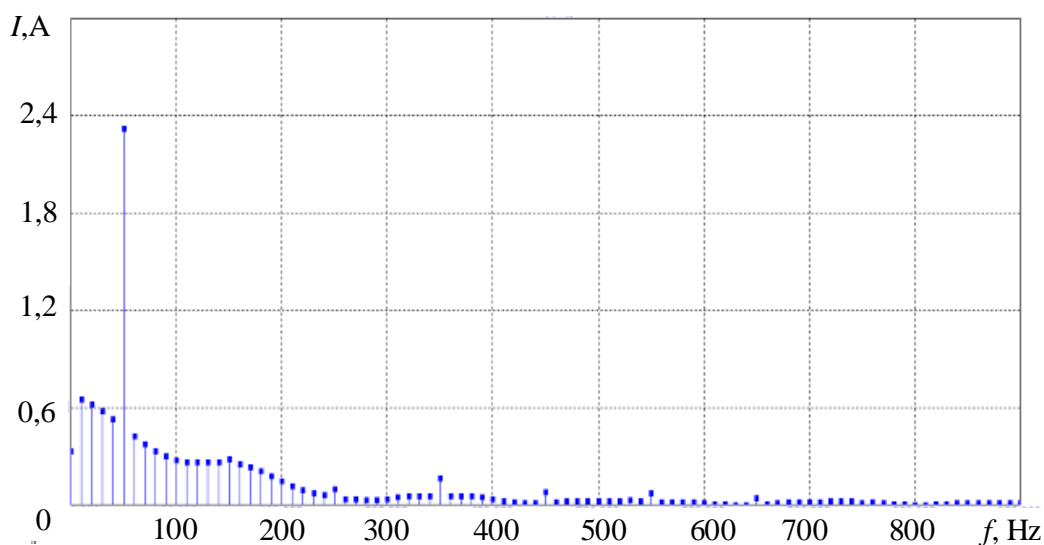
MicroCap circuit simulation package is used to implement the model [6].

The feature of most electrical devices is the presence of a non-magnetic gap in its magnetic core. The magnetic core WAC of 3100B material is put into the model with two options of non-magnetic gap i.e. 0.4 mm and 0.9 mm, which leads to different WAC inclination.

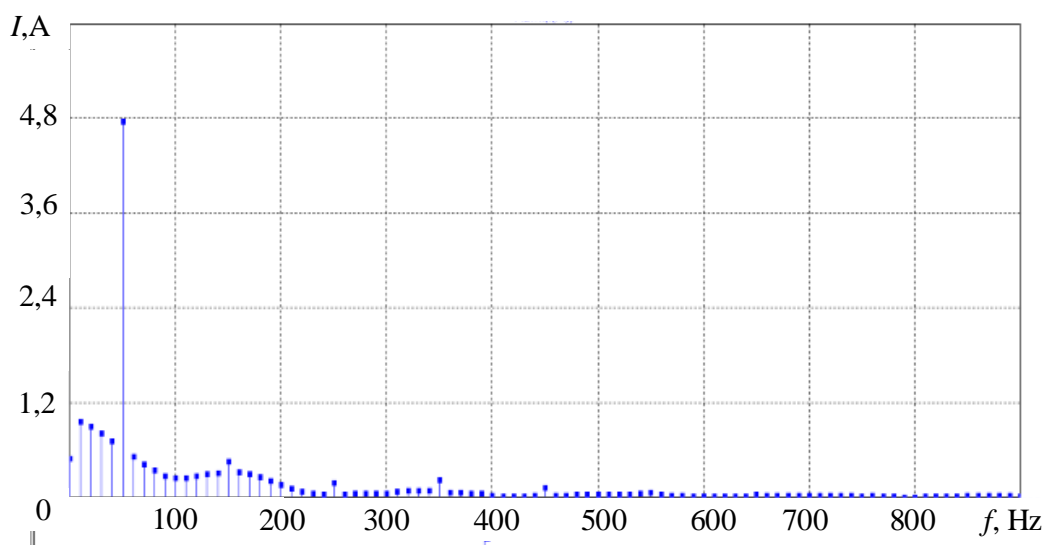
The electrical device model comprises sequentially connected nonlinear inductance with the number of turns equal to 95 and the active resistance  $R$  of 0.15 and 0.1 ohms for the abovementioned non-magnetic gaps, respectively. The electrical device

is connected to the sine voltage source with the wave frequency of 50 Hz and the amplitude  $U_a$  equal to 1.65 and 1.7 V for the abovementioned non-magnetic gaps, respectively.

The figures 1 and 2 show the current spectra for the selected non-magnetic gaps, which demonstrate that it is possible to determine the upper current harmonics from the first to the thirteenth one. At that the current harmonics flowing through an electrical device with the gap of 0.9 mm have a greater amplitude than the current harmonics flowing through an electrical device with the gap of 0.4 mm. The error calculation results at WAC determination for electrical devices with a variety of non-magnetic gaps of 0.4 mm and 0.9 mm, calculated using different amounts of current harmonics are shown in Table 1.



**Figure 1. Current spectrum for a non-magnetic gap of 0.4 mm**



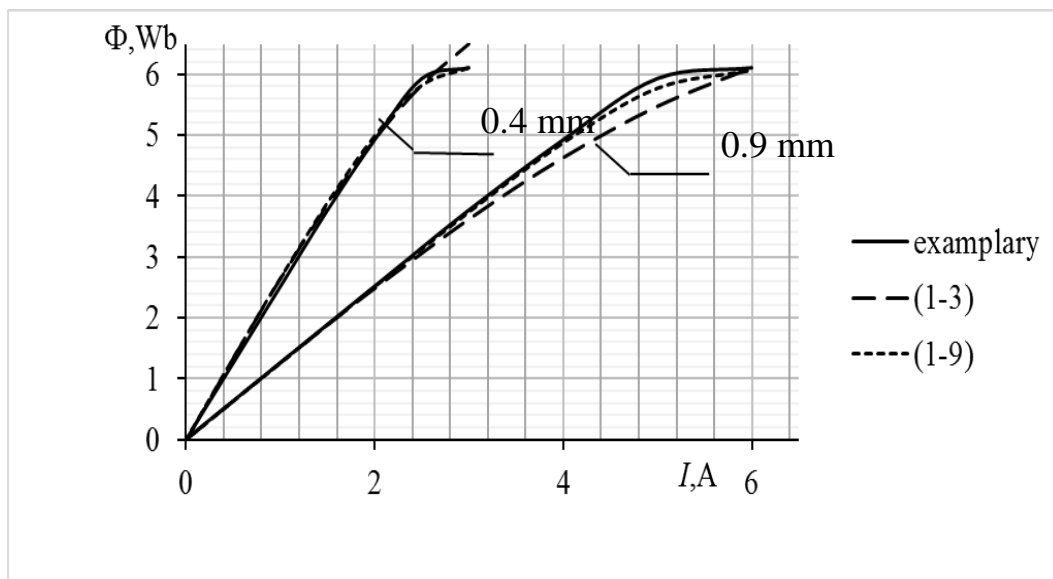
**Figure 2. Current spectrum for a non-magnetic gap of 0.9 mm**

The measurement error was calculated by the method described in [7]. This method is based on finding the maximum distance between two compared characteristics.

**Table 1 - WAC determination errors**

Maximum current harmonic		3	5	7	9	11	13	
Error, %	weber-ampere characteristic with the gap of 0,4 mm	$\delta_I$	5,3	2,8	1,3	1,4	1,1	0,9
		$\delta_\Phi$	4,5	2,4	2,8	2,4	2,1	1,8
		$\delta$	7,0	3,7	3,1	2,8	2,4	2,0
	weber-ampere characteristic with the gap of 0,9 mm	$\delta_I$	6,2	5,9	5,6	4,8	4,3	4,1
		$\delta_\Phi$	2,1	1,3	0,9	0,9	0,8	0,8
		$\delta$	6,5	6,0	5,7	4,9	4,4	4,2

The measured WAC of electrical devices are shown in Figure 3, the amount of current harmonics taken into account is shown in parentheses.



**Figure 3. WAC of electric device**

The obtained results demonstrate that the method application success depends on the correct choice of the harmonic number. The popular approach is used to determine the required amount of current harmonics involved in the calculation of  $k_m$  equation ratio for WAC approximation (9) [8,9]. Successively increasing the amount of current harmonics, we control the residual dispersion change. The addition of current harmonics termination moment is determined by testing the hypothesis of difference absence concerning the residual dispersions using the Fisher's ratio test.

The results of current harmonic required amount determination at the significance level of  $\alpha = 5\%$  are shown in Table 2, which show that the use of the first to eleventh odd current harmonic is required for the first WAC, and the the first to eleventh odd current harmonic for the second WAC.

### 3.2. Natural experiment

The natural experiment measuring the WAC of electrical devices is tested. The following electrical devices were used: electromagnetic AC relay PIY-1, induction motor КД 1 - 2 and the toroidal transformer CTT - 12A.

**Table 2– Results of current harmonic amount determination**

Parameter	$2p+1$	$S_{ocr}^2$	$p$	$f_1$	$f_2$	$F_p$	$F_{kp}$
weber-ampere characteristic with the gap of 0,4 mm	<b>3</b>	0,091	2	18	16	-	-
	<b>5</b>	0,022	3	18	15	4,09	2,39
	<b>7</b>	0,0081	4	18	14	2,75	2,44
	<b>9</b>	0,0031	5	18	13	2,67	2,51
	<b>11</b>	0,0012	6	18	12	3,00	2,6
	13	0,00072	7	18	11	1,43	2,71
weber-ampere characteristic with the gap of 0,9 mm	<b>3</b>	0,063	2	18	16	-	-
	<b>5</b>	0,021	3	18	15	3,00	2,39
	<b>7</b>	0,0084	4	18	14	2,50	2,44
	9	0,0058	5	18	13	1,45	2,51

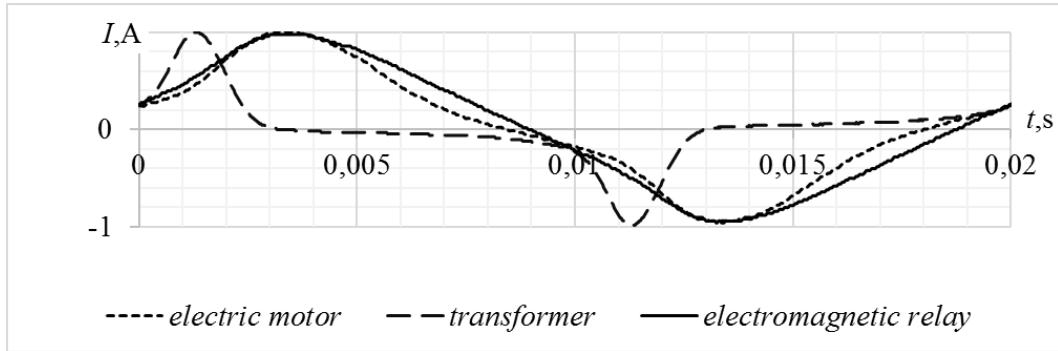
The following devices were used for the experiment: the laboratory autotransformer Wusley tdgc 8A, two RMS multimeters Fluke 289, the digital oscilloscope Tektronix 2024b, personal computer.

To determine the reference WAC of electrical devices the ammeter and voltmeter method were used [10]. The measuring coils are applied on magnetic cores of electrical devices [11]. The determination of the calculated WAC for electrical devices are carried out using the proposed method based on the inverse problem solution for harmonic balance. The initial data to solve the inverse problem of the harmonic balance are shown in Table 3.

**Table 3– Initial data**

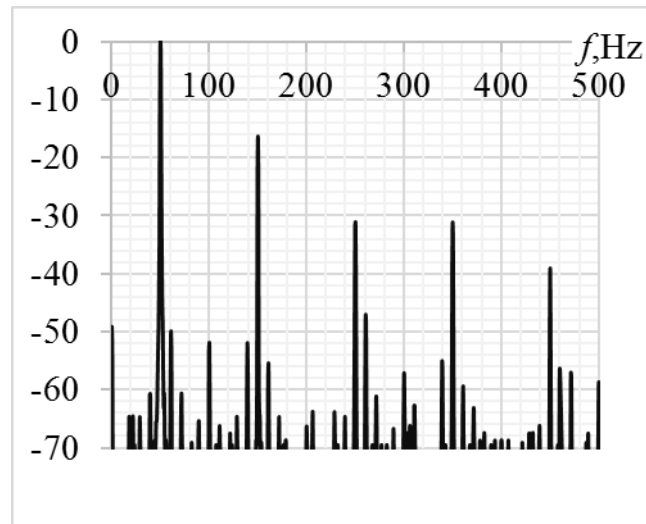
Parameter	$U_a, B$	$R, Om$	$I_1, A$	$I_3, A$	$I_5, A$	$I_7, A$	$I_9, A$
Electromagnetic relay	260	488	0,49	0,012	0,00049	0,00042	0,00005
Electric motor	97	64,5	1,47	0,092	0,0018	0,0014	0,00015
Transformer	200	512	0,38	0,004	0,00038	0,00004	0,000039

Figure 4 shows the current shapes of the electrical device coils. The current flowing through the electrical device coils has different degrees of unsinusoidality.

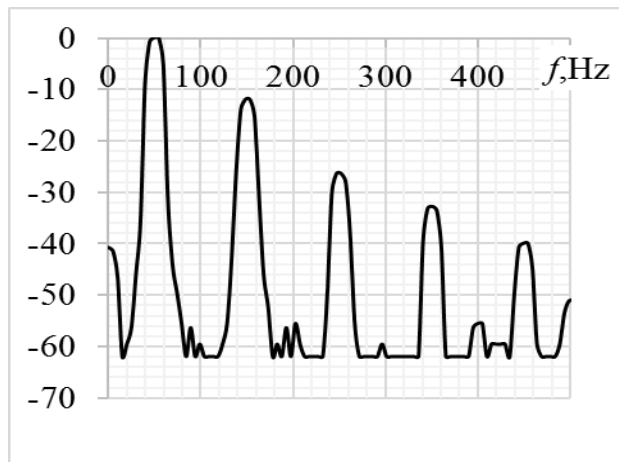


**Figure 4. Current shapes during the study of electrical devices.**

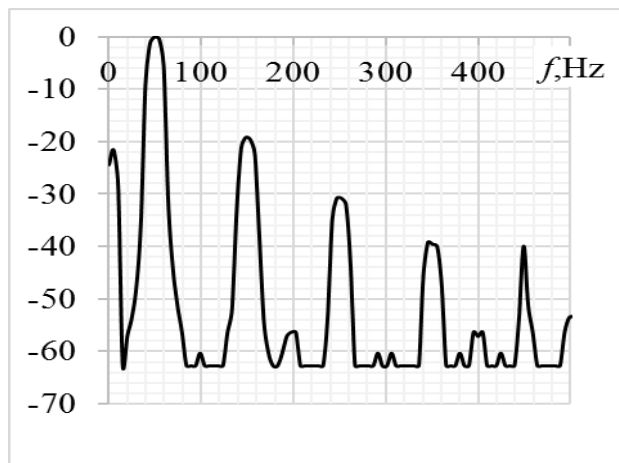
Figures 5 - 7 show the current spectrograms in the coils of electric devices. The current harmonics with attenuation of no more than 40 dB are used for calculation. The obtained results using the first and third and the first to ninth current harmonics are shown by Figures 8 - 10, and Table 4 shows the results of error calculation during the determination of WAC for electrical devices.



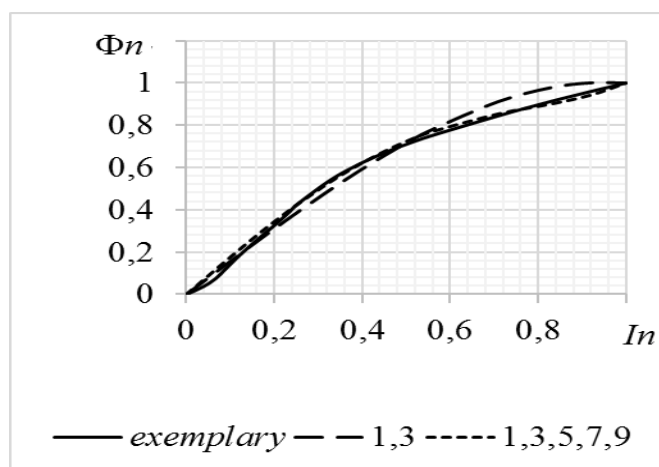
**Figure 5. The range of current during electromagnetic relay study**



**Figure 6.** The range of current during electric motor study



**Figure 7.** The range of current during transformer study



**Figure 8.** Electric motor WAC



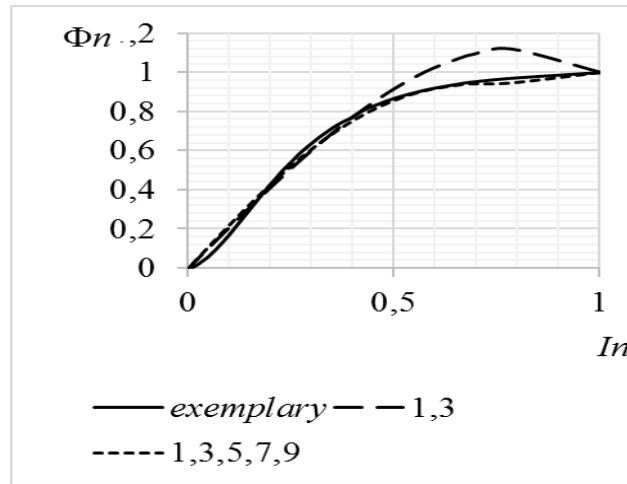


Figure 9. Toroidal transformer WAC

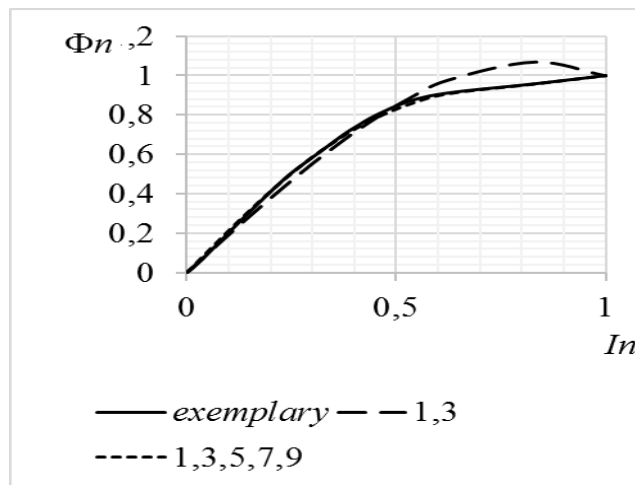


Figure 10. Electromagnetic relay WAC

Table 4 – WAC determination errors

Maximum current harmonic		Electric motor	Transformer	Electromagnetic relay	
Error, %	3	$\delta_I$	21	9	18
		$\delta_\Phi$	14	7	12
		$\delta$	25	11	22
	9	$\delta_I$	2,5	2,1	1,2
		$\delta_\Phi$	1,8	1,5	0,9
		$\delta$	3,1	2,6	1,5

Figures 8 - 10 show that the use of the first and third current harmonics provides deviations from reference characteristic throughout WAC and the current har-

monic use from the first to the ninth one allows to achieve the acceptable accuracy of WAC for electric devices.

#### **4. Conclusion:**

The article showed that the WAC of the operation cycle is an important diagnostic feature of electrical devices. To obtain this feature it is suggested to use the solution of the harmonic balance inverse problem. The mathematical analysis of the harmonic balance inverse problem is performed. The calculation and natural experiments are performed using the mathematical model of Micro-cap circuit simulation package and three types of electrical devices (electromagnetic relay, electric motor and toroidal transformer). The results of which suggest that the proposed method of WAC measuring based on the solution of the harmonic balance inverse problem, allows to get the description of the electrical device at the error which does not exceed 3%. This method may be used for testing electrical equipment during its production as well as during its operation.

#### **Acknowledgements:**

The operation results were obtained due to the support of the project № 1.2690.2014/K "Methods of inverse problems solution for complex systems diagnostics (in engineering are and in medicine) based on natural-model experiment" performed within the project part of the state task with the use of equipment CCP "Diagnosis and energy-efficient electrical equipment" YURGPU (SPE).

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