International Journal of Applied Engineering Research.
ISSN 0973-4562 Volume 10, Number 3 (2015), pp. 6447–6457

http://www.ripublication.com/ijaer.htm

Multiphase Closure Properties of Compressible Fluid Mixing Models[1]

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Abstract

We discuss compressible multiphase flow models with improved physics. An essential difficulty for multiphase flow models is to define averages of nonlinear terms. This is known as the closure problem. The purpose of this paper is to examine all the closure constraints which should be imposed on the two-phase flow model. We identify the problem associated with the entropy of averaging and derive an entropy inequality constraint as opposed to entropy conservation for microphysically adiabatic processes.

AMS subject classification: 76Txx, 76F25, 76N99, 76Axx.
Keywords: Multiphase flow, Closure, Entropy of averaging, Constitutive laws, Averaged equations.

1. Introduction

The problem of multiphase flow has received much attention in recent years due to its strong effect on basic science and engineering applications. Multiphase flow is a prototypical multiscale problem, with a cascade of length scales generally too broad to be modeled effectively. Averaging is one of the most basic tools to deal with multiscale science [5]. The equations of multiphase flow are derived by an averaging process applied to a microphysical description of distinct fluids separated by sharp interfaces. This process introduces an essential difficulty, the closure problem, to define averages of nonlinear functions of the primitive variables. These quantities must be modeled by

[1]This work was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(No. NRF-2010-0010164).

some expressions written in terms of the averaged variables, to close the system of averaged equations, including proper physics requirements. Different choices of closures, appropriate for distinct flow regimes, contribute to the number of different multiphase flow equations.

We consider compressible multiphase flow with surface tension and transports. We discuss the multiphase flow models of the type proposed by [3, 4, 9, 13]. The main result of the present work is to examine all the closure constraints of conservation requirements for the continuity, momentum and energy equations, boundary conditions at the edges of the mixing zone and an entropy inequality. We also identify the problem associated with the entropy of averaging and derive an entropy inequality constraint as opposed to entropy conservation for microphysically adiabatic process.

When an ensemble average is applied to the micro equations to derive macro equations, there is a choice of averaging the total energy [16], internal energy [3, 8] or entropy [15, 17] equations. These averages give distinct equations, which differ by triple correlations only, and so they should have similar solutions. We find that the triple correlations which mark the difference between the three sets of equations occur in the energy equation. Thus the same incompressible limit discussed in [6] applies all three averaged equations.

In other words, there are three possible energy equations, directly averaged from different formulations of the microphysical energy equations of these three equations; and only one is to be used. Obviously, the total energy closure and entropy closure show total energy and phase entropy conservation, respectively. But the entropy in the total energy closure, the total energy and entropy in the internal energy closure and the total energy in the entropy closure are not obviously conserved. Thus conservation is a constraint on the allowed closures. In §3.1, we discuss the conservation constraints and examine conservation of the energy and entropy for the total energy closure.

Conservation of phase mass, total momentum and total energy is a fundamental restriction on any nondissipative system; similarly conservation of phase entropy is fundamental for adiabatic (smooth) flows. Conservation of mass and total momentum is a consequence of derivation of the averaged equations from the conservative microphysical equations. However, phase entropy conservation is a separate problem. Entropy should not be conserved because averaging is nonadiabatic. In §3.2 we derive an inequality constraint as a necessary and sufficient condition for the positivity of an entropy of averaging. The closures which impose the constraint guarantee conservation of the phase entropy for smooth flows, interpreted as an inequality giving positivity for the entropy of averaging.

Multiphase closures have been studied systematically, see *e.g.*, [5]. The interfacial forces between the phases are often divided into separate effects to reflect added mass, drag, and buoyancy force terms. A remarkable feature of the equations proposed here is the fact that they are solvable in closed form in the incompressible limit [7]. Specifically, the closed form expression for the interfacial pressure [7, 8] contains three types of terms, readily identified as corresponding to added mass, drag, and buoyancy related to the edge motions, for Z_k. We assume $(-1)^k V_k = (-1)^k \dot{Z}_k \geq 0$ so that mixing zone is expanding. At each edge $z = Z_k$ of the mixing zone, the closures of the interfacial quantities must equal the phase average of phase which vanishes there. This boundary condition at the mixing zone edge for the closures is summarized in §3.3.

The governing equations of the multiphase flow model we are interested here describe multi-fluid mixing layers which grow out of acceleration driven instabilities, including the classical cases of Rayleigh-Taylor instability, driven by a steady acceleration and Richtmyer-Meshkov instability, driven by an impulsive acceleration. In [11], we have proposed a closure for the compressible multiphase flow model by derivation of a natural formulation for the constitutive laws. The closure satisfies all the closure constraints which are examined here. Spatial homogeneity closures have been compared [12] in a validation study to spatial averages of direct numerical simulations, *i. e.*, simulation solutions of the microphysical equations.

2. The Averaged Equations

In [13, 11] and earlier papers in this series, we have proposed averaged equations for the interior of the mixing zone, coupled to the buoyancy drag equations for the motion of the edges of the mixing zone. The equations for the interior have a functional form derived from a mathematical analysis of the averaged, unclosed, equations. Closure comes from assigning values to unknown parameters in these functional forms.

Let the function X_k be the phase indicator for material k ($k = 1, 2$); *i.e.*, $X_k(t, \mathbf{x})$ equals 1 if position $\mathbf{x}$ is in fluid k at time t, zero otherwise. Multiphase equations for the phase k are obtained by multiplying the microphysical Navier-Stokes equations for compressible fluid flow by X_k and performing an ensemble average. We denoted the ensemble average $\langle\cdot\rangle$. The average of the indicator function X_k is denoted $\beta_k \equiv \langle X_k\rangle$; $\beta_k(z, t)$ is then the expected fraction of the horizontal layer at height z that is occupied by fluid k at time t. Averaging removes most discontinuities from the solution. Specifically, although X_k is discontinuous, β_k is not.

For other variables, we use two types of averaging. The phase average of a variable f is defined by

$$f_k = \frac{\langle X_k f\rangle}{\beta_k}, \tag{1}$$

and the phase mass-weighted average is defined as

$$f_k = \frac{\langle X_k \rho f\rangle}{\langle X_k \rho\rangle}. \tag{2}$$

In the averaged equations, only two variables, the averaged densities and the averaged pressures, are defined by the phase weighted average (1). All other averaged variables are defined by the phase mass-weighted average (2).

In the remainder of this paper, we choose a preferred direction normal to the mixing layer and integrate the primitive equations over two other directions tangent to it. This procedure yields one dimensional averaged equations. We follow Drew [5] and earlier papers in the present series [2, 3, 11] to obtain the ensemble averaged equations

$$\frac{\partial \beta_k}{\partial t} + v^* \frac{\partial \beta_k}{\partial z} = 0\,, \tag{3}$$

$$\frac{\partial (\beta_k \rho_{ki})}{\partial t} + \nabla^s (\beta_k \rho_{ki} v_k) = \beta_k \mathcal{D}_{ki}\,, \tag{4}$$

$$\frac{\partial (\beta_k \rho_k v_k)}{\partial t} + \nabla^s (\beta_k \rho_k v_k^2) = -\frac{\partial}{\partial z}(\beta_k p_k) + p_k^* \frac{\partial \beta_k}{\partial z} + \beta_k \rho_k g + \beta_k \mathcal{M}_k\,, \tag{5}$$

$$\frac{\partial (\beta_k \rho_k E_k)}{\partial t} + \nabla^s (\beta_k \rho_k v_k E_k) = -\nabla^s (\beta_k p_k v_k) + (p_k v)^* \frac{\partial \beta_k}{\partial z} + \beta_k \rho_k v_k g + \beta_k \mathcal{E}_k \tag{6}$$

for the volume fraction β_k, velocity v_k, density ρ_k, pressure p_k, and total energy E_k of phase k. Here $g = g(t) > 0$ is the gravity and the geometry indicator $s = 0, 1, 2$ corresponds to the planar, cylindrical and spherical form of the primitive equations. Generalizations of these equations, not shown here in the interest of simplicity, allow for circular or spherical averages in a cylindrical or spherical geometry. Summing the equation (4) over i, we get the equations for total mass

$$\frac{\partial (\beta_k \rho_k)}{\partial t} + \nabla^s (\beta_k \rho_k v_k) = 0\,, \tag{7}$$

where

$$\nabla^s f(z) = \frac{1}{z^s} \frac{\partial z^s f(z)}{\partial z} \tag{8}$$

is the curvilinear divergence. For convenience, we use the following symbols to represent the source terms of (4)-(6)

$$\mathcal{D}_{ki} = (\nabla \cdot \mathbf{j}_i)_k \tag{9}$$

$$\mathcal{M}_k = (\nabla \cdot \tau')_{k,3} + f_k^s\,, \tag{10}$$

$$\mathcal{E}_k = (\nabla \cdot \tau' \mathbf{v})_k + \left(\nabla \cdot \left(\sum_i h_i \mathbf{j}_i \right) \right)_k + (\nabla \cdot (\kappa \nabla T))_k, \tag{11}$$

in which τ' is the viscous stress tensor, f_k^s is the averaged geometrical source term; $f_k^s = 0$ for rectangular coordinates [14]. h_i is the specific enthalpy of species i, $\mathbf{j}_i$ is the diffusion flux [1], and κ is the heat conductivity. $(\nabla \cdot \tau')_{k,3}$, and v_3 mean the third component of $(\nabla \cdot \tau')_k$ and $\mathbf{v}$. Therefore, the source terms $\mathcal{D}_{ki}$, $\mathcal{M}_k$, and $\mathcal{E}_k$ represent the effects of mass diffusion, viscosity and heat conduction, as well as geometrical source terms. These effects are negligible in the models for the closure in the interior of the mixing zone. To simplify the discussion, they could henceforth set to zero. The Reynolds stress relating to the combined phases is large, the effect of this term is included within the model equations. In contrast, turbulence within a single phase is typically much smaller, and on this basis we neglect the in phase Reynolds stress. Stated alternatively, the velocity differences between the heavy and light fluids are large, and the associated Reynolds stress is already present in the equations. The velocity fluctuation within a light or heavy fluid bubble or droplet is small and can be neglected in the averaged equations. *i.e.*, for example, we are taking advantage of the fact that the light fluid velocity fluctuations about the light fluid mean velocity is small.

Three interfacial terms are defined by

$$v^* = \frac{\langle \mathbf{v} \cdot \nabla X_k \rangle}{\langle \mathbf{n}_3 \cdot \nabla X_k \rangle} , \quad p_k^* = \frac{\langle p_k \mathbf{n}_3 \cdot \nabla X_k \rangle}{\langle \mathbf{n}_3 \cdot \nabla X_k \rangle} , \quad (p_k v)^* = \frac{\langle p_k \mathbf{v} \cdot \nabla X_k \rangle}{\langle \mathbf{n}_3 \cdot \nabla X_k \rangle} , \tag{12}$$

where $\mathbf{n}_3$ is the unit normal vector in the preferred direction. These quantities represent averages of microscopic quantities. The definitions (12) are fundamental to all that follows. They are mathematically exact consequences of the averages of the primitive equations and specify the quantities (the right hand side of (12)) that are to be approximated in a definition of closure to complete the averaged equations (3)-(6).

In the non-zero surface tension case, pressure is discontinuous at the interface ∂X_k, and p_k is the value of the pressure defined by continuity from the interior of X_k. These limiting pressures at the microphysical level, i.e. before ensemble averaging, are related by the equation

$$p_1 - p_2 = (K\sigma\mathbf{n} + \nabla_{\|}\sigma) \cdot \mathbf{n} + \mathbf{n}^T[\tau']\mathbf{n} = K\sigma + \mathbf{n}^T[\tau']\mathbf{n} \tag{13}$$

where K is the mean curvature and σ is the surface tension. It is convenient to define

$$p^* = \frac{1}{2}(p_1^* + p_2^*) , \quad (pv)^* = \frac{1}{2}\left[(p_1 v)^* + (p_2 v)^*\right] \tag{14}$$

and the capillary pressure

$$p_c^* = p_1^* - p_2^* , \quad (p_c v)^* = (p_1 v)^* - (p_2 v)^*. \tag{15}$$

For later use we define

$$\begin{aligned} \widetilde{p_k} &= p_k + (-1)^k \frac{p_c^*}{2} \\ \widetilde{p_k v_k} &= p_k v_k + (-1)^k \frac{(p_c v)^*}{2}. \end{aligned} \tag{16}$$

In [11] we derive a mathematically exact expression for each q^*, $q = v,\ p,\ pv$, independently of any closure assumptions. The derivation leads to a natural formulation of closures for the constitutive laws.

3. The Closure Constraints

The chunk mix two-phase flow model is a stochastic description of chaotic interpenetration of two viscid heat-conduction fluids. We are here concerned with chunk mix, a flow regime characterized by large scale coherent mixing structures (bubbles of light fluid, *etc.*), on the order of the thickness of the mixing zone, and by short time scales. An ensemble average is applied to the micro equations to derive macro equations. The entropy in the energy averaged equations and the energy in the entropy conserved equations are not obviously conserved. These models differ in the variables after averaging and in the closure assumption. Thus conservation is a constraint on the allowed closures. We discuss the conservation constraints in §3.1. We examine conservation of the phase entropy and derive an inequality constraint for the positivity of an entropy of averaging in §3.2. The boundary conditions at the mixing zone edge for the closures are summarized in §3.3.

We specify boundary conditions for compressible and incompressible flow. We assume a container with a slab of heavy fluid of density ρ_2 lying beneath a slab of light fluid of density ρ_1 and separated by an interface. This configuration is then accelerated downwards with an acceleration larger than the earth gravity, reversing the direction of gravity. We assume existence of rigid wall at the top of a finite but large domain $\mathcal{D}$. Then velocity is zero and the pressure is unknown there. At the bottom of this domain, we have conceptually an open container. This fixes the pressure at some ambient value, but not the velocity at the bottom of $\mathcal{D}$. This leads to the boundary conditions

$$v_1(z^{+\infty}) = 0\,, \tag{17}$$

$$p_2(z^{-\infty}) = \text{const}, \tag{18}$$

where $z = z^{+\infty}$ ($z = z^{+\infty}$) denotes the position of the upper (lower) wall of the domain $\mathcal{D}$.

For later use, we introduce the notation

$$\Delta q \equiv q_1 - q_2 \tag{19}$$

for any quantity q and the notation of the phase k convective derivative

$$\frac{D_k}{Dt} \equiv \frac{\partial}{\partial t} + v_k \nabla^s. \tag{20}$$

3.1. Conservation Constraints

In this section we examine conservation of the total energy or phase entropy for the case of the total energy closure. We believe that the total energy closure, with conservation of total energy is the most attractive of the three closure alternatives, and that the species entropy should be conserved only after an entropy of averaging has been imposed [11]. We derive a conservation constraint which is required to conserve the entropy in the total energy closure (6). Conservation of phase k mass, total momentum and total energy is a fundamental restriction on any nondissipative system; similarly conservation of phase entropy is fundamental for adiabatic (smooth) flows. Aside from entropy, this conservation property results from the derivation of (3)-(7) from the conservative microphysical equations. Conservation of phase k mass is seen from the absence of source terms in (7). Conservation of total momentum is seen from the reversal of sign in the (5) source terms, namely $p^*\partial\beta_1/\partial z = -p^*\partial\beta_2/\partial z$ since $\beta_1 + \beta_2 = 1$ and $\partial(\beta_1 + \beta_2)/\partial z = 0$. Similarly conservation of total energy results from the reversal of signs in (6). However, conservation of phase k entropy, likewise a basic property of smooth (adiabatic) flows, raises basic questions addressed as a central issue of this paper.

We assume the thermodynamic relation is satisfied for the averaged quantities,

$$T_k d\mathbb{S}_k = de_k + p_k\, d\left(\frac{1}{\rho_k}\right). \tag{21}$$

The total energy closure (6) reduces to the internal energy equation

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\beta_k\rho_k e_k\right) + \nabla^s\left(\beta_k\rho_k v_k e_k\right)\\ &= -\beta_k p_k \nabla^s v_k - p_k^* v_k \frac{\partial\beta_k}{\partial z} + (p_k v)^* \frac{\partial\beta_k}{\partial z} + \beta_k\mathcal{F}_k,\end{aligned} \tag{22}$$

where

$$\mathcal{F}_k \;=\; \mathcal{E}_k - v_k\mathcal{M}_k. \tag{23}$$

From (22) and (21), we derive the entropy equation

$$\beta_k\rho_k T_k \frac{D\mathbb{S}_k}{Dt} = [-p_k v^* + p_k v_k - p_k^* v_k + (p_k v)^*]\frac{\partial\beta_k}{\partial z} + \beta_k\mathcal{F}_k \tag{24}$$

for smooth solutions. Here $\mathbb{S}_k = \mathbb{S}_k(e_k, \rho_k)$ is the entropy expressed from the fluid k EOS and the directly averaged total energy E_k and density ρ_k of fluid k using the additional definition of the internal energy $e_k \equiv E_k - v_k^2/2$. Using (14) and (15), Eq. (24) can be reformulated as

$$\beta_k\rho_k T_k \frac{D\mathbb{S}_k}{Dt} = [-\widetilde{p}_k v^* - p^* v_k + \overline{p_k v_k} + (pv)^*]\frac{\partial\beta_k}{\partial z} + \beta_k\mathcal{F}_k, \tag{25}$$

where we define

$$\overline{p_k v_k} \;=\; \widetilde{p}_k v_k + \frac{(-1)^k}{2}[p_c^* v^* - (p_c v)^*]. \tag{26}$$

Assuming the source term $\mathcal{F}_k = 0$, from (25), the entropy $\mathbb{S}_k$ of each phase is conserved if

$$-\widetilde{p}_k v^* - p^* v_k + \overline{p_k v_k} + (pv)^* = 0. \tag{27}$$

The difference between the internal energy and entropy closure results from the triple correlation in (12). In [2, 3], the right hand side of the internal energy closure is derived by use of the identity

$$\begin{aligned}-\langle X_k p\nabla\cdot\mathbf{v}\rangle &= -\langle p\nabla\cdot(X_k\mathbf{v})\rangle + \langle p\mathbf{v}\cdot\nabla X_k\rangle\\ &= -p_k\langle\nabla\cdot(X_k\mathbf{v})\rangle - \langle(p-p_k)\nabla\cdot(X_k\mathbf{v})\rangle + \langle p\mathbf{v}\cdot\nabla X_k\rangle.\end{aligned} \tag{28}$$

The second term in (28) represents a truncated correlation and it can be set to zero because the numerical data shows that this truncated correlation is small. We examine the energy equation when the same correlation is used in the last term of (28). Calculations show

$$\begin{aligned} -\langle X_k p \nabla \cdot \mathbf{v} \rangle = &- p_k \langle \nabla \cdot (X_k \mathbf{v}) \rangle - \langle (p - p_k) \nabla \cdot (X_k \mathbf{v}) \rangle \\ &+ p_k \langle \mathbf{v} \cdot \nabla X_k \rangle + \langle (p - p_k) \mathbf{v} \cdot \nabla X_k \rangle \\ = &- p_k \frac{\partial (\beta_k v_k)}{\partial z} + p_k v^* \frac{\partial \beta_k}{\partial z} \end{aligned} \tag{29}$$

by assuming $\langle (p - p_k) \nabla \cdot (X_k \mathbf{v}) \rangle$ and $\langle (p - p_k) \mathbf{v} \cdot \nabla X_k \rangle$ are negligible. The replacement of (28) by (29) gives the internal energy equation derived from the microscopic entropy equation. Thus the entropy closure and internal energy closure differ by triple correlations only.

3.2. The Entropy Constraint

Conservation of the total energy for the averaged equations is a direct consequence of the choice of total energy as a primitive variable and the total energy microscopic equation as the equation to be averaged. Similarly, conservation of mass and total momentum is a consequence of these same properties for the microphysical equations. However, phase entropy conservation is a separate question. We distinguish between two notions of entropy. The first is the direct average of the microphysical entropy in phase k, S_k, which is preserved for processes which are microphysically adiabatic. The second entropy $\mathbb{S}_k$ is defined as a thermodynamic function from the equation (21) of state and the averaged primitive variables (density and energy). These two definitions of entropy are not the same. The quantity $\mathbb{S}$ includes a (positive) entropy of averaging resulting from the in phase average in the definitions of ρ_k and E_k. Thus $\mathbb{S}_k \geq S_k$. Assuring this inequality (as a constraint on the closure) is a central result of this paper.

In general, the entropy $\mathbb{S}_k$ of each phase fails to be conserved because the macro entropy

$$\mathbb{S}_k = \mathbb{S}_k(E_k, \rho_k) \equiv \mathbb{S}_k \left(\frac{\langle X_k \rho E \rangle}{\langle X_k \rho \rangle}, \frac{\langle X_k \rho \rangle}{\langle X_k \rangle} \right), \tag{30}$$

expressed *via* the equation of state from the macro (averaged) energy is not the same as the macro entropy, $S_k \equiv \langle X_k \rho S \rangle / \langle X_k \rho \rangle$, expressed directly as an average of the micro entropy. The difference

$$\delta S_k \equiv \mathbb{S}_k(E_k, \rho_k) - S_k \tag{31}$$

is due to the averaging of within phase fluctuations and is thus identified as an entropy of averaging, within a single phase. In other words, averaging of the ρ_k and E_k variables is not an isentropic process. It increases the entropy of the system.

With an ensemble average, the entropy of averaging reflects a loss of information within the averaging process. One could also define the ensemble in terms of spatial and/or temporal averages. The characteristic averaging length could then reflect the length scale associated with a measuring probe and again the entropy of averaging reflects loss of information via the average. But the spatial average could also reflect some omitted physical process, such as heat and mass diffusion within a single phase. From this perspective, the entropy of averaging is actually an entropy of mixing, and thus reflects a physical process not modeled in the original equations (3)-(6).

We expect the inequality $\delta S_k \geq 0$ to be maintained. From (6) we derive conservation of the microphysical entropy and its average S_k for adiabatic processes. Using this fact together with (25), we obtain the equation

$$\frac{\partial(\beta_k \rho_k \delta S_k)}{\partial t} + \frac{\partial(\beta_k \rho_k v_k \delta S_k)}{\partial z} = \frac{1}{T_k}[-\widetilde{p}_k v^* - p^* v_k + \overline{p_k v_k} + (pv)^*]\frac{\partial \beta_k}{\partial z}. \tag{32}$$

Summing Eqs. (32) over $k = 1, 2$, we see that a necessary condition for the $k = 1$ and $k = 2$ flows to have nonnegative entropy of averaging source terms is that

$$\sum_{k=1}^{2}[-\widetilde{p}_k v^* - p^* v_k + \overline{p_k v_k} + (pv)^*]\frac{\partial \beta_k}{\partial z} = \left[-v^* \Delta \widetilde{p} - p^* \Delta v + \Delta(\overline{pv})\right] \frac{\partial \beta_1}{\partial z} \geq 0. \tag{33}$$

Notice that the closure for $(pv)^*$ makes no contribution to (33). Thus, independently of a closure for $(pv)^*$, the condition (33) is required to have nonnegative source terms in (32) for $k = 1, 2$. As a sufficient condition, we require the inequality

$$(-1)^k (pv)^* \frac{\partial \beta_1}{\partial z} \leq (-1)^k \left(\widetilde{p}_k v^* + p^* v_k - \overline{p_k v_k}\right) \frac{\partial \beta_1}{\partial z}, \tag{34}$$

equivalent to nonnegative source terms (RHS) in (32). Let

$$\mathcal{B}_k \equiv \left(\widetilde{p}_k v^* + p^* v_k - \overline{p_k v_k}\right) \frac{\partial \beta_1}{\partial z}. \tag{35}$$

Then (34) is rewritten as

$$\mathcal{B}_1 \leq (pv)^* \frac{\partial \beta_1}{\partial z} \leq \mathcal{B}_2. \tag{36}$$

Therefore, the inequality constraint (36) is required as a necessary and sufficient condition for the positivity of an entropy of averaging, which should be imposed in the closures v^*, p^*, $(pv)^*$ of the governing equations (3)-(6). In [12, 11], we propose a new definition of v^*, p^*, $(pv)^*$, which imposes (36) and gurantees conservation of the total energy.

The entropy inequality constraint (36) is a restriction on the model system. The model depends on the motions $Z_k(t)$ of the edges of the mixing zone. It was shown in [10] that given the edge velocities V_k of the mixing zone, the system of the compressible equations is closed mathematically and the solutions depend on V_k. Thus, (36) couples two edge motions of the mixing zone.

3.3. The Boundary Conditions

We introduce mixing zone boundary conditions which closures for q^*, $q = v, p, pv$, should satisfy. The height at which β_1 (β_2) vanishes is labelled the lower (upper) edge of the mixing zone, and it corresponds to the tip of the frontier portion of light (heavy) fluid in the microscopic flow. Therefore the average q^* of the fluid quantity q must equal q_1 (q_2) at the lower (upper) mixing zone edge $z = Z_1(t)$ ($z = Z_2(t)$). Consistency with the microphysical equations leads to mixing zone boundary constraints

$$v^* = v_k\,, \quad p^* = p_k, \quad (pv)^* = p_k v_k \quad \text{at } z = Z_k. \tag{37}$$

The property of hyperbolic stability (real characteristics) follows from the general form of Eqs. (3)-(6) in the inviscid case since in the characteristic analysis the q^* terms and the volume fraction equation decouple from the other equations, and these remaining ones have the same characteristic analysis as two copies of the Euler equations of a compressible fluid.

4. Conclusions

All required closure constraints of boundary conditions and conservations are identified for the multiphase flow model (3)-(6). The boundary constraints are given in (37). Total energy is conserved according to (6). Entropy is not be conserved because averaging is nonadiabatic, but the entropy inequality constraint (36) should be enforced for the positivity of an entropy of averaging.

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