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Direct Product Of Semiprime Ideals In Ternary Semigroups

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Abstract

This Paper deals with direct product of ternarysemigroups. The general properties and characteristics of complete rightideal of a ternarysemigroup. In this paper we have mainly proved that R be a complete rightideal of a ternarysemigroup T = $\prod_{i \in I} T_i$. Then P_i(R) is a complete rightideal of T_i and $\prod_{i \in I} P_i(R)$ is a complete rightideal of T_i.

Keywords : Direct product of ternarysemigroups, projection, semi prime.

Introduction

Anjaneyulu.A [1] investigated the structure of prime ideals in semi groups, which I have used in this paper. In this paper I used the concepts on the direct product of semi groups written by [3] R.croisot introduced [2] the following condition: An element b of a semi group S satisfies the condition (m, n) if there exist an element $y \in$ S such that $b = b^m y b^n$; m, n are positive integers. The set of all elements satisfying the condition (m, n) is called a class of regularity and it is denoted by R_S(m, n). By means of this notion some properties of ternarysemigroup have been studied. In this paper now I have introduce the definition of direct product and projection of ternarysemigroups.

Definition 1:
Let { T_i }, $i \in$ I be an arbitrary system of ternarysemigroups. Denote by T the set of all functions ξ, defined on I such that $\xi(i) \in T_i$. Introduce in T a multiplication in this way: If $\alpha, \beta, \gamma \in$ T are arbitrary elements of T, then the product $\omega = \alpha\beta\gamma$ is given by $\omega(i) = \alpha(i)\beta(i)\gamma(i)$ for every $i \in$ I. The set T with this multiplication is a ternarysemigroup, which is called a direct product of ternarysemigroups { T_i }, $i \in$ I and is denoted by T = $\prod_{i \in I} T_i$.

Definition 2:
A rightideal R of the ternarysemigroup T is known as a semi prime rightideal of T if for any element $a \in$ T an arbitrary odd integer n the relation $a^n \in$ R implies that $a \in$ R.

Theorem 1:
Let R_i be a semi prime rightideal of a ternarysemigroup T_i for every $i \in$ I. Then R = $\prod_{i \in I} R_i$ is a semi prime rightideal of T = $\prod_{i \in I} T_i$.

Proof :
Let $\beta \in$ T = $\prod_{i \in I} T_i$ be an arbitrary element and let $\beta^n \in$ R = $\prod_{i \in I} R_i$. Then $[\beta(i)]^n \in R_i$ for every $i \in$ I and n is an odd integer. Since R_i is a semi prime rightideal of $T_{i,}$ we have $\beta(i) \in R_i$ for every $i \in$ I. Hence $\beta \in R = \prod_{i \in I} R_i$.

Definition 3:
Let N $\subseteq$ T = $\prod_{i \in I} T_i$. The set of all elements $x_i \in T_i$ for which there exist at least one element $\xi \in$ N such that $\xi(i) = x_i$, will be indicated by P_i(N) and known as the projection of the set N into the semi group S_i.

Theorem 2:
Let R = $\prod_{i \in I} R_i$ be a semi prime rightideal of a ternarysemigroup T = $\prod_{i \in I} T_i$. Then P_i(R) is a semi prime rightideal of T_i.

Proof :
Let R = $\prod_{i \in I} R_i$ be a semi prime rightideal of T = $\prod_{i \in I} T_i$. The fact that P_i(R) is a rightideal of T_i, It is only necessary to prove that it is semi prime. Let $a_i \in T_i$, $a_i^n \in P_i$(R) where $i \in$ I is arbitrary, but fixed. It is necessary to show that $a_i \in P_i$(R) Since $a_i^n \in P_i$(R) it follows that $\exists\ \beta \in$ R $\ni \beta(i) = a_i^n$, put $\beta(j) = b_j$ for $j \neq i$, $j \in$ I. Let $\alpha \in$ T such that $\alpha(i) = a_i$, $\alpha(j) = b_j$ for $j \neq i$,, $j \in$ I. Since $\beta \in$ R, then $\beta(j) = b_j \in P_j$(R) for every $j \neq i$. But P_j(R) is a rightideal, hence $b_j^n \in P_j$(R) for $j \neq i$.

And corresponding to the assumption $a_i^n \in P_i$(R). That is $[\alpha(i)]^n \in P_i$(R) for every $i \in$ I, hence $\alpha^n \in$ R (since R = $\prod_{i \in I} R_i$). But since R is a semi prime rightideal of T, then $\alpha \in$ R, hence $\alpha(i) = a_i \in P_i(R)$, i.e $P_i(R)$ is a semi prime rightideal of T_i.

Theorem 1 and 2 implies :

Corollary 1:
A rightideal R = $\prod_{i \in I} R_i$ of a ternarysemigroup T = $\prod_{i \in I} T_i$ is a semi prime $\Leftrightarrow$ The rightideal R_i for every $i \in$ I is semi prime.

Definition 4:
A rightideal R of a ternarysemigroup T is complete if RTT = R.

A rightideal R of a ternarysemigroup T is complete if for any $a \in$ R there exist $x, y \in$ T, $b \in$ R such that $bxy = a$.

Theorem 3:
The set union of two complete rightideals of a ternarysemigroup T is a complete rightideal of T.

Proof :
Assume R_1, R_2 are two complete rightideals of T. Then $R_1TT = R_1$, $R_2TT = R_2$. Hence $(R_1 \cup R_2)TT = R_1TT \cup R_2TT = (R_1 \cup R_2)$. Which proves our assumption.

The question arises, whether the intersection of two complete rightideals is a right complete ideal.

The next example gives a negative answer.

Example 1:
Let T = {a, b, c, d} be a ternarysemigroup with the multiplication table.

.	*a*	*b*	*c*	*d*
a	*a*	*a*	*a*	*a*
b	*a*	*a*	*a*	*a*
c	*a*	*a*	*c*	*d*
d	*a*	*a*	*c*	*d*

R_1 = {a, b, c}; R_2 = {a, b, d} are complete rightideals of T, but $R_1 \cap R_2 = R_3$ = {a, b} is not a complete rightideal of T.

Definition 5:
A rightideal R of a ternarysemigroup T is called a minimal if there exist no rightideal of T properly contained in R.

Evidently every minimal rightideal of a ternarysemigroup T is a complete rightideal of T.

Theorem 4:
Every rightideal of a ternarysemigroup T is complete rightideal of T if and only if T = $R_T(0,1)$.

Proof :
Let R = $\bigcup_{a\in R} a$ be a rightideal of T. Then RTT $\supset \{\bigcup_{a\in R} a\}\{\bigcup_{a\in R} x_a\}\{\bigcup_{a\in R} y_a\} \supset \bigcup_{a\in R} a x_a y_a = \bigcup_{a\in R} a$ = R. Hence RTT = R.

Conversely suppose that every rightideal of T be complete. Let $a \in$ T be any element of T. The rightideal $a \cup a$TT satisfies ($a \cup a$TT)TT = $a \cup a$TT. i.e aTT $\cup$ aTTTT = $a \cup a$TT. Hence aTT = $a \cup a$TT. Therefore $a \in a$TT which proves that T = $R_T(0,1)$.

Remark :
Clearly the following assumptions hold.

a) If T contains a right unit, then every rightideal is complete.

b) T = $R_T(0,1)$ = $R_T(1,0)$ if and only if every rightideal of T is complete.

c) If T = $R_T(1,1)$ then every rightideal of T is complete.

d) If all rightideals of T are complete, then then T^3 = T.

The next example of a ternarysemigroup shows that the converse of the assumption (*d*) need not hold.

Example 2:
Let T be an additive ternarysemigroup of positive numbers. Then T^3- T. Let R = (a, ∞) with $a > 0$. Then RTT = $(a, \infty) \subset (a, \infty)$. So that R is not complete.

Theorem 5:
Let R_i for every $i \in$ I is a complete rightideal of the ternarysemigroup T_i then R = $\prod_{i \in I} R_i$ is a complete rightideal of T = $\prod_{i \in I} T_i$.

Proof :
Let R_i be a complete rightideal of a ternarysemigroup T_i, hence $R_i T_i T_i = R_i$, we have to prove that for any $\omega \in$ R, there exist $\vartheta \in$ R and $\beta, \gamma \in$ T such that $\vartheta\beta\gamma = \omega$.

Since R_i is a complete rightideal of T_i, there exist for every $\omega(i) = a_i \in R_i$, three elements $b_i \in R_i$ and x_i, $y_i \in T_i$ such that $b_i x_i y_i = a_i$. The functions ϑ, α defined by $\vartheta(i) = b_i$; $\beta(i) = x_i$; $\gamma(i) = y_i$ satisfy $\vartheta\beta\gamma = \omega$.

Theorem 6:
Let R be a complete rightideal of a ternarysemigroup T = $\prod_{i \in I} T_i$.

Then

i) P_i(R) is a complete rightideal of T_i.

ii) $\prod_{i\in I} P_i\,(R)$ is a complete rightideal of T_i.

Proof :
Let R be a complete rightideal of T = $\prod_{i \in I} T_i$. Clearly $P_i(R)$ is a rightideal of T_i. It is only necessary to prove that it is complete. Let $a_i \in P_i(R)$. To prove that $P_i(R)$ is a complete rightideal. i.e we have to show that there exist $b_i \in P_i(R)$ and x_i, $y_i \in T_i$ such that $b_i x_i y_i = a_i$.

Since $a_i \in P_i(R)$, it follows that there exist an element $\mu \in$ R such that $\mu(i) = a_i$. Since R is a complete rightideal of T = $\prod_{i \in I} T_i$, there exist elements $\vartheta, \alpha, \beta \in$ R and $\mu \in$ T such that $\vartheta\alpha\beta = \mu$. This means that for every $i \in$ I, we have $\vartheta(i)\alpha(i)\beta(i) = \mu(i)$, where $\mu(i) = a_i$, $\vartheta(i) = b_i \in P_i(R)$ and $\alpha(i) = x_i$, $\beta(i) = y_i \in T_i$. Therefore, we have $b_i x_i y_i = a_i$. This proves (i).

(ii) The statement (ii) follows from (i) and theorem 5.

Theorem 7:
A ternarysemigroup T = $\prod_{i \in I} T_i$ satisfies the condition (m, n) if and only if each of the ternarysemigroup T_i satisfies this condition.

Proof :
Let us assume that every ternarysemigroup T_i satisfies the condition (m, n). Let $\alpha \in$ T be an arbitrary element. Then $\alpha(i) = a_i \in T_i$ for every $i \in$ I. Since T_i satisfies the condition (m, n), there exists an $x_i \in T_i$ such that $a_i = a_i^m x_i a_i^n$ (1).

Define $\beta \in T$ by the requirement that $\beta(i) = x_i$ for every $i \in$ I. The relation (1) can be written in the form $\alpha(i) = [\alpha(i)]^m \beta(i) [\alpha(i)]^n$ for every $i \in$ I. This means $\alpha = \alpha^m \beta \alpha^n$. But the last relation says that T = $\prod_{i \in I} T_i$ satisfies the condition (m, n).

Conversely suppose that let T = $\prod_{i \in I} T_i$ satisfies the condition (m, n). Let $a_i \in T_i$ be an arbitrary element. Then there exist atleast one element $\alpha \in$ T such that $\alpha(i) = a_i$. Since T satisfies the condition (m, n), there exists an element $\beta \in$ T such that $\alpha = \alpha^m \beta \alpha^n$. Hence for our i, $a_i = a_i^m x_i a_i^n$. This means that T_i satisfies condition (m, n).

Corollary 2:
Every rightideal of the ternarysemigroup T = $\prod_{i \in I} T_i$ is complete if and only if every rightideal of the ternarysemigroup T_i for every $i \in$ I is complete.

Proof :
The proof follows from theorem 4 and 7.

Corollary 3:
The following statements are equivalent :

i) Each of the ternarysemigroup T_i for every $i \in$ I satisfies the condition (0,1).

ii) The ternarysemigroup T = $\prod_{i \in I} T_i$ satisfies the condition (0,1).

iii) Every rightideal of T_i for every $i \in I$ is complete.
iv) Every rightideal of $T = \prod_{i \in I} T_i$ is complete.

Proof :

(i) implies (ii) according to theorem 7.
(ii) implies (iii) according to theorem 7 and theorem 4.
(iii) implies (iv) according to corollary 2.
(iv) implies (i) according to corollary 2 and theorem 4.

Definition 6:

A rightideal R of a ternarysemigroup T is known as semi prime if for every element $a \in T$ and an arbitrary integer n the relation $a^n \in R$ implies $a \in R$.

Theorem 8:

Let R_i be a right semi prime ideal of T_i for every $i \in I$. Then $R = \prod_{i \in I} R_i$ is a right semi prime ideal of $T = \prod_{i \in I} T_i$.

Proof :

Let $\alpha \in T = \prod_{i \in I} T_i$ be an arbitrary element and let $\alpha^n \in R = \prod_{i \in I} R_i$. Then $[\alpha(i)]^n \in R_i$ for every $i \in I$. Since R_i is a semi prime ideal of T_i we have $\alpha(i) \in R_i$ for every $i \in I$. Hence $\alpha \in R = \prod_{i \in I} R_i$.

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