International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 3 (2015) pp. 6293-6304

http://www.ripublication.com

Chemical Reaction And Radiation Effects On A Parabolic Flow Past An Infinite Vertical Plate With Variable Temperature And Uniform Mass Diffusion

R.Muthucumaraswamy[1] and P.Sivakumar[2*]

[1]Department of Applied Mathematics, Sri Venkateswara College of Engineering Irungattukottai, Sriperumbudur-602 117, INDIA, Phone: +9144-27152000
[2] Department of Mathematics, P.B.College of Engineering Irungattukottai, Sriperumbudur-602 117, INDIA.
**Corresponding author: sivakumarpushparaj@gmail.com*

Abstract

The effects of heat and mass transfer on a parabolic flow past an infinite vertical plate with variable temperature and uniform mass diffusion with radiation and chemical reaction has been discussed. The dimensionless governing equations were solved using the Laplace transform method and for graphs we used MATLAB software. Approximate solutions have been derived for the velocity, temperature, concentration profiles. The obtained results are discussed with the help of the graphs to observe the effect of various parameters like radiation parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time on the velocity, temperature and concentration.

Keywords: parabolic, radiation, chemical reaction, vertical plate, heat and mass transfer.

INTRODUCTION

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid.

We are particularly interested in cases in which diffusion and chemical reaction occur at roughly the same speed. When diffusion is much faster than chemical reaction, then only chemical factors influence the chemical reaction rate; when diffusion is not much faster than reaction, the diffusion and kinetics interact to produce very different effects. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction. Due to the fast growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight.

There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of the chemical reaction effect. These processes are observed in nuclear reactor safety and combustion systems, solar collectors, as well as metallurgical and chemical engineering. Their other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, and combustion of atomized liquid fuels.

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young [1] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al* [2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das *et al* [3]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level. The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [4]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [5]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. [6].Muthucumaraswamy and Ganesan [7] investigated the diffusion and first order chemical reaction on impulsively started infinite vertical plate with variable temperature. The effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection was studied by Kandasamy et al [8]. Muthucumaraswamy and Janakiraman [9] investigated the mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction. Kafousias and Raptis [10] extended this problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by the Singh and Singh [11]. Basant kumar Jha and Raindrop Prasad [12] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

MATHEMATICAL ANALYSIS

A chemical reaction and radiation effect on a parabolic flow past an infinite vertical plate with variable temperature and uniform mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x'-axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ .At time $t' > 0$, the plate is started with a velocity $u = u_0.t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. The plate is infinite in length all the terms in the governing equations will be independent of x' and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - k_l (C' - C'_\infty) \tag{3}$$

With the following initial and boundary conditions:

$$u = 0,\ T = T_\infty,\ C' = C'_\infty \text{ for all } y, t' \le 0$$

$$t' > 0:\ u = u_0.t'^2,\ T = T_\infty + (T_W - T_\infty)At',\ C' = C'_W \text{ at } y = 0 \tag{4}$$

$$u \to 0\ \ T \to T_\infty,\ C' \to C'_\infty \text{ as } y \to \infty$$

Where,

$$A=\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y}=-4a^*\ \sigma(T_\infty^4-T^4) \quad (5)$$

It is assumed that the temperature differences with in the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\,T-3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p\ \frac{\partial T}{\partial t'}=k\ \frac{\partial^2 T}{\partial y^2}+16a^*\sigma T_\infty^3\ (T_\infty-T) \quad (7)$$

On introducing the following non-dimensional quantities:

$$U=u\left(\frac{u_0}{\nu^2}\right)^{1/3},\ t=\left(\frac{u_0{}^2}{\nu}\right)^{1/3}t',\ Y=y\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta=\frac{T\ -T_\infty}{T_w-T_\infty},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gr=\frac{g\beta(T_w-T_\infty)}{\nu.u_0{}^{1/3}},$$

$$Gc=\frac{g\beta(C'_w-C'_\infty)}{\nu.u_0{}^{1/3}}, \quad (8)$$

$$R=\frac{16a^*\sigma.T_\infty{}^3}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}},\ \Pr=\frac{\mu C_p}{k},\ Sc=\frac{\nu}{D}$$

The equations (1), (3) and (7), reduces to the following dimensionless form

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{11}$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0,\ \theta = 0,\ C = 0 \text{ for all } Y, t \le 0$$

$$t > 0: \quad U = t^2,\ \theta = t\ ,\ C = 1 \text{ at } Y = 0 \tag{12}$$

$$U \to 0,\ \theta \to 0,\ C \to 0 \quad \text{as } Y \to \infty$$

The dimensionless governing equations (9) to (11) and the corresponding initial and boundary conditions (12) are tackled using Laplace transform technique.

$$C = \frac{1}{2}\left[\exp\left(2\eta\sqrt{Sc\,K t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \tag{13}$$

$$\theta = \frac{t}{2}\left[\exp\left(2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right]$$

$$- \frac{\eta\sqrt{\Pr}\sqrt{t}}{2\sqrt{a}}\left[\exp\left(-2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right) - \exp\left(2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right)\right] \tag{14}$$

$$U = 2\left[\frac{t^2}{6}\left[\left(3 + 12\eta^2 + 4\eta^4\right) erfc(\eta) - \frac{\eta}{\sqrt{\pi}}\left(10 + 4\eta^2\right)\exp(-\eta^2)\right]\right]$$

$$+d\begin{bmatrix} erfc(\eta) - \frac{\exp(bt)}{2}\left[\exp\left(2\eta\sqrt{bt}\right) erfc\left(\eta + \sqrt{bt}\right) + \exp\left(-2\eta\sqrt{bt}\right) erfc\left(\eta - \sqrt{bt}\right)\right] \\ -\frac{1}{2}\left[\exp\left(2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \\ + \frac{\exp(bt)}{2}\left[\exp\left(2\eta\sqrt{\Pr\,(a+b)\,t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{(a+b)\,t}\right) + \exp\left(-2\eta\sqrt{\Pr\,(a+b)\,t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{(a+b)\,t}\right)\right] \end{bmatrix}$$

$$+e\begin{bmatrix} erfc(\eta) - \frac{\exp(ct)}{2}\left[\exp\left(2\eta\sqrt{ct}\right) erfc\left(\eta + \sqrt{ct}\right) + \exp\left(-2\eta\sqrt{ct}\right) erfc\left(\eta - \sqrt{ct}\right)\right] \\ -\frac{1}{2}\left[\exp\left(2\eta\sqrt{Sc\,K t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\ + \frac{\exp(ct)}{2}\left[\exp\left(2\eta\sqrt{Sc\,(K+c)\,t}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{(K+c)\,t}\right) + \exp\left(-2\eta\sqrt{Sc\,(K+c)\,t}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{(K+c)\,t}\right)\right] \end{bmatrix}$$

$$+d\left[b\left(\begin{array}{l} t\left[\left(1+2\eta^{2}\right) erfc(\eta)-\dfrac{2\eta}{\sqrt{\pi}}\exp(-\eta^{2})\right] \\ -\dfrac{t}{2}\left[\exp\left(2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr}-\sqrt{at}\right)\right] \\ +\dfrac{\eta\sqrt{\Pr}\sqrt{t}}{2\sqrt{a}}\left[\exp\left(-2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr}-\sqrt{at}\right)-\exp\left(2\eta\sqrt{\Pr a t}\right) erfc\left(\eta\sqrt{\Pr}+\sqrt{at}\right)\right] \end{array}\right)\right] \quad (15)$$

Where

$$a=\frac{R}{\Pr}, b=\frac{R}{1-\Pr}, c=\frac{KSc}{1-Sc}, d=\frac{Gr}{b^{2}(1-\Pr)}, e=\frac{Sc}{c(1-Sc)} \text{ and } \eta=\frac{y}{2\sqrt{t}}$$

RESULTS AND DISCUSSION

In order to get a physical view of the problem the numerical values of the velocity, temperature and concentration for different values of radiation parameter, chemical reaction parameter, Schmidt number and time. The value of the Schmidt number *Sc* is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number *Pr* are chosen such that they represent air (*Pr* = 0.71). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, chemical reaction parameter, radiation parameter, Schmidt number and time are studied graphically The Laplace transform solutions are in terms of exponential and complementary error function.

Fig.1. Illustrates the effect of velocity for different values of the chemical reaction parameter (K = 2, 5, 10), *Gr* = *Gc* = 5, *R* = 2 and *t* = 0.2. This shows that the increase in the chemical reaction parameter leads to a fall in the velocity. Fig.2. Represents the effect of velocity for different values of the radiation parameter (R= 2, 5, 10), Gr = Gc = 5 and t = 0.4. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that there is a fall in velocity in the presence of high thermal radiation.

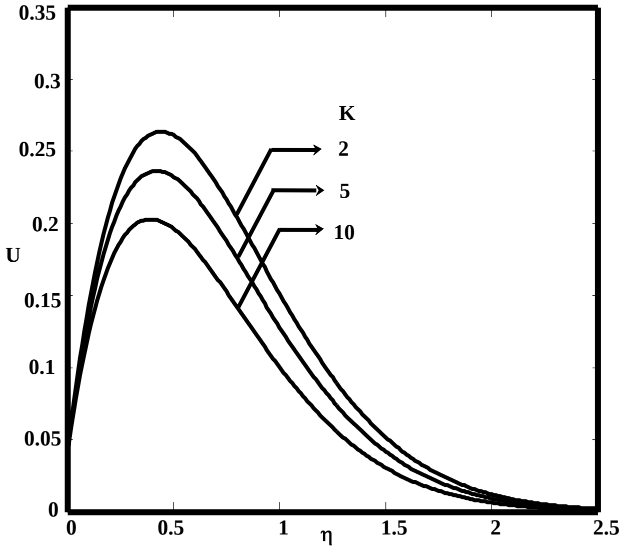

Fig.1. Velocity profiles for different K

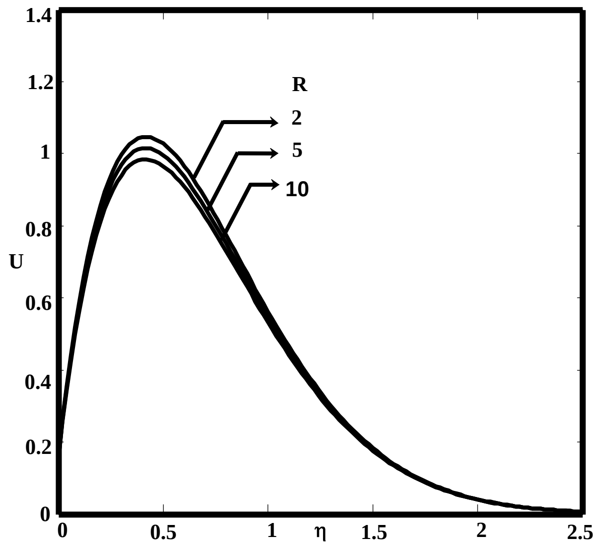

Fig.2. Velocity profiles for different R

The velocity profiles for different values of (t = 0.2, 0.4, 0.6, 0.8), K = 2, Gr = Gc = 5 are studied and presented in Fig.3. It is observed that the velocity increases with increasing values of the time t. The effect of velocity for different values of the Schmidt number (Sc = 0.16, 0.3, 0.6,) and time t = 0.2 are shown in Fig.4. The trend shows that the velocity increases with decreasing Schmidt number. The relative variation of the velocity with the magnitude of the time and the Schmidt number is observed.

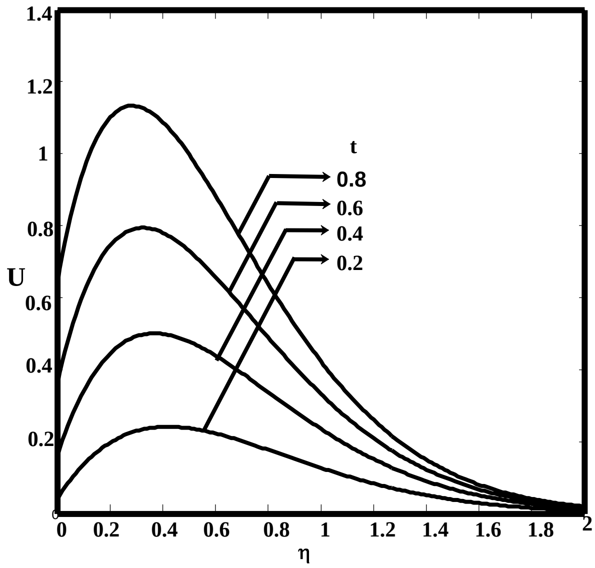

Fig.3. Velocity profiles for different t

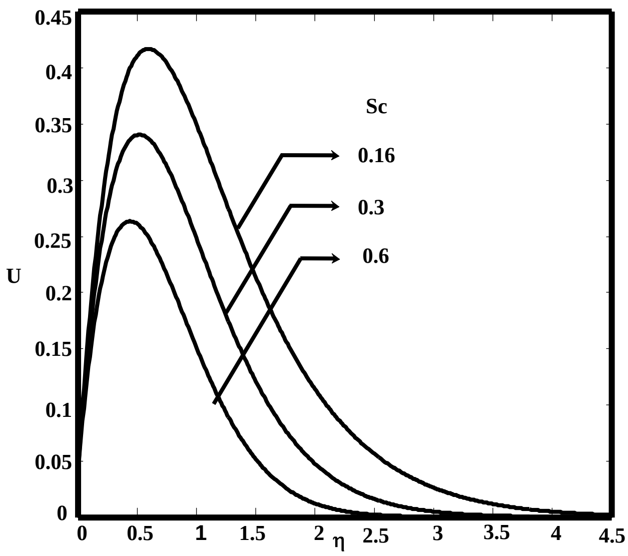

Fig.4. Velocity profiles for different Sc

Fig.5.demonstrates the effect velocity fields for different thermal Grashof number (Gr = 2,5), mass Grashof number (Gc = 2,5), K = 2, R = 2 and t = 0.4 . It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. The temperature profiles calculated for different values of time (t = 0.2, 0.4, 0.6, 0.8) are shown in Fig.6.for water (Pr = .71). The effect of a thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with increasing the time t.

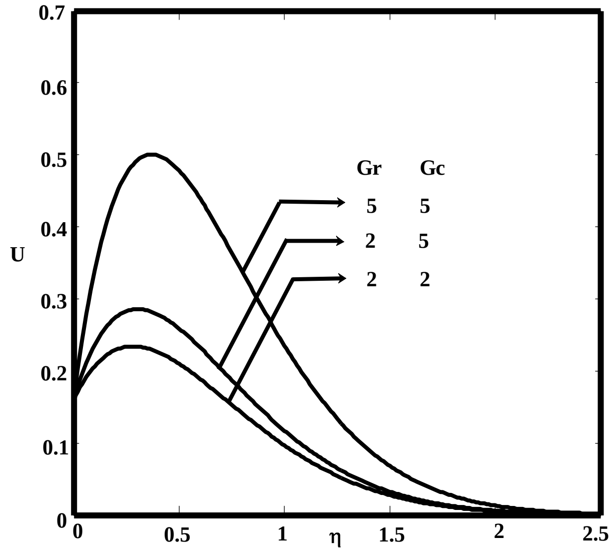

Fig.5. Velocity profiles for different Gr & Gc

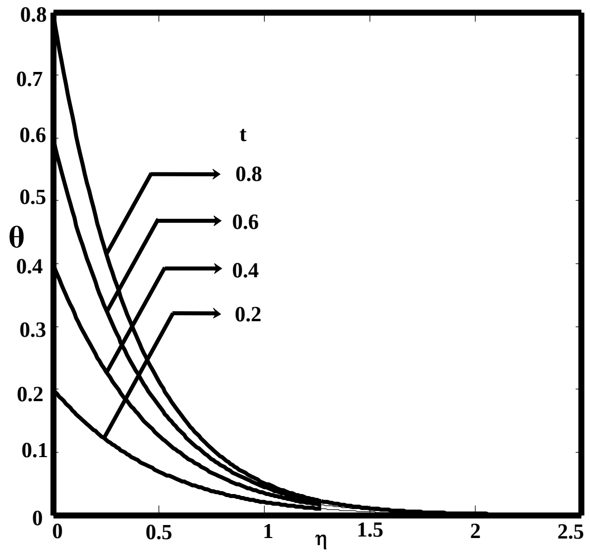

Fig.6. Temperature profiles for different t

The concentration profiles for different time (t = 0.2, 0.4, 0.6, 0.8), Sc = 0.6, and t = 0.2 are shown in Fig.7. The trend shows that the wall concentration increases with increasing values of the time. The concentration profiles for different values of the Schmidt number (Sc=0.16, 0.3, 0.6), K=0.2 and time t = 0.2 are presented in Fig.8. The effect of the Schmidt number is dominant in concentration field. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

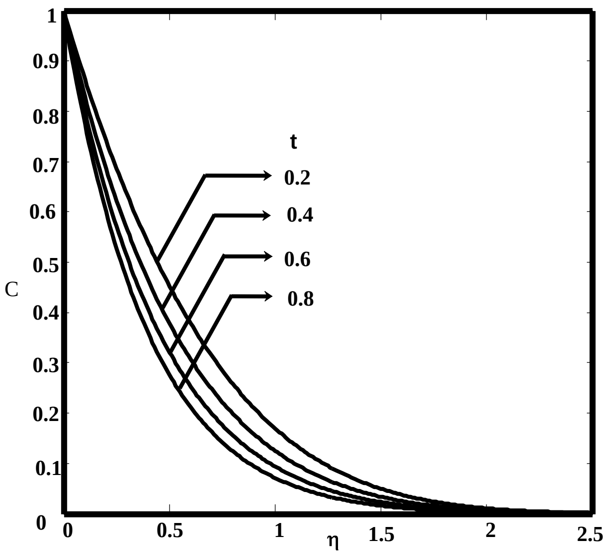

Fig.7. Concentration profiles for different t

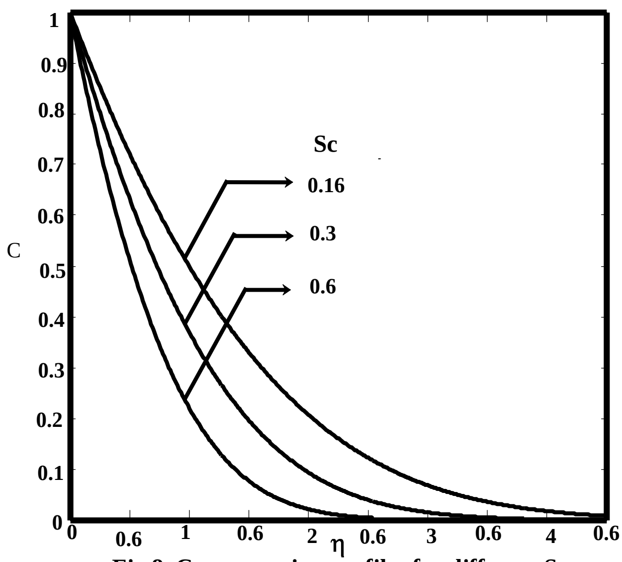

Fig.8. Concentration profiles for different Sc

CONCLUSION

The theoretical solution of parabolic flow past an infinite vertical plate with variable temperature and uniform mass diffusion with radiation and chemical reaction has been studied. The dimensionless governing equations were solved by the usual Laplace-transform technique. The effect of different physical parameters like chemical reaction parameter, radiation parameter, thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr,Gc and t . But the trend is just reversed with respect to the chemical reaction parameter or radiation parameter

NOMENCLATURE

A Constants
C' species concentration in the fluid $kg\,m^{-3}$
C dimensionless concentration
C_p specific heat at constant pressure $J.kg^{-1}.k$
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity

Greek symbols

β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions

REFERENCES

1. Chambre, P. L., Young, J. D., On the Diffusion of a Chemically Reactive Species in a Laminar Boundary Layer Flow, The Physics of Fluids, 1 (1958), 1, pp. 48-54
2. Das, U. N., Deka, R. K., Soundalgekar,V. M., Effects of Mass Trans fer on Flow Past an Impulsively Started Infinite Vertical Plate with Chemical Reaction, The Bulletin, GUMA, 5 (1999), 1, pp. 13-20
3. Das, U. N., Deka, R. K., Soundalgekar,V. M., Effects of Mass Trans fer on Flow Past an Impulsively started infinite Vertical Plate with Constant Heat Flux and Chemical Reaction, Forschung im Ingenieurwesen, 60 (1994), 10, pp. 284-287
4. Soundalgekar, V. M., Free Convection Effects on the Flow Past a Vertical Oscillating Plate, Astrophysics Space Science, 64 (1979), 2, pp. 165-172
5. Soundalgekar, V. M., Akolkar, S. P., Effects of Free Convection Currents and Mass Transfer on the Flow Past a Vertical Oscillating Plate**,** Astrophysics Space Science, 89 (1983), 2, pp. 241-254
6. Soundalgekar, V. M., *et al.*, Ef fects of Mass Trans fer on the Flow Past an Oscillating Infinite Vertical Plate with Constant Heat Flux, Thermo physics and Aero Mechanics, 1 (1994), 3, pp. 119-124
7. Muthucumaraswamy R and Ganesan P (2002). Diffusion and first order chemical reaction on impulsively started infinite vertical plate with variable temperature**,** International Journal of Thermal Sciences, 41(5), pp. 475-479.
8. Kandasamy R, Wahib B, Md Raj and Azme B Khamis (2006). Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in presence of suction or injection. Theatrical Applied Mechanics Belgrade, 33(2), pp. 123-148
9. Muthucumaraswamy R and Janakiraman B (2008). Mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction. International Journal of Applied Mathematics and Mechanics, 491, pp.66-74.
10. Kafouias,N.G.; Raptis, A.A. 1981. Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection , Rev Roum. Sci. Techn.- Mec.Apl.,26: 11-22.
11. Singh, A.K.; Singh, J. 1983. Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux, Astrophysics and Space Science**,** 97:57-61.
12. Basanth Kumar Jha; Ravindra Prasad; Surendra Rai. 1991. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science, 181:125-134.