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Contextual ISO-Triangular Array P System models

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Abstract

Membrane computing is an area of computer science aiming to abstract computing models from the structure and functioning of living cells. In this paper, the study of Contextual iso-triangular array P system is continued to generate the two dimensional picture languages. The main interesting study is a class of languages describes digitized number seven, which is generated by contextual iso-triangular array P system and investigated its generative powers.

Keywords: Iso-triangular tiles, Iso-triangular arrays, Contextual iso-triangular array rules, Array P system, Two dimensional picture language.

I. Introduction

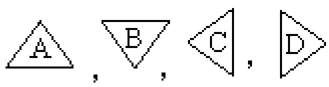
In formal language theory, theoretical string grammar models have been introduced, investigated. Theoretical array grammar models are introduced as a extension of string grammar models in [7] to generate two dimensional picture languages. The contextual string grammar model was introduced by Marcus and studied by Gh. Paun in [11]. Motivated by different problems in the frame work of image analysis and picture generation, Freund et al [6] introduced contextual d dimensional array grammars. The contextual and Chomsky grammars are differentiated in such a way that, in contextual grammars the symbols cannot be rewritten, but the symbols can be adjoin to the current string and the introduced symbols cannot be altered, it remains in the generated string finally. In contextual array grammars, starting from an axiom array, a production rule of the form (s, c) can be applied where s, c are finite patterns.

Membrane computing is a biological inspired computational model introduced by Gh. Paun [12]. Its devices are called P systems and they perform computations by applying a finite set of rules. P systems are applied in computational topology within the context of digital images. Many variants of p system use string object and context-free rules for processing them. P system consists of membranes with multisets of objects which evolve according to the given rules. Ceterchi et al and C. Ferretti [4], [5] introduced array P systems using rewriting of arrays in context- free type rules. P systems with context-free iso- array grammar rules have been defined in [1], [2], also it generates iso-pictures and the power of context-free iso-array rewriting P systems is examined. The contextual way of handling string objects in p systems have been considered in [8, 10, 11, 13] and the contextual array P systems are found to be a rich frame work than the string contextual array grammars. In [14] array P systems and a proper hierarchy of language classes described by contextual array grammars are considered.

In section two, iso-triangular tiles, d-dimensional arrays and translation of vertices and shapes are defined. In section3, contextual iso-triangular array grammar is defined. The mode of generating two dimensional picture language is given by f = {*, t}. The languages generated by contextual iso- triangular array rules with the mode * and t are illustrated with examples. In section 4, it introduced the contextual iso-triangular array P system with iso-triangular array objects and with the rules of adjoining or erasing rules in the way of contextual form. The main result is a class of languages of digitized seven is generated by contextual iso-triangular array p system and also the generative power of contextual iso-triangular array P system is discussed.

2. Prerequisites

In this section we recall the notion of iso-triangular tilling systems defined in [9], d dimensional arrays and translation of vertices and shapes and iso-triangular co-ordinate system in [3].

Definition 2.1: Let ∑= { 

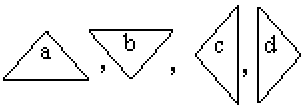
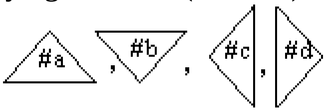
} be a finite set of labeled isosceles right angled triangular tiles of dimension1/√2, 1/√2 and 1 unit are obtained by intersecting a unit square by its diagonals. Tile A can be glued with tile B by the pasting rules $\{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}$ with tile C by the rule $\{(a_3, c_1)\}$ and with tile D by the rule $\{(a_1, d_3)\}$. In a similar way the gluable rules can be defined for the tiles B, C and D.

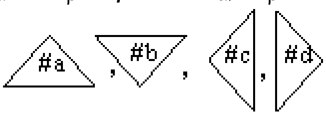
Definition 2.2. Let $\mathbb{Z}$ and $\mathbb{N}$ denote the set of integers and positive integers respectively and let d ∈ N. Then a d-dimensional array $\mathcal{A}$ over an alphabet V is a function $\mathcal{A}$: $\mathbb{Z}^d \longrightarrow V \cup \{\#\}$ where Shape ($\mathcal{A}$) = $\{v \in \mathbb{Z}^d / \mathcal{A}(v) \neq \#\}$ is finite $\# \notin V$ is called the empty symbol.

We write $\mathcal{A} = \{v, \mathcal{A}(v) / v \in \text{shape}(\mathcal{A})\}$. The sets of all d-dimensional arrays over V is denoted by V^{*d}. The empty array in empty shape is denoted by Λ_d. Moreover, we define $V^{+d} = V^{*d} - \{\Lambda_d\}$. Any subset of V^{+d} is called a Λ free d-dimensional array language.

Definition 2.3. Let $v \in \mathbb{Z}^d$, then the translation $T_v : \mathbb{Z}^d \rightarrow \mathbb{Z}^d$ is defined by $T_v(w) = w + v$ for all $w \in \mathbb{Z}^d$ and for any $\mathcal{A} \in V^{*d,}$ we define $T_v(\mathcal{A})$, the corresponding d-dimensional array translated by $T_v(\mathcal{A})(\ w)\ ) = \mathcal{A}(w - v)$ for $w \in \mathbb{Z}^d$.

3. Contextual Iso-Triangular Array Grammar

A contextual iso-triangular array grammar (CITAG) is a construct G =({  }, # = {  }, P, A) where A is a finite set of axioms in V^{+2} and P is a finite set of rules of the form$(U_\alpha, \alpha, U_\beta, \beta)$ where (i) $U_\alpha, U_\beta \subseteq \mathbb{Z}^2$, $U_\alpha \cap U_\beta = \phi$ and U_α, U_β are finite and at least U_α is non-empty.

(ii) $\alpha: U_\alpha \rightarrow V \cup \{$  $\}$, $\beta : U_\beta \rightarrow V$ such that card (shape(α)) $\neq 0$, and shape (α) = $\{w \in U_\alpha | \alpha(w) \in V\}$. (U_α, α) is called the selector and (U_β, β) the context of the production (α, β), U_α is called the selector area and U_β is the context area.

For arrays $\mathcal{C}_1, \mathcal{C}_2 \in V^{+2}$, we can find a sub-array that corresponds to the selector (U_α, α) and if the place corresponding to (U_β, β) are labeled only by the blank symbol, then we can add the context (U_β, β) and thereafter we get the iso-triangular array $\mathcal{C}_2$.

Formally, $\mathcal{C}_2$ is derivable from $\mathcal{C}_1$ by the contextual iso-triangular array production p $\in$ P, p = $(U_\alpha, \alpha, U_\beta, \beta)$ can be written as $\mathcal{C}_1 \Rightarrow_p \mathcal{C}_2$. If there exists a $v \in \mathbb{Z}^2$ such that $\mathcal{C}_2$ is derivable from $\mathcal{C}_1$ denoted by $\mathcal{C}_1 \Rightarrow_G \mathcal{C}_2$ if and only if $\mathcal{C}_1 \Rightarrow_p \mathcal{C}_2$ for some p $\in$ P.

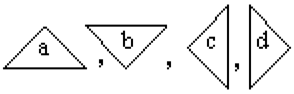
- $\mathcal{C}_1(w) = \mathcal{C}_2(w)(w) = \alpha(T_{-v}(w))\ \forall\ w \in T_v(U_\alpha)$
- $\mathcal{C}_1(w) = \#$ for all $w \in T_v(U_\beta)$
- $\mathcal{C}_2(w) = \beta(T_{-v}(w))$ for all $w \in T_v(U_\beta)$
- $\mathcal{C}_1(w) = \mathcal{C}_2(w)$ for all $w \in \mathbb{Z}^2 - T_v(U_\alpha \cup U_\beta)$

By $\Rightarrow{}^*_G$ we note that the reflexive transitive closure of $\Rightarrow_G$ and by $\Rightarrow^t_G$ (t mode) for arbitrary $\mathcal{A}, \mathcal{B} \in V^{+2}$ is defined by $\mathcal{A} \Rightarrow^t_G \mathcal{B}$ if and only if $\mathcal{A} \Rightarrow{}^*_G \mathcal{B}$ and there is no $\mathcal{C} \in V^{+2}$ such that $\mathcal{B} \Rightarrow_G \mathcal{C}$. For a given CITAG G, the relation $\Rightarrow^t_G$ corresponds to the derivation which cannot be continued is called maximal mode (or) t-mode and the collection of all possible derivations is commonly named as the $*$-mode of derivation. From these two derivation modes we get two kinds of iso-triangular aray languages associated with CITAG G, $L_*(G) = \{\mathcal{B} \in V^{+2} / \mathcal{A} \Rightarrow{}^*_G \mathcal{B}$ for some $\mathcal{A} \in A\}$, L_t G)= $\{\mathcal{B} \in V^{+2} / \mathcal{A} \Rightarrow{}^*_G \mathcal{B}$ for some $\mathcal{A} \in A\}$.

Example 3.1. Refer [3]

4. Contextual iso-triangular array P system

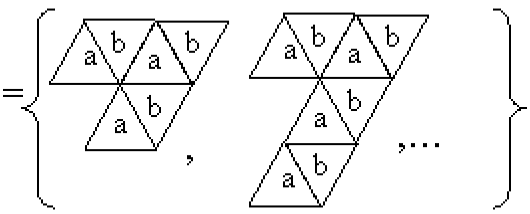
P system is a class of distributed parallel computing model which is inspired from the biological way of living cell structure. In membrane computing, P system has found to be a rich frame work for dealing with several types of computing related problems. Ceterchi et al [4] introduced an array P system linking the areas of membrane computing and picture grammars and investigated its powers in generating array languages. Here it is introduced the iso-triangular array P system with the rules of contextual iso-triangular array grammar, the iso-triangular arrays are adjoined or erased with the target indications. In this system a halting computation is one that reaches a configuration when no rule can be applied. The resultant picture is obtained in a specified membrane and it is a halting computation.

Definition 4.1: A contextual iso-triangular array P system of degree m ≥ 1 is a construct $\prod = (V, \#, \mu, A_1, A_{2,} \ldots, A_m, P_1, P_2, \ldots, P_m, i_0)$, where V = {  }, # = {  } , μ is a membrane structure with m membranes with m regions, labeled 1, 2, . . . , m in a one-to-one way, $A_1, A_2, \ldots, A_m$ are finite sets of iso-triangular arrays over V respectively associated with the m regions of μ. The rules P_i, $1 \leq I \leq m$, are finite sets of adjoining and erasing rules given in the following forms $((\alpha_i, \beta_i), tar)_a$ (adjoining rules), $((\alpha_i, \beta_i), tar)_e$ (erasing rules) and in both cases $((\alpha_i, \beta_i)$ is a contextual iso-triangular array grammar rules and *tar* ∈ *{here, in, out}*, i_0 is the label of the elementary membrane (output membrane) and in which the resultant picture pattern is collected.

In this computation, the successful computation of a picture is a halting one. For each array $\mathcal{A}$ in each region of the system, the contextual iso-triangular array rules are associated with *tar* ∈ *{ here, in, out}* with regions are applied. A computation is successful only if no rule can be applied to the existing iso-triangular arrays. The result of a halting computation consists of the iso-triangular arrays collected in the membrane with label i_0, output membrane. The set of all such iso-triangular arrays computed by a system is denoted by L(CITA). The family of all iso-triangular array languages L(CITA) generated by with at most m membrane is denoted by $CITAP_m$.

4.2. A class of Language digitized seven

We now define a language class of iso-triangular array with two a tiles on the head by contextual iso-triangular array P system and the language is denoted by $L_{7, 2}$.The language is given below.

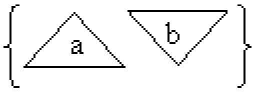
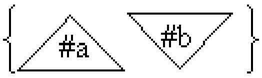
$L_{7, 2}$ = 

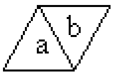
Theorem 4.2.1. L (CITAG) = $CITAP_1$

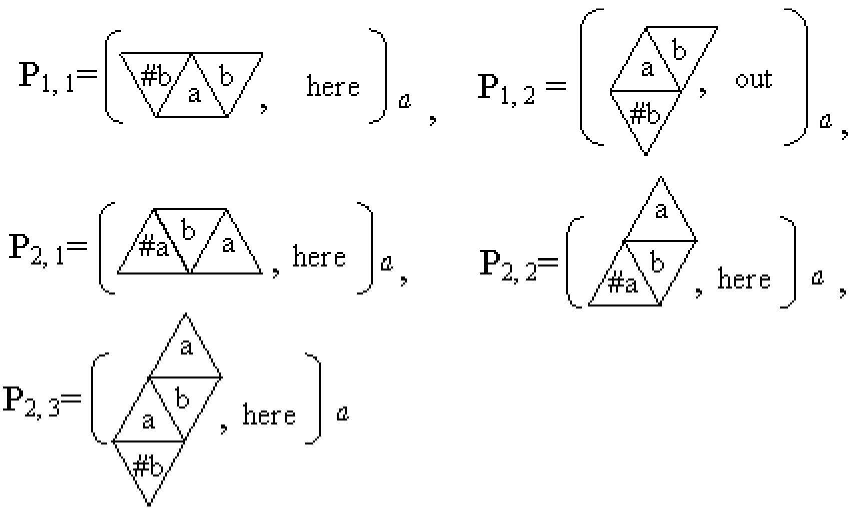
It is obvious from the definitions.

Theorem 4.2.2. $CITAP_1 \subset CITAP_2$

It is very clear that the language $L_{7,2}$ is generated by $CITAP_1$. In order to prove that $CITAP_1 \subset CITAP_2$,

We define $\prod_{7,2}$= ( , # =  , $_2[_1]_1]_2$,

A_1, A_2, P_1, P_2, 2) where A_1=  , $A_2=\phi$ and 2 is the output membrane. The rules are $P_1 = \{P_{1,1}, P_{1,2}\}$, $P_2 = \{P_{2,1}, P_{2,2}, P_{2,3}\}$ and in each rules, in the place of the empty symbols, the non-empty symbols are adjoined. The rules are given below:

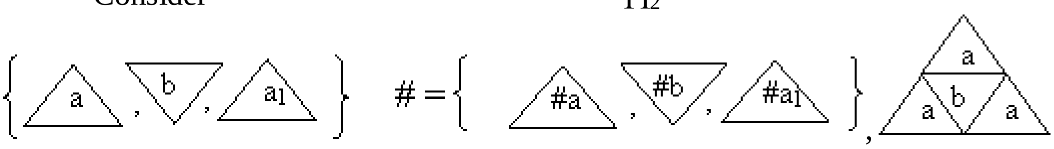


It has been easily shown that the computation starts with the axiom A_1, the rules $P_{1,1}$ and $P_{1,2}$ of P_1 in region one of the membrane structure are adjoined and the target indication out sent the resultant picture into the region two. The rules $P_{2,1}$, $P_{2,2}$ are applied with the resultant picture pattern, a member of the language is generated which is the halting computation and if the interest is to generate the next member of the language, the rules $P_{2,3}$ and $P_{2,2}$ are applied in the region two. In this way one can generate the members of the language $L_{7,2}$.

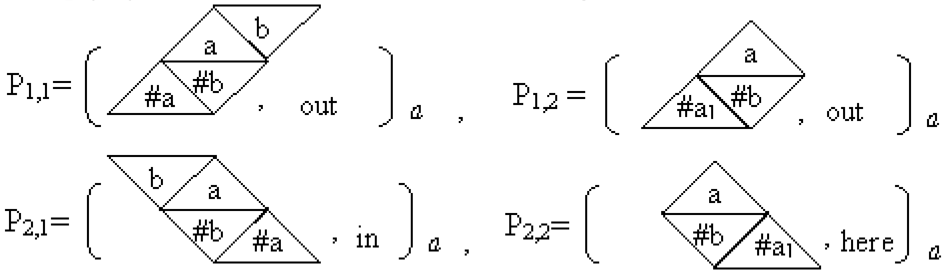
Theorem 4.2.3. $CITAP_2 \subset CITAP_3$

Proof: The inclusion follows from the definition. For the proper inclusion, let we consider a class of language $\wedge$ generated by the adjoining rules of contextual iso triangular arrays in P system $\prod_2$.

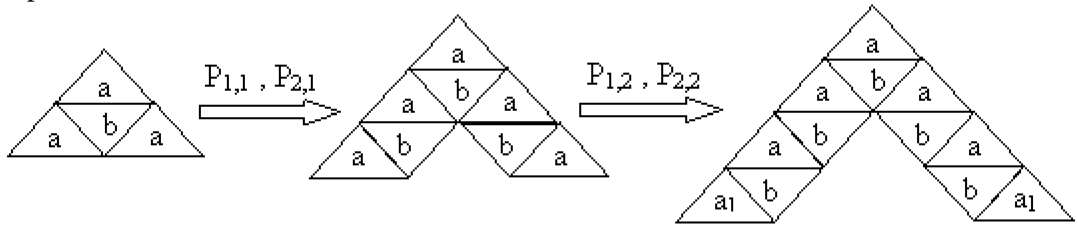
Consider $\prod_2$= (

 ,

$[_2[_1]_1]_2$, A_1, A_2, P_1, P_2, 2) where A_1= , ,A_2=ϕ and 2 is the out put membrane. The rules are $P_1 = \{P_{1,1}, P_{1,2}\}$, $P_2 = \{P_{2,1}, P_{2,2}\}$ and in each rules in place of the empty symbols, the non-empty symbols are adjoined. The rules are given below:

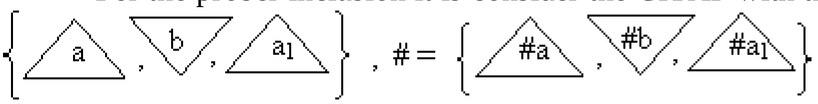


The derivation of a member of the language is as follows with the derivation steps:

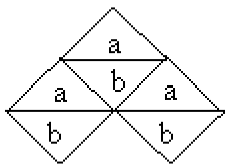


The number of choice of $P_{1,1}$ and $p_{2,1}$ will decide the length of the picture pattern and hence noticed that the picture pattern has equal arms in left down and right down..The number of tiles in each arms (left or right) is 2(n+1) + 3 and the number of tiles of the picture is 4 (n+2).

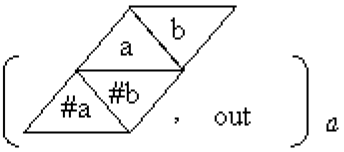
For the prober inclusion it is consider the CITAP with three membranes $\prod_3$= (

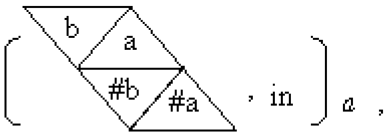


, $[_3[_2]_2[_1]_1]_3$, A_1, A_2, A_3, P_1, P_2, P_3, 3), where A_1=



, $A_2 = \phi$, $A_3 = \phi$ and 3 is the out put membrane. The rules are

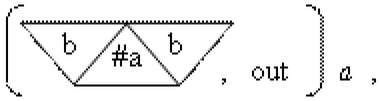
$P_{1,1}=$ 

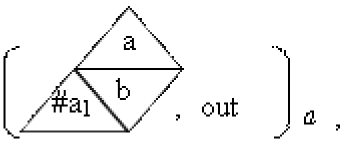
$P_{3,1}=$ 

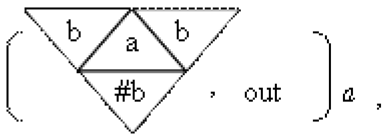
a

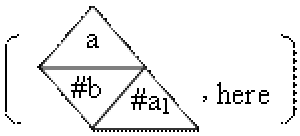
$P_1 = \{P_{1,1}, P_{1,2}\}$, $P_2 = \{P_{2,1}, P_{2,2}\}$, $P_3 = \{P_{3,1}, P_{3,2}\}$ and in each rules in the place of the empty symbols, the non-empty symbols are adjoined. The rules are given below:

,

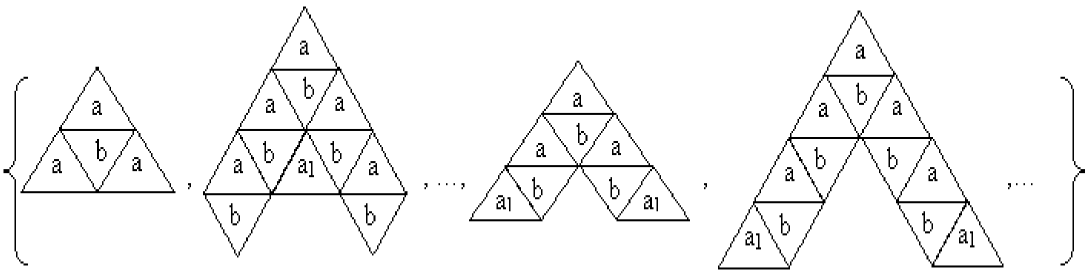
$P_{1,2}=$ 

$P_{2,1}=$ 

$P_{2,2}=$ 

$P_{3,2}=$ 

The member of picture language is generated by the following steps. In the first step of the derivation, the rule $P_{1,1}$ is applied with the axiom A_1 and the resultant array is sent out to the region three, in which the rule $P_{3,1}$ is applied. The target indication allows the picture enter into the region one or two as a choice. With the choice, the resultant picture entered into the region 2 there the rule $P_{2,1}$ is applied and hence the resultant picture comes out to the region three and then the rule $P_{3,2}$ is applied.Thereafter no rule can be applicable therefore the computation gets halt in the membrane 3. It is clear that L(CIATG)= $CITAP_3$. We Identified that $CIATP_3$ consists of the iso picture languages which is shown below which cannot be generated by any $CITAP_2$.



Hence it observed that the result is true.

5. Conclusion:

In this paper the study of iso-triangular array P system for generating some two dimensional picture languages is continued. The important result is that iso-triangular P system is generated a class language of digitized number seven and discussed the expressive power of contextual iso-triangular P system, which strictly increases with the number of membrane in the membrane structure.

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