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MHD Boundary Layer Flow With Heat Transfer, Variable Conductivity and Nonlinear Rosseland Approximation Thermal Radiation Effects Past A Nonlinearly Stretching Porous Surface With A Power-Law Velocity

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Abstract

Nonlinear hydromagnetic two dimensional steady, laminar, boundary layer flow of a viscous, incompressible, electrically conducting and radiating fluid with variable conductivity and nonlinear Rosseland approximation thermal radiation past a nonlinearly stretching porous surface is analysed in the presence of a variable magnetic field. The fluid is assumed to be a gray, emitting, absorbing but non-scattering medium. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. Fourth-Order Runge-Kutta shooting method along with the Nachtsheim-Swigert iteration for satisfaction of asymptotic boundary conditions is utilized for obtaining the numerical solution of the resulting nonlinear differential equations. The numerical results for velocity and temperature distribution are obtained for different values of velocity exponent parameter, magnetic interaction parameter, porosity parameter, radiation parameter, surface temperature parameter, variable thermal parameter and Prandtl number and are shown graphically. Numerical values of Skin friction coefficient and dimensionless and rate of heat transfer are also obtained and tabulated.

Keywords: MHD Flow, Porous Stretching Surface, Nonlinear Rosseland Approximation Thermal Radiation Effects, Variable Conductivity.

Introduction

The study on flow past a stretching plate is of great importance in many industrial applications such as in polymer industry to draw plastic films and artificial fibers. In special, studies' pertaining to boundary layer flow and heat transfer past a stretching porous plate have derived the attention of researchers due to their abundant applications. However in many of the practical situations, thermal conductivity of the fluid need not be constant. Hence this paper is devoted to a study on variable conductivity in these types of problems.

The boundary layer flow past a stretching sheet whose velocity is proportional to the distance from the slit was first investigated by Crane[1]. Carragher[2] considered the same problem of Crane[1] to study heat transfer and calculated the Nusselt Number for the entire range of Prandtl number.

Abdelhafez[3] investigated numerically the skin friction and heat transfer on a continuous flat surface moving in a parallel free stream. Later, Chappidi and Vajravelu [4] presented the boundary layer behavior on a continuous porous flat surface moving in a parallel free stream with internal heat generation/absorption. Further, analytical and numerical investigations of the problem of Abdelhafez[3] were obtained by Chappidi and Gunnerson[5]. Vajravelu and Rollins[6] examined the heat transfer in a hydromagnetic flow over a stretching sheet.

Ahmad and Mubeen[7] studied the boundary layer flow and heat transfer over the stretching plate with suction. A linear stretching plate with pores whose velocity is proportional to the distance from the slit and a boundary layer of constant thickness was generated by the motion of the porous sheet.

In all the above investigations, the fluid was considered to have constant properties only. Kays[8] outlined the significance of variation of thermal conductivity with temperature in fluids. Owing to this aspect, Arunachalam and Rajappa[9] studied the steady state laminar thermal boundary layer in liquid metals with thermal conductivity, which was assumed to vary linearly with temperature. They approximated the velocity components in the energy equation by those of the inviscid outer flow in order to simplify the equation. They then employed the regular perturbation technique and obtained closed form analytical solutions, up to second order, to the resulting linearized equations.

Chiam[10] considered heat transfer with variable conductivity in stagnation point flow towards a stretching sheet and obtained closed form perturbation solution for temperature profile and also presented the numerical solution. The effect of heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet was investigated by Chiam[11] . Naseem Ahmad and Naved Khan[12] have analysed boundary layer flow past a stretching plate with suction and heat transfer with variable conductivity.

The effect of boundary layer flow and heat transfer past a stretching plate with variable thermal conductivity were analysed by Naseem Ahamad et al[13] and Anjali devi and Thiagarajan[14] for various physical parameters of the problem.

No contribution from author's observation is so far made to study the nonlinear hydromagnetic boundary layer flow with heat transfer with variable conductivity and nonlinear radiation effects past a porous surface stretching with a power-law velocity in the presence of transverse variable magnetic field. Here the nonlinear Rosseland approximation is taken for radiation effect which is applicable for small and large temperature difference between the fluid and the plate, whereas a linearized Rosseland is applicable only for small temperature difference between the fluid and the plate.

Formulation of the Problem

Steady, two-dimensional, laminar nonlinear MHD boundary layer forced convection flow of a viscous, incompressible, electrically conducting and radiating fluid with variable conductivity caused by a nonlinearly stretching sheet in the presence of variable magnetic field and nonlinear radiation has been considered. The fluid is assumed to be gray, emitting, absorbing and electrically conducting but non-scattering medium at temperature T_∞. The stretching surface is subjected to porosity. Cartesian coordinate system is chosen and u and v are the velocity components in the x and y directions respectively.

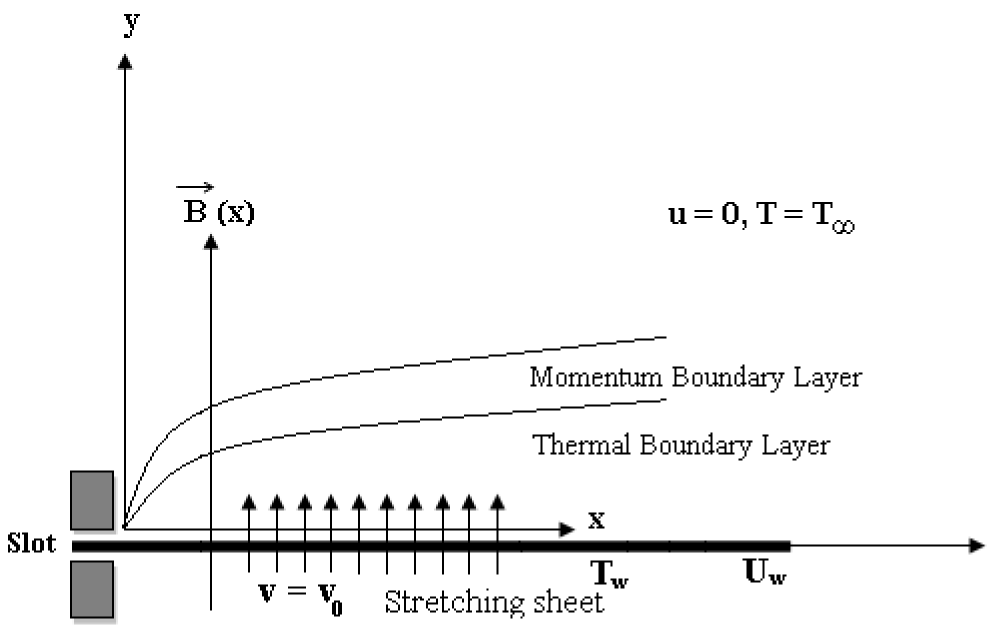

Figure 1: Schematic diagram of the problem

The following assumptions are made:

- The fluid properties are assumed to be constant, except for the conductivity of the fluid.
- The usual boundary layer assumptions are made [Ali.M.E[15]].
- Magnetic Reynolds number is assumed to be small. Under this assumption, the induced magnetic field is assumed to be negligible.
- Since the flow is steady, $curl\vec{E} = 0$. Also $div\vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$ is assumed.
- The viscous and Joule's dissipation are considered to be negligible.
- The radiation heat flux in the *x*-direction is considered to be negligible in comparison to that in the *y*-direction.

Under the above assumptions, the continuity, momentum, and energy equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B(x)^2 u}{\rho} \tag{2}$$

$$\rho C_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\frac{\partial}{\partial y}(K\frac{\partial T}{\partial y})-\frac{\partial q_r}{\partial y} \tag{3}$$

where $\mathrm{B}(x)=\mathrm{B}_o x^{\frac{(m-1)}{2}}$

with the associated boundary conditions

$$\begin{aligned} &u = u_w = u_o x^m, v = v_0, \quad T = T_w && at\ y = 0 \quad (u_o > 0),(v_o > 0) \\ &u = 0, \qquad T = T_\infty && as\ y \to \infty \end{aligned} \tag{4}$$

where u_w is the velocity of the stretching surface, u_0 is the dimensional constant and v_0 is the variable injection velocity and the quantities *u*, *v*,.. etc have the meanings as mentioned in the nomenclature. The radiative heat flux term is simplified by using the Rosseland diffusion approximation (Hossian et al[16]) .Here the nonlinear Rosseland approximation is taken into consideration which is applied for both small and large difference between the fluid and the plate. Further, this assumptions leads to a new parameter θ_w, called surface temperature parameter in our study.

$$q_r = -\frac{16\sigma^* T^3}{3\alpha^*}\frac{\partial T}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzmann constant, α^* is the Rosseland mean absorption coefficient.

A stream function $\psi(x, y)$ is chosen in the following form so that the equation of continuity is satisfied.

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Introducing the usual similarity transformation (Ali.M.E[15])

$$\eta(x,y) = y\sqrt{\frac{m+1}{2}}\sqrt{\frac{u_o x^{m-1}}{\nu}},\ \psi(x,y) = \sqrt{\frac{2}{m+1}}\sqrt{\nu u_0 x^{m+1}}\ f(\eta) \tag{7}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \theta_w = \frac{T_w}{T_\infty} \tag{8}$$

where θ_w is the surface temperature parameter, the velocity components are obtained as

$$u = u_w f'(\eta) \tag{9}$$

$$v = -\sqrt{\frac{m+1}{2}}\sqrt{\frac{\nu u_w}{x}}\left[f(\eta) + \frac{m-1}{m+1}\eta f'(\eta)\right] \tag{10}$$

Define dimensionless variables by

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{11}$$

$$K = K_s(1+\varepsilon\theta) \tag{12}$$

$$\varepsilon = \frac{(K_w - K_s)}{K_s} \tag{13}$$

Where ε is the variable thermal parameter, K_w is the conductivity of the wall and K_s is the conductivity of the surrounding fluid.

Equations (2) and (3) are obtained in the following form utilizing (7) to (13)

$$f''' + ff'' - \frac{2m}{m+1} f'^2 - M^2 f' = 0 \tag{14}$$

$$\begin{aligned} &\left\{1 + \varepsilon\theta(\eta) + \frac{4}{3R^*}(1 + (\theta_w - 1)\theta(\eta))^3\right\}\theta''(\eta) + \\ &\left\{\varepsilon + \frac{4}{R^*}(1 + (\theta_w - 1)\theta(\eta))^2(\theta_w - 1)\right\}\theta'^2(\eta) + \Pr f(\eta)\theta'(\eta) = 0 \end{aligned} \tag{15}$$

where $M = \sqrt{\dfrac{2\sigma B_o^2}{\rho u_o (m+1)}}$ is the magnetic interaction parameter

$R^* = \dfrac{K\alpha^*}{4\sigma^* T_\infty^3}$ is the radiation parameter

The boundary conditions in dimensionless variables are given by

$$f(0) = -S, \quad f'(0) = 1, \quad \theta(0) = 1$$
$$f'(\infty) = 0, \quad \theta(\infty) = 0 \qquad (16)$$

where

$S = \sqrt{\frac{2}{(1+m)}}$ C is the porosity parameter

(S > 0 injection for and < 0 for suction), where C is a non-dimensional constant.

Numerical Solutions

Equations (14) and (15) are nonlinear ordinary differential equations which constitute the nonlinear boundary value problem. As no prescribed method is available to solve nonlinear boundary value problem, it has to be reduced to an initial value problem. Shooting method is utilized for doing this.

Equations (14) and (15) are solved numerically subject to (16) using Fourth-Order Runge-Kutta based shooting method along with Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions. To initiate the shooting process, Initial guesses for the values of $f''(0)$ and $\theta'(0)$ are made. The success of the procedure depends very much on how good this guess is. Different initial guesses were made for different values of physical parameters in accordance with convergence. Numerical results are obtained for several values of the physical parameters *M, m, S,* θ_w, *R*,* ε and *Pr*.

Results and Discussion

Numerical solution of the problem concerned with the effects of nonlinear hydromagnetic boundary layer flow with heat transfer with variable conductivity and nonlinear radiation effects considering nonlinear Rosseland approximation for the heat flux term past a porous surface stretching with a power-law velocity in the presence of variable magnetic field are obtained for various values of the physical parameters involved in the problem such as magnetic interaction parameter, velocity exponent parameter, porosity parameter, radiation parameter, surface temperature parameter, variable thermal parameter and Prandlt number. The numerical results are displayed through graphical illustrations.

In the absence of radiation, the results have been compared with that of Anjali Devi and Thiagarajan[14]. These comparison results are portrayed in Figs.2 to 6. From these figures, it is clearly observed that the results are in good agreement with that of Anjali Devi and Thiagarajan[14] when there is no radiation.

The numerical results for velocity and temperature of the present problem are illustrated through Figs.7 to 15.

Figure 7 displays the effects of magnetic field over the velocity. For increasing values of *M*, the velocity is decreased. It can also be noted that the momentum boundary layer thickness is also reduced as we move away from the wall due to the effect of *M*.

The effect of magnetic field over temperature is depicted in Fig. 8. As M increases, the temperature also decreases elucidating the fact that the effect of magnetic interaction parameter over temperature is to reduce it when Pr = 0.71. On the other hand temperature increases for increasing M when Pr =7.0.

The velocity distribution for various values of m is portrayed in Fig. 9. For increasing values of m, the velocity decreases. This is due to the fact the effect of m is to decrease the velocity.

The nondimensional velocity profiles for different values of both suction and injection in the case of nonlinearly stretching surface are shown through Fig.10. In the case of injection, dimensionless velocity $f'(\eta)$ increases as injection parameter increases where as the opposite trend is observed in the case of suction so as to decrease the velocity for increasing values of suction parameter in magnitude.

Figure 11, shows the effect of suction or injection S over dimensionless temperature $\theta(\eta)$ for a nonlinearly stretching surface. In the presence of magnetic field, it is evident from figure 11, that the effect of suction parameter S in magnitude is to decrease the temperature $\theta(\eta)$. On the other hand, the effect of injection is to increase the temperature. Furthermore, the thickness of the thermomagnetic layer is reduced due to the effect of suction whereas it thickness due to the effect of injection.

The temperature distribution for various values of θ_w which is obtained due to the nonlinear Rosseland approximation is depicted in Fig.12. The effect of θ_w over temperature is to increase it.

For increasing values of Prandtl number the temperature is decreased elucidating the fact that the temperature is suppressed. This effect is clearly portrayed in Fig.13.

The temperature distribution for various values of ε is depicted in Fig.14. The temperature is enhanced due to the increase in ε.

The effect of R^* over temperature is depicted in Fig.15. For increasing values of R^*, the temperature is decreased.

Table 1 to 4 display the results of skin friction coefficient and non-dimensional rate of heat transfer for various values of physical parameters.

In case of suction, the skin friction coefficient decreases for increasing values of m. Similarly when the value of M increases the skin friction coefficient get decreased. This shows that both 'm' and 'M' has a reducing effect over the coefficient of skin friction which is clearly shown through table 1.

The effect of R^* and ε over the non- dimensional rate of heat transfer is elucidated through table 2. The non-dimensional rate of heat transfer decreases as the value of R^* increases whereas ε has an increasing effect over the rate of heat transfer.

Table 3, depicts the effect of m and M over the skin friction coefficient for the case of injection. Both 'm' and 'M' has same effect over the skin friction coefficient. As the values of 'm' and 'M' increases, the coefficient of skin friction decreases.

The effect of R^* and ε over the non- dimensional rate of heat transfer is portrayed through table 4. For higher values of R^* the dimensionless rate of heat transfer is getting decreased. This portrayed that R^* decreases the dimensionless rate of heat transfer. Unlike R^*, ε increases the dimensionless rate of heat transfer

Conclusion

In this work the problem of nonlinear hydromagnetic boundary layer flow with heat transfer with variable conductivity and nonlinear radiation effects past a porous surface stretching with a power-law velocity is investigated. The results are presented for various values of physical parameters including the magnetic interaction parameter, velocity exponent parameter, porosity parameter, radiation parameter, surface temperature parameter, variable thermal parameter and Prandlt number. A systematic study of the effects of the various parameters on flow, heat transfer, skin friction coefficient and non dimensional rate of heat transfer is carried out.

In the absence of radiation the present results are in good agreement with that of Anjali Devi and Thiagarajan[14].

The following conclusions are drawn in view of the above results and discussions made.

- It is seen that the effect of magnetic interaction parameter is to reduce the velocity.
- The temperature is found to decrease for increasing M when Pr = 0.71 whereas the temperature enhances for increasing M for Pr = 7.0.
- The effect of velocity exponent parameter is to decrease the velocity.
- When the suction parameter increases in magnitude, the non dimensional velocity decreases whereas it increases for increasing values of injection.
- It is observed that the effect of suction is to reduce the temperature. On the other hand, the effect of injection is to increase the temperature.
- It is observed that the temperature is found to increase for increasing surface temperature parameter and variable thermal parameter.
- The thermal boundary layer thickness decreases sharply with increasing Prandtl number.
- The effect of radiation parameter is to reduce the dimensionless temperature.
- Skin friction Coefficient decreases due to increasing values of m and M in both cases of suction and injection.
- The effect of radiation parameter is to suppress the dimensionless rate of heat transfer where as the effect of variable thermal parameter is to enhance the non-dimensional rate of heat transfer in both the cases of suction and injection.

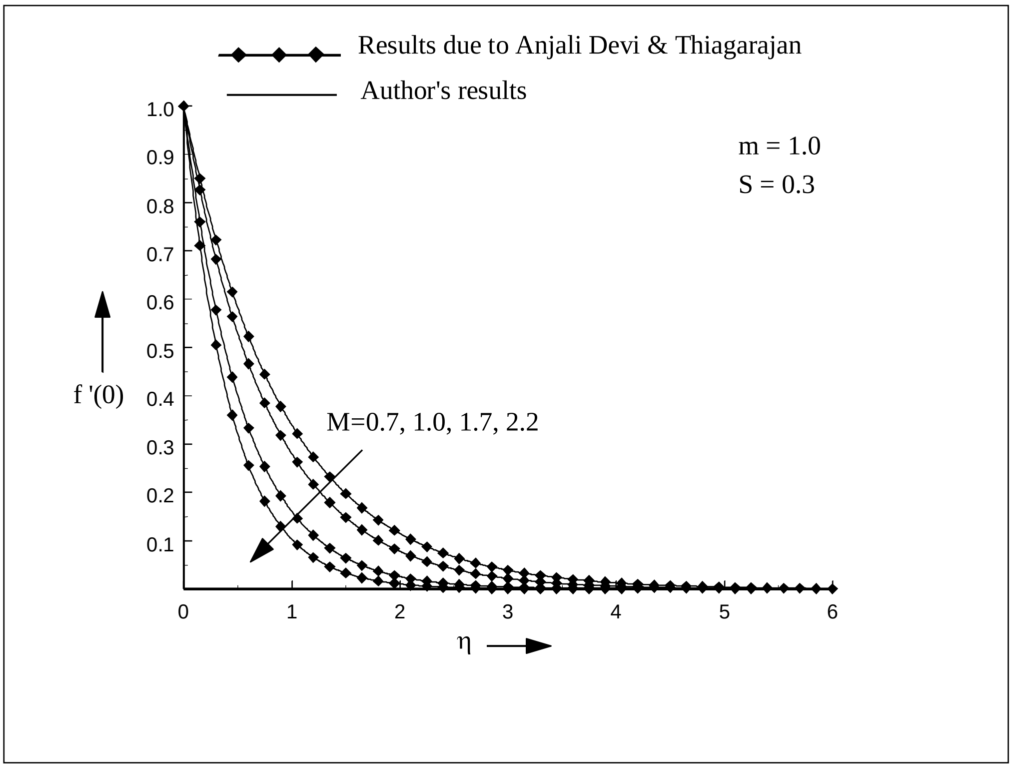

Figure 2: Velocity profiles for different M

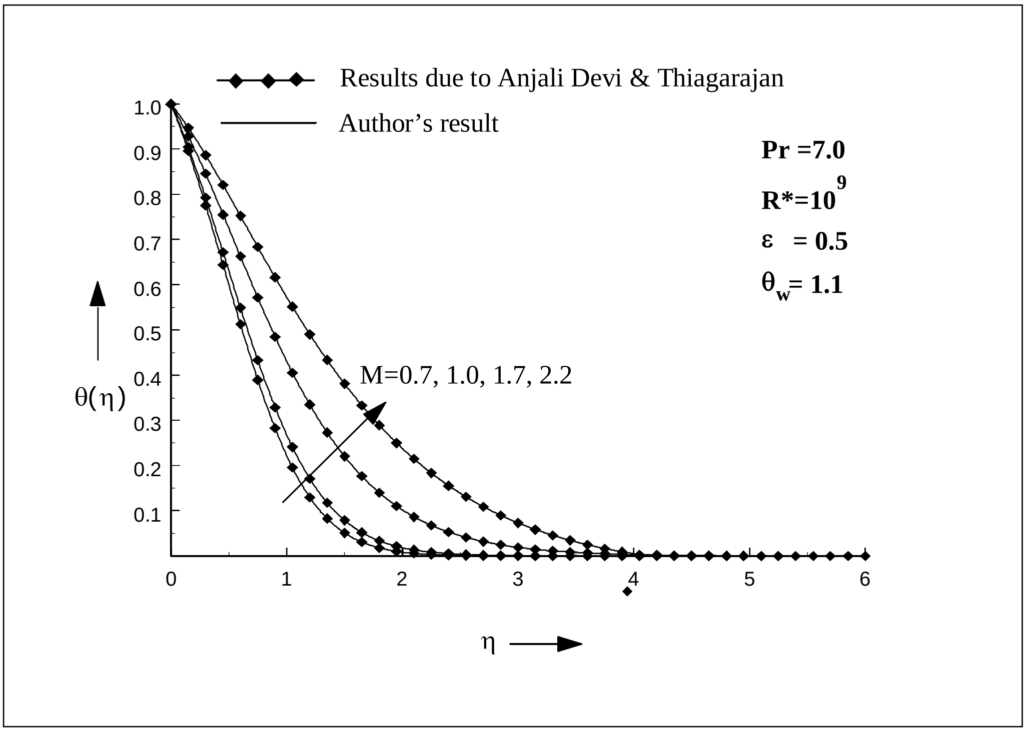

Figure 3: Temperature profiles for different M

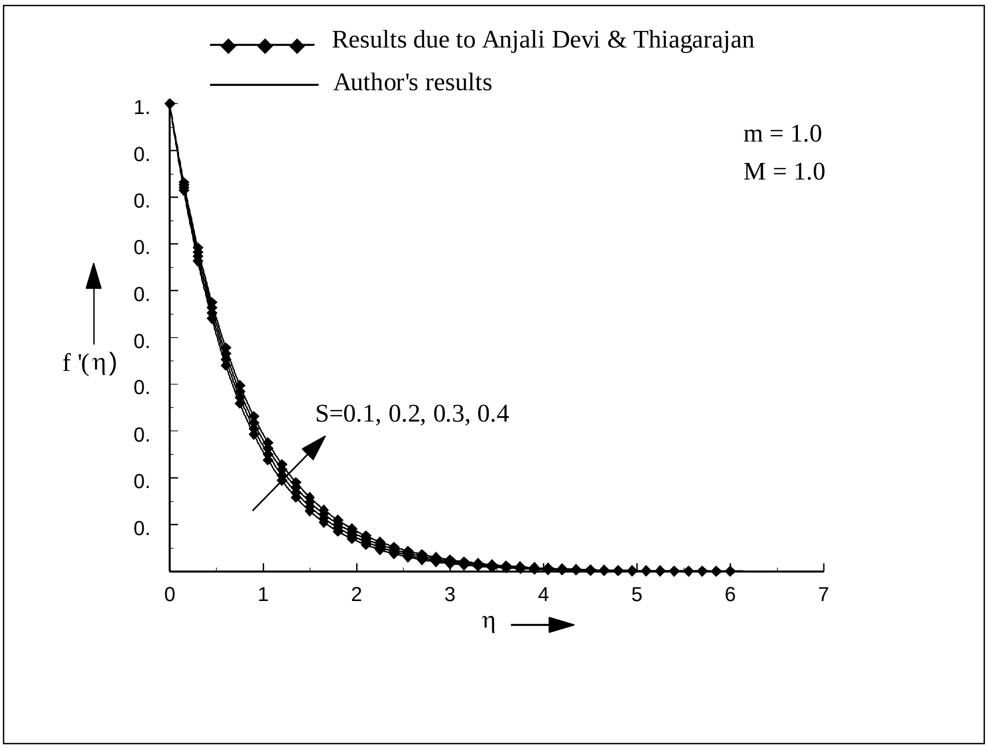

Figure 4: Velocity profiles for different S

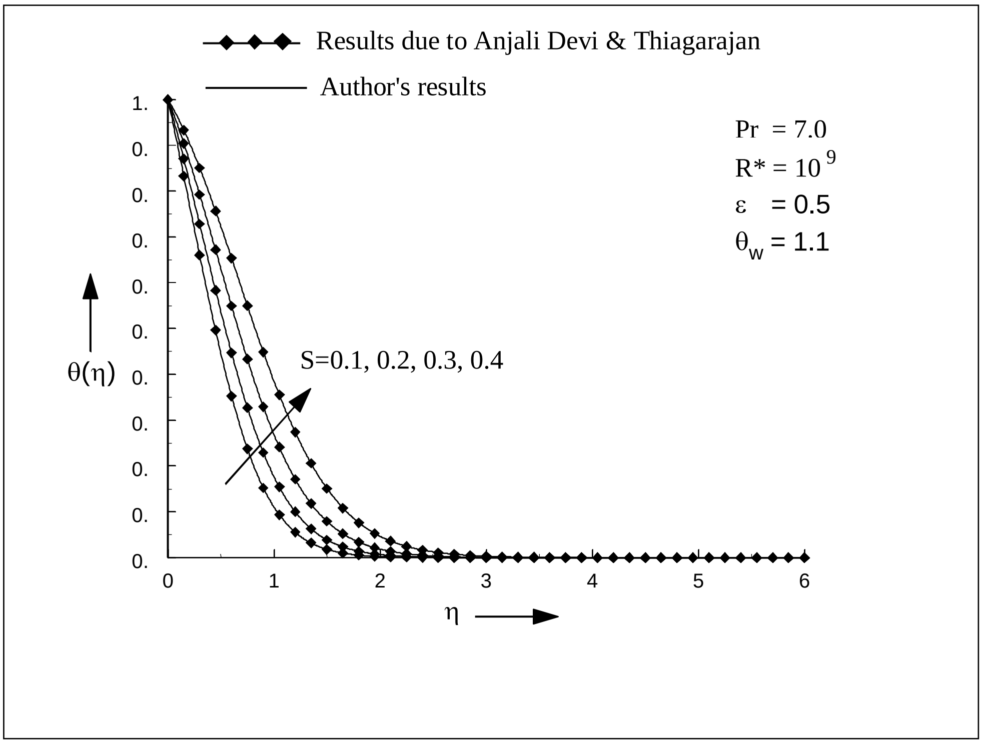

Figure 5: Temperature profiles for different S

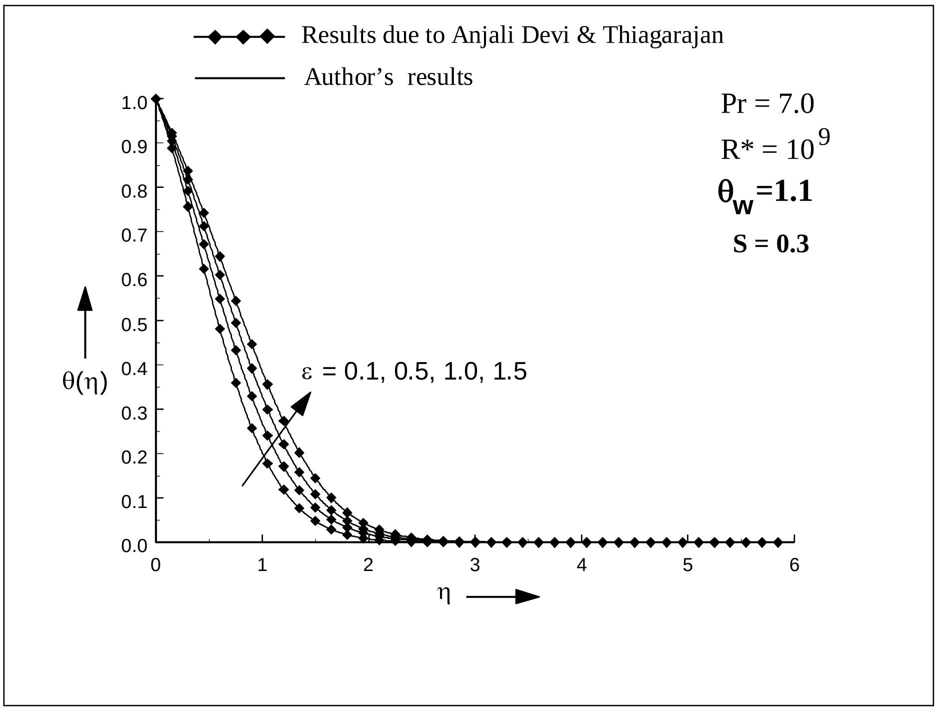

Figure 6: Temperature profiles for different ε

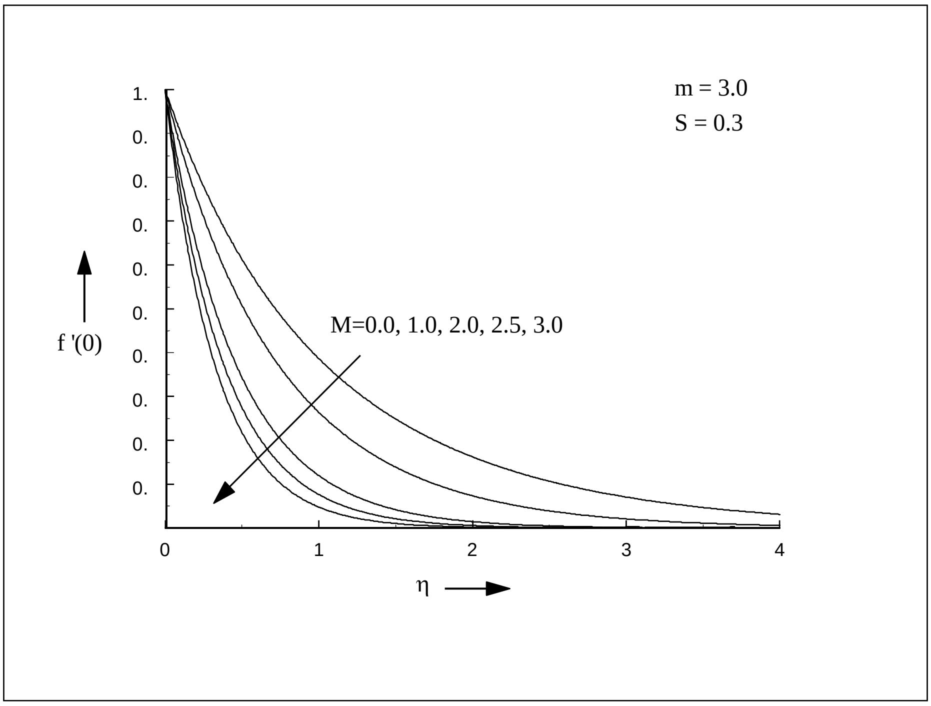

Figure 7: Velocity profiles for different M

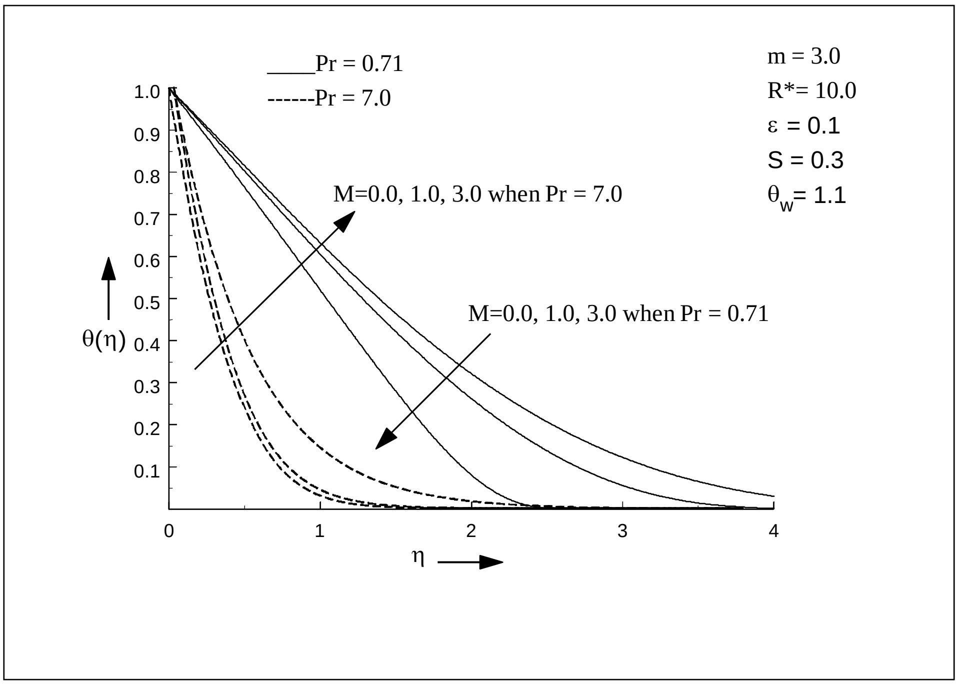

Figure 8: Temperature profiles for different M

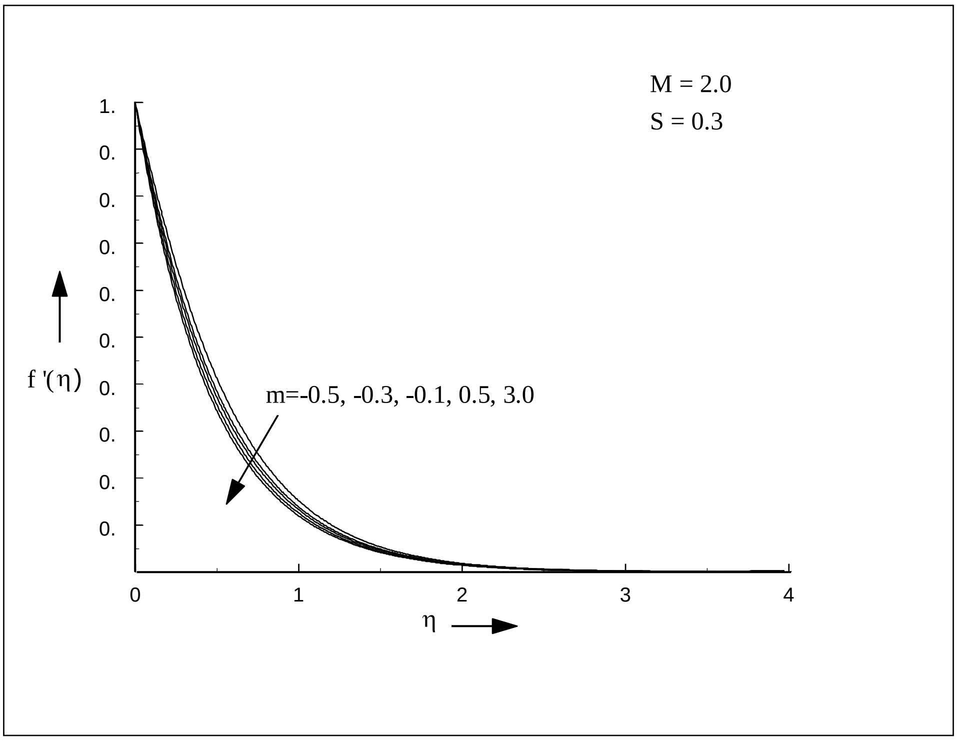

Figure 9: Velocity profiles for different m

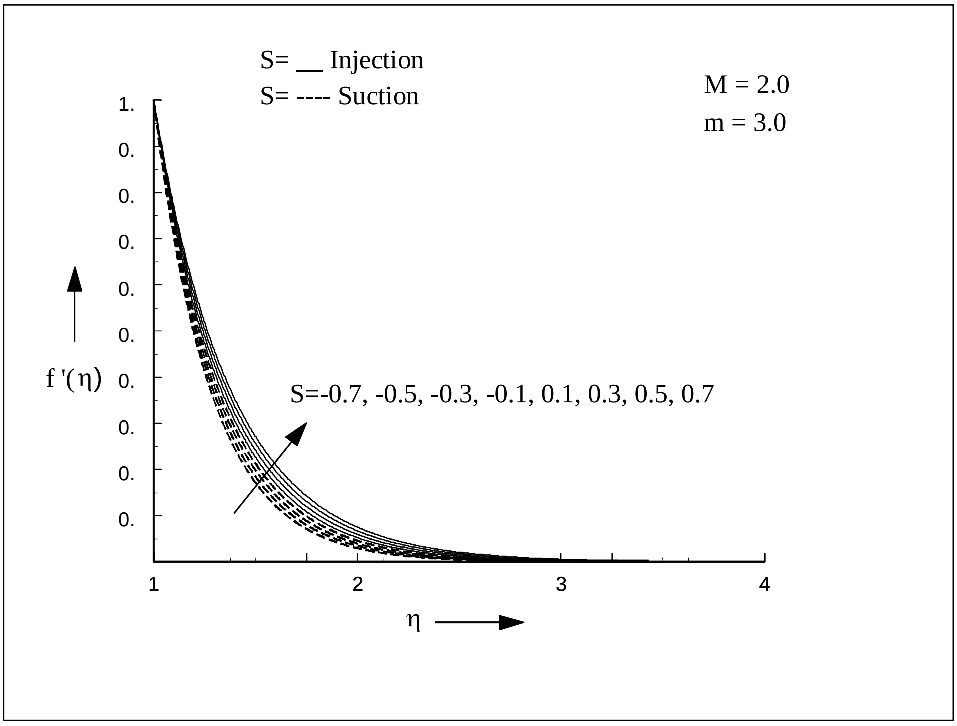

Figure 10: Velocity Profiles for Different S

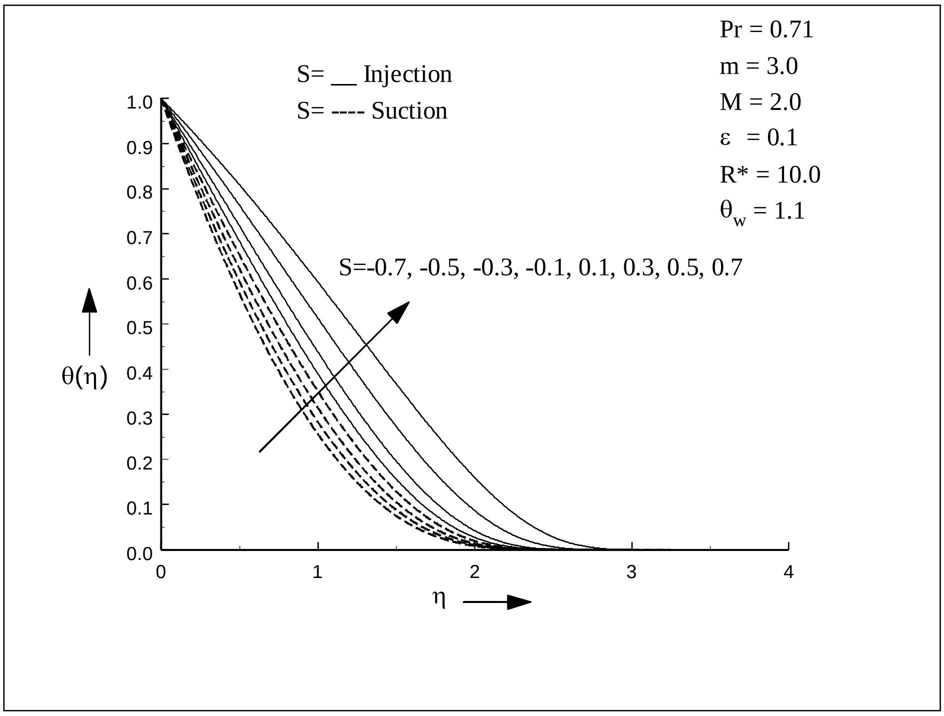

Figure 11: Temperature profiles for different S

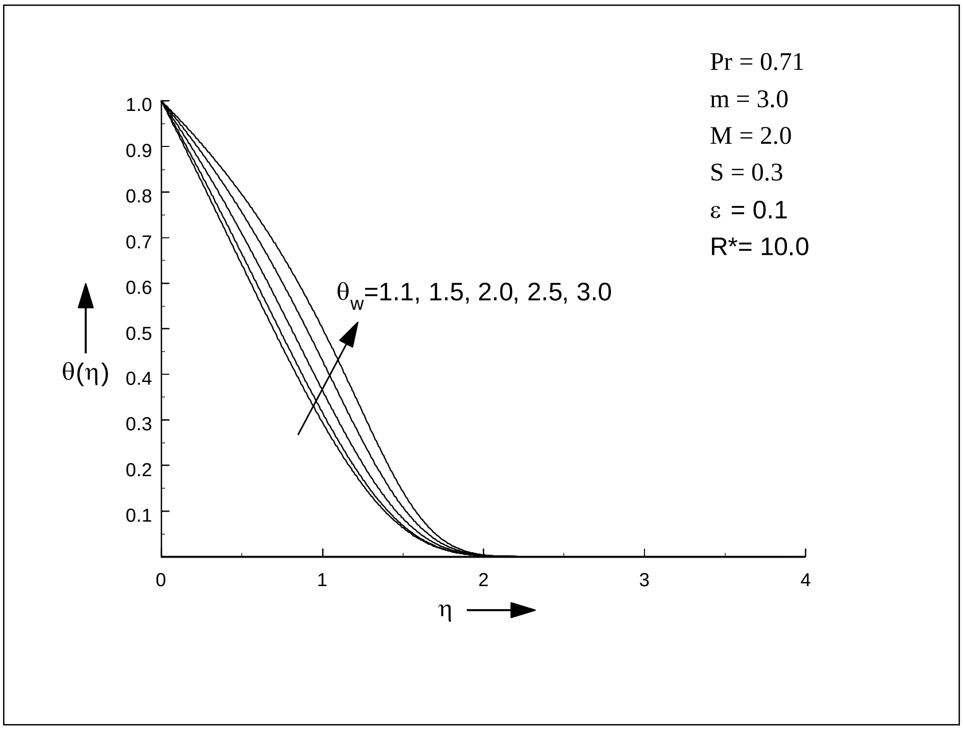

Figure 12: Temperature profiles for different θ_w

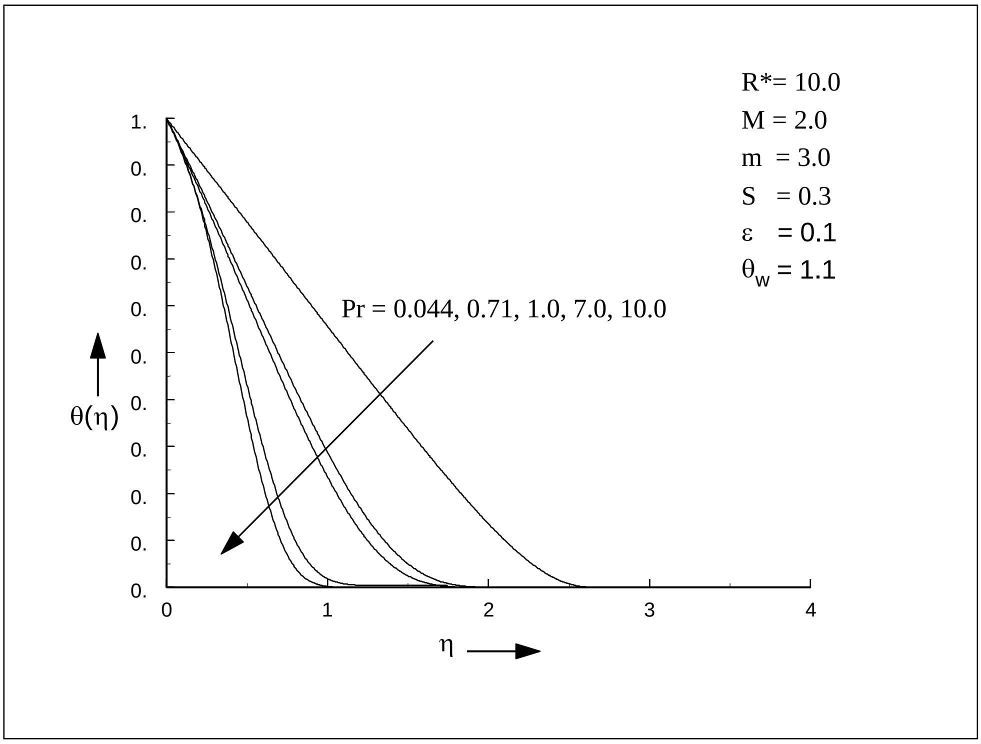

Figure 13: Temperature profiles for different Pr

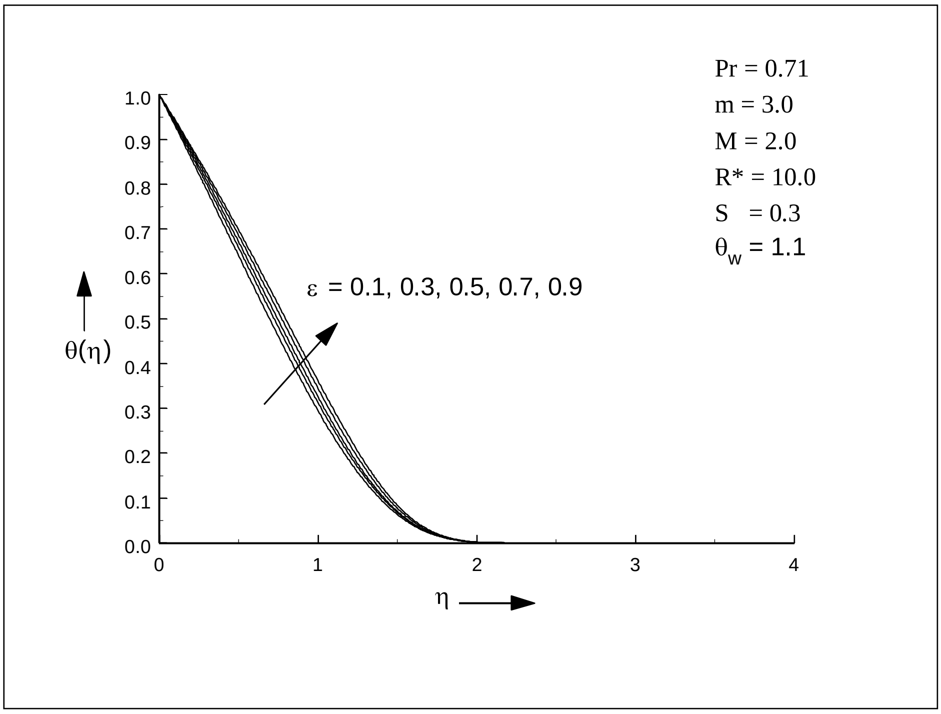

Figure 14: Temperature profiles for different **ε**

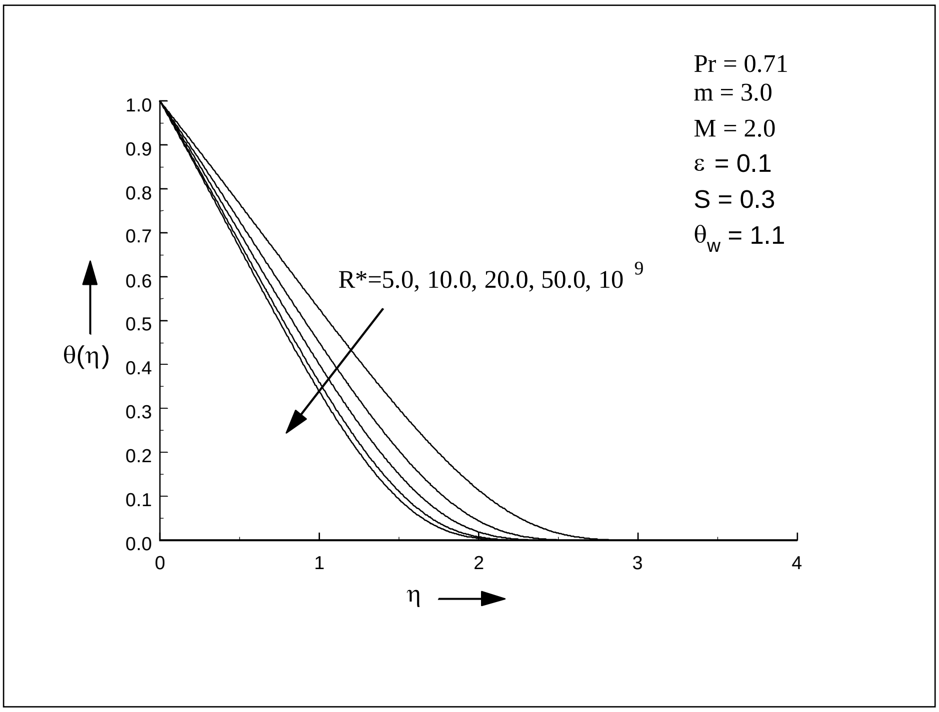

Figure 15: Temperature profiles for different R*

Table 1: For the case of suction, variation of $f''(0)$ for different values of m and M

M	**M**	**-S**	$f''(0)$
-0.5	2	-0.3	-1.90968
-0.3			-2.10614
-0.1			-2.20766
0.5			-2.34249
3.0			-2.46213
3.0	0	-0.3	-1.34084
	1		-1.52138
	2		-2.46213
	2.5		-2.90644
	3		-3.36703

Table 2: For the case of suction, variation of $\theta'(0)$ for different values of R^* and ε

R^*	ε	**-S**	$\theta'(0)$
5.0	0.1	-0.3	-0.79726
10.0			-0.85319
20.0			-0.88090
50.0			-0.89935
10^9			-0.90805
10.0	0.1	-0.3	-0.85319
	0.3		-0.81324
	0.5		-0.77729
	0.7		-0.74007
	0.9		-0.71005

Table 3: For the case of injection, variation of $f''(0)$ for different values of m and M

M	**M**	**S**	$f''(0)$
-0.5	2	0.3	-1.57982
-0.3			-1.79066
-0.1			-1.89838
0.5			-2.04023
3.0			-2.16527
3.0	0	0.3	-1.01497
	1		-1.38633
	2		-2.16527
	2.5		-2.60864
	3		-3.06865

Table 4: For the case of injection, variation of $\theta'(0)$ for different values of R^* and ε

R^*	ε	S	$\theta'(0)$
5.0	0.1	0.3	-0.66354
10.0			-0.69564
20.0			-0.72072
50.0			-0.73655
10^9			-0.73663
10.0	0.1	0.3	-0.69055
	0.3		-0.64488
	0.5		-0.61303
	0.7		-0.58162
	0.9		-0.55613

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