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Signed Edge Domination in Circulant Graphs

G. Kalaimurugan and R. Jayakumar

*Department of Mathematics,
Thiruvalluvar university,
Vellore - 632115, Tamil Nadu, India
Department of Mathematics,
Indian arts and science college,
Kondam - 606802, Tamil Nadu, India
Email addresses: kalaimurugan@gmail.com (G. kalaimurugan),
rjayakumarmaths@gmail.com (R. Jayakumar)*

Abstract

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. A function $f: E(G) \rightarrow \{-1,1\}$ is called the signed edge domination fuction (SEDF) of G if $\sum_{e' \in N[e]} f(e') \geq 1$ for every $e \in E(G)$, where $N[e] = N(e) \cup \{e\}$, $N(e)$ is the set of all edges adjacent to e. The signed edge domination number $\gamma_S'(G)$ of G is defined as $\gamma_S'(G) = \min \{\sum_{e \in E(G)} f(e) \mid f$ is an SEDF of $G\}$. In this paper, we obtain exact values for the signed edge domination number of certain class of circulant graphs.

Keywords: signed edge domination, signed edge dominating number, circulant graph.

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1. Introduction

We use [5, 3] for terminology and notation which are not defined here and consider finite, simple and undirected graphs without isolated vertices only. Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The *order* of G denotes the number of vertices of G. For any $v \in V(G)$, $d(v)$ is the degree of v and $E(v)$ is the set of all edges incident with v.

Two edges e_1, e_2 of G are called adjacent if they are distinct and have a common vertex. The open neighborhood $N(e)$ of an edge $e \in E(G)$ is the set of all edges adjacent to e. Its closed neighborhood is $N[e] = N(e) \cup \{e\}$. Let

G be a graph and $f: E(G) \to \{+1, -1\}$ be a function. For every vertex v, we define $s_v = \sum_{e \in E(v)} f(e)$ and $N_i = \{v \in V(G) | s_v = i\}$.

A function $f: E(G) \to \{-1, 1\}$ is called the signed edge domination function (SEDF) of G if $\sum_{e' \in N[e]} f(e') \geq 1$ for every $e \in E(G)$. The signed edge domination number $\gamma'_S(G)$ of G is defined as $\gamma'_S(G) = \min\{\sum_{e \in E(G)} f(e) \mid f$ is an SEDF of $G\}$. In the past few years, several types of domination problems in graphs have been studied by various authors, most of these belonging to the vertex domination. Xu [6] initiated the study of signed edge domination numbers of graphs and several papers have been published on bounds of the signed edge domination number of graphs, one can refer [1, 4, 7, 8]. We use the following lemma for signed edge domination of graphs.

Lemma 1. [1] Let $f: E(G) \to \{-1, 1\}$ be a function. Then f is an SEDF of G, if and only if for any edge $e = uv, s_u + s_v - f(e) \geq 1$. Moreover, if f is an SEDF, then $s_u + s_v \geq 0$.

The circulant graphs are an important class of graphs, which can be used in the design of interconnection networks. Let $1 \leq a_1 < a_2 < \ldots < a_m \leq n/2$, where n and a_j, $j = 1, 2, \ldots, m$, are positive integers. A circulant graph $C_n(a_1, a_2, \ldots, a_m)$ is a regular graph with the vertex-set $V = \{v_0, v_1, \ldots, v_{n-1}\}$ and the edge-set $E = \left\{v_i v_{i+a_j} \middle| i = 0, 1, \ldots, n-1 \text{ and } j = 1, 2, \ldots, m\right\}$, where indices are taken modulo n. It is easy to see that if $a_m < n/2$ then $C_n(a_1, a_2, \ldots, a_m)$ is a $2m$-regular circulant graph with mn edges. On the other hand if $a_m = n/2$ then the circulant graph is a $(2m + 1)$-regular one of size $n(2m + 1)/2$.

The circulant graph $C_n(a_1, a_2, \ldots, a_m)$ is connected, see [2], if and only if for the greatest common divisor of the numbers $a_1, a_2, \ldots, a_m, n$ holds that gcd($a_1, a_2, \ldots, a_m, n$)= 1. More precisely $C_n(a_1, a_2, \ldots, a_m)$ has h = gcd($a_1, a_2, \ldots, a_m, n$) connected components which are isomorphic to $C_{n/h}(a_1/h, a_2/h, \ldots, a_m/h)$.

In this paper we focus on circulant graphs and we obtain exact values for the signed edge domination number of certain class of circulant graphs.

2. Signed edge domination in graphs

In this section we present the lemma for a minimum signed edge domination function of graphs. It will play a role for the proof of theorem in next section.

Lemma 2. Let G be a graph of order n. Let $f: E(G) \to \{-1, 1\}$ be a function and $|N_1| = n$. Then f is a minimum SEDF.

Proof. Let $f: E(G) \to \{-1, 1\}$ be a function. Since $|N_1| = n$, for every $e = uv \in E(G)$ we have $s_u + s_v - f(e) \geq 1$. By the lemma 1 f is a SEDF.

Suppose f is not a minimum SEDF, then there exist a minimum SEDF f'. Also at least one edge $e = uv \in E(G), f(e = uv) = 1$ of f replace by a SEDF f' with $f'(e = uv) = -1$, then we get $s'_u + s'_v = \sum_{e \in E(u)} f'(e) + \sum_{e \in E(v)} f'(e) = -1 - 1 = -2 \ngeq 0,$ which is a contradiction. Hence f is minimum SEDF.

Lemma 3. Let G be a graph of order n. Let $f : E(G) \to \{-1,1\}$ be a function and $|N_0| = |N_2| = \frac{n}{2}$. If no edges $e = uv \in E(G)$, where $u, v \in N_0$ such that $f(uv) = 1$ and every $u \in N_0$ there exist a vertex $v \in N_2$ such that $f(uv) = 1,$ then f is a minimum SEDF.

Proof. Let $f : E(G) \to \{-1,1\}$ be a function and $|N_0| = |N_2| = \frac{n}{2}$. Since no edges $e = uv$, where $u, v \in N_0$ such that $f(uv) = 1$, therefore every edge $e = uv \in E(G)$, we have $s_u + s_v - f(e) \geq 1$. By the lemma 1, f is SEDF. Now, to prove f is minimum. Suppose f is not a minimum, there exist a minimum SEDF f' such at least one edge $e = uv \in E(G), f(e = uv) = 1$ of f replace by f' with $f'(e = uv) = -1$. Now, we have the following two cases, Case(i): Suppose for every $u, v \in N_2$, we get $s'_u = \sum_{e \in E(u)} f'(e) = 0$ and $s'_v = 0$. Which is a contradiction to $|N_0| = |N_2| = \frac{n}{2}$ and every $u \in N_0$ there exist a vertex $v \in N_2$ such that $f(uv) = 1$. Case(ii): If $u \in N_0$ and $v \in N_2$, then $s'_u + s'_v = -2 + 0 \ngeq 0$, which is a contradiction. Hence f is minimum SEDF.

3. Signed edge domination number in circulant graphs

In this section, we give the signed edge domination number for the certain class of circulant graph $C_n(a_1, a_2, \ldots, a_m)$.

Theorem 4. Let $a_j, j = 1,2, \ldots, m$ be positive integers and $1 \leq a_1 < a_2 < \ldots < a_m \leq \frac{n}{2}$. If $n \geq 4, m \geq 1$ and $G = C_n\left(a_1, a_2, \ldots, a_m, \frac{n}{2}\right)$ be a $2m + 1$ regular circulant graph, then $\gamma'_S(G) = \frac{n}{2}$.

Proof: Let $n \geq 4, m \geq 1$ and let $\{v_i \mid i = 0, 1, \ldots, n - 1\}$ be the vertices of the $2m + 1$- regular circulant graph $G = C_n\left(a_1, a_2, \ldots, a_m, \frac{n}{2}\right)$. Define a function $f : E(G) \to \{-1,1\}$ by

$$f\left(v_i v_{i+a_j}\right) = (-1)^{j-1} \ for \ 1 \leq j \leq m \ and \ 0 \leq i \leq n-1.$$

$$f\left(v_i v_{i+\frac{n}{2}}\right) = (-1)^m \ for \ \ 0 \leq i \leq n-1.$$

It can be easily verified that for every vertex $v \in V(G)$, $s_v = \sum_{e \in E(v)} f(e) = 1$. By Lemma 1 and 2, for every edge $e \in E(G)$, $\sum_{e' \in N[e]} f(e') \geq 1$ and f is a minimum SEDF on G. Thus $\gamma'_S(G) = \sum_{e \in E(G)} f(e) = \frac{n}{2}$.

Theorem 5. Let n be positive integers. If $n \geq 8, n \equiv 0 \ (mod\ 4)$ and $G = C_n(1,2)$ be a 4- regular circulant graph, then $\gamma'_S(G) = \frac{n}{2}$.

Proof: Let $n \geq 8$, $n \equiv 0 \ (mod\ 4)$ and let $\{v_i \mid i = 0,1,\ldots,n-1\}$ be the vertices of the 4- regular circulant graph $G = C_n(1,2)$. Define a function $f : E(G) \to \{-1,1\}$ by

$$f(v_i v_{i+1}) = \begin{cases} 1 & for \ \ n \equiv 0 \ (mod\ 4) \\ -1 & otherwise \end{cases}$$

$$f(v_i v_{i+2}) = 1 \ \ for \ \ 0 \leq i \leq n-1.$$

It can be easily verified that for $i = 0,1,2,\ldots,n-1$ and $v_i \in V(G)$,

$$s_{v_i} = \begin{cases} 2 & for \ \ n \equiv 0,1 \ (mod\ 4) \\ 0 & otherwise \end{cases}$$

Therefore, $|N_0| = |N_2| = \frac{n}{2}$. Also, no edges $e = uv \in E(G)$, where $u, v \in N_0$ such that $f(uv) = 1$ and every $u \in N_0$ there exist a vertex $v \in N_2$ such that $f(uv) = 1$. By Lemma 1 and 3, for every edge $e \in E(G)$, $\sum_{e' \in N[e]} f(e') \geq 1$ and f is a minimum SEDF on G. Thus $\gamma'_S(G) = \sum_{e \in E(G)} f(e) = \frac{n}{2}$.

4. Concluding remark

As a final remark, we present some problems that are raised from this paper. Note that circulant graphs are Cayley graph on cyclic group $\mathbb{Z}_n$.

1. Find the signed edge domination number of $2m$ - regular circulant graph $G = C_n\left(1 \leq a_1 < a_2 < \ldots < a_m < \frac{n}{2}\right)$ for $m \geq 2$.
2. Find the signed edge domination number of Cayley graph.

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