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Investigating the possibility of using piecewise constant functions in spectral analysis of signals

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Abstract
A new way to improve spectral analysis of signals is discovered. Given method reduces the number of multiplications while calculating result, witch in many cases is very important. Especially it is important in real-time systems.

Keywords: Fourier transform algorithms, discrete Fourier transform, Walsh-Hadamard transform, real-time calculating.

Introduction

There are various spectral analysis problems that require obtaining the result as quickly as possible.

Nondestructive testing methods [1],[2] are some of such problems. Simply put, the time between obtaining the last signal value and obtaining a result should be minimal.

A particular analysis method is chosen depending on the signal length, number of analyzed frequencies and permissible accuracy of result.

The following are possible options:

If the number of analyzed frequencies is approximately equal to the length of the signal and high accuracy is required, then one of the fast Fourier transform (FFT) algorithms (Cooley-Tukey FFT, Grapes experiment and so on) is used. If there is lower requirement for frequency, the Hadamard transform [3] is used. If the number of analyzed frequencies is significantly less than the signal length for obtaining a highly accurate result, it is recommended to use direct discrete Fourier transform (DFT) or transform with a basis in the form of single functions [1].

Let's consider the principles of some of these transforms on the signal S(t).

Walsh-Hadamard transform

Analysis of a specific-frequency signal using the Walsh-Hadamard transform [4] can be represented as:

$$A(p) = \int_0^T S(t) * wal(p,t)\, dt$$

Where p is the parameter that determines the type of Walsh function $wal(p,t)$

The following relations between Walsh functions and trigonometric functions $\sin(2\pi ft), \cos(2\pi ft)$ are true.

TABLE.1. Comparison of Walsh functions and trigonometric functions.

$\boldsymbol{wal(p,t)}$	$\boldsymbol{\sin(x), \cos(x)}$
$wal(0,t)$	$\cos(0)$
$wal(1,t)$	$\sin(2\pi t)$
$wal(2,t)$	$\cos(2\pi t)$
$wal(3,t)$	$\sin(4\pi t)$
$wal(4,t)$	$\cos(4\pi t)$
$wal(5,t)$	$\sin(6\pi t)$
$wal(6,t)$	$\cos(6\pi t)$
…	…

forp of type 2k

$wal(p,t) => cos(p * \pi t)$

for p of type 2k+1

$wal(p,t) => \sin((p+1) * \pi t)$

where k=0,1,2, ...

$$reA(p) = \int_0^T (S(t) * wal(p,t))\, dt$$

$$imA(p) = \int_0^T (S(t) * wal(p-1,t))\, dt$$

where$wal(-1,t) = const\ 0$

Fourier transform

The real and imaginary parts are calculated by the formulas

$$re\ A(f,t) = \int_0^T (S(t) * sin(2\pi ft))\, dt$$

$$im\ A(f,t) = \int_0^T (S(t) * cos(2\pi ft))\, dt$$

Generalized Fourier transform

Generalized Fourier transform [5] where basis functions take the values +-1

Similar to Fourier transform, we can write the expressions:

$$re\,A(f,t) = \int_0^T S(t) * sign[sin(2\pi f t)]\,dt$$

$$im\,A(f,t) = \int_0^T S(t) * sign[cos(2\pi f t)]\,dt$$

It can be said that the function $sign(sin(x))$ is the simplest approximation of the function $sin(x)$.
As can be seen, in all the methods considered, different ways are used to approximate trigonometric function $sin(x)$
It is proposed to use piecewise constant approximation of $sin(x)$: $st(N,x)$
Where N is the number of pieces in the period where $st(N,x)$ is constant.
Let's prove the possibility in principle of using the functions in spectral analysis.

Generalized Fourier transform
We will approximate trigonometric function $\sin 2\alpha x$ with periodic piecewise constant function f(x). Let n be the number of steps used for approximation in the period $\left[0,\frac{\pi}{\alpha}\right]$. For the *i*-th step, we choose a value of the approximating function as follows:

$$f(x) = \sin\left(2\alpha \cdot \frac{1}{2}\left(\frac{\pi(i-1)}{\alpha n} + \frac{\pi i}{\alpha n}\right)\right) = \sin\frac{\pi(2i-1)}{n}.$$

Then$f(x) = sin\frac{\pi(2i-1)}{n}$, as $x \in \left[\frac{\pi(i-1)}{\alpha n}, \frac{\pi i}{\alpha n}\right]$, where $i = 1, \dots, n$.
Let $S(t) \in C(\mathbb{R})$ and $k = \max_{j\in\mathbb{N}, \frac{\pi j}{\alpha} \le a} j$

Then the following estimate holds:

$$\left|\int_0^a S(t)\sin 2\alpha t dt - \int_0^a S(t) f(t) dt\right| \le \max_{t\in[0,a]} S(t) \cdot \frac{(k+1)\pi^2}{\alpha n}$$

Proof. We do the necessary estimate:

$$\left|\int_0^a S(t)\sin 2\alpha t dt - \int_0^a S(t) f(t) dt\right| \le \left|\int_0^a S(t)(\sin 2\alpha t - f(t)) dt\right| \le$$

$$\le \max_{t\in[0,a]} S(t) \int_0^a |\sin 2\alpha t - f(t)| dt \le$$

$$\le \max_{t\in[0,a]} S(t) \left(\int_0^{\frac{k\pi}{\alpha}} |\sin 2\alpha t - f(t)| dt + \int_{\frac{k\pi}{\alpha}}^a |\sin 2\alpha t - f(t)| dt \right). \quad (1)$$

We estimate the integrals in (1) separately. Since

$$t \in \left[\frac{\pi(i-1)}{\alpha n}, \frac{\pi i}{\alpha n}\right]$$

the inequality

$$|\sin 2\alpha t - f(t)| \le \max_{t\in\left[\frac{\pi(i-1)}{\alpha n}, \frac{\pi i}{\alpha n}\right]} \left|\frac{d}{dt}\sin 2\alpha t\right| \left|t - \frac{\pi(2i-1)}{\alpha n}\right|$$

holds. Then

$$\int_0^{\frac{k\pi}{\alpha}} |\sin 2\alpha t - f(t)| dt \le k \int_0^{\frac{\pi}{\alpha}} |\sin 2\alpha t - f(t)| dt =$$

$$= k \sum_{i=1}^n \int_{\frac{\pi(i-1)}{\alpha n}}^{\frac{\pi i}{\alpha n}} \left|\sin 2\alpha t - \sin\frac{\pi(2i-1)}{n}\right| dt \le$$

$$\le k \sum_{i=1}^n \int_{\frac{\pi(i-1)}{\alpha n}}^{\frac{\pi i}{\alpha n}} \max_{t\in\left[\frac{\pi(i-1)}{\alpha n}, \frac{\pi i}{\alpha n}\right]} \left|\frac{d}{dt}\sin 2\alpha t\right| \cdot \left|t - \frac{\pi(2i-1)}{\alpha n}\right| dt \le$$

$$\le k \cdot n \cdot 2\alpha \cdot 2\left(\frac{\pi}{2\alpha n}\right)^2 = \frac{k\pi^2}{\alpha n}. \quad (2)$$

Using similar reasoning, we have:

$$\int_{\frac{k\pi}{\alpha}}^a |\sin 2\alpha t - f(t)| dt \le \int_{\frac{k\pi}{\alpha}}^{\frac{\pi(k+1)}{\alpha}} |\sin 2\alpha t - f(t)| dt \le \frac{\pi^2}{\alpha n}.$$

The theorem is confirmed by combining estimates (1), (2) and (3). Q.E.D.

Modeling
Application of various basis functions based on a 16-point Hadamard transform was modeled.
The analyzed signal was formed as follows:

$$s(t) = a_1 sin(2\pi t) + a_2 cos(2\pi t) + a_3 sin(4\pi t) + a_4 cos(4\pi t) + a_5 sin(6\pi t) + a_6 cos(6\pi t) + a_7 sin(8\pi t) + a_8 cos(8\pi t) + a_9 sin(10\pi t) + a_{10} cos(10\pi t) + a_{11} sin(12\pi t) + a_{12} cos(12\pi t) + a_{13} sin(14\pi t) + a_{14} cos(14\pi t)$$

Variables $a_1, \dots, a_{14}$ took values from the set {0,1,2}
The reference result was an array of 7 values:

$$A_{et} = \left[\sqrt{a_1^2 + a_2^2}, \sqrt{a_3^2 + a_4^2}, \dots, \sqrt{a_{13}^2 + a_{14}^2}\right]$$

Piecewise constant approximation of the sine had 32 constant areas for the period.
The transform results with different bases were contained in arrays

$$R_{wal} = [r_{wal}0,\ r_{wal}1, \dots, r_{wal}15]$$
$$R_{trg} = [r_{trg}0,\ r_{trg}1, \dots, r_{trg}15]$$
$$R_{sgn} = [r_{sgn}0,\ r_{sgn}1, \dots, r_{sgn}15]$$
$$R_{cst} = [r_{cst}0,\ r_{cst}1, \dots, r_{cst}15]$$

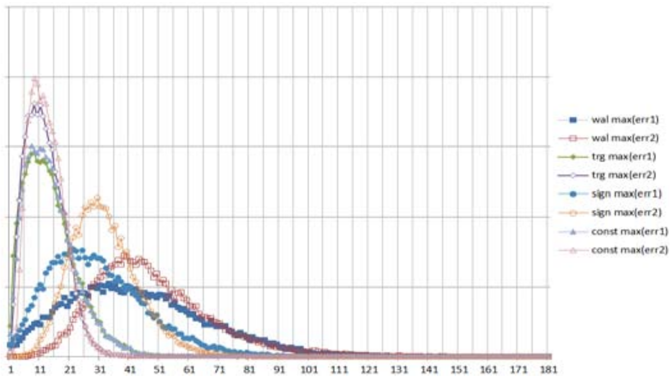

Fig. Result of modeling

Error calculating method

Replacing the trigonometric functions with their piecewise constant approximations introduced an error in the modeling result.
The error of result was calculated by the formulas:

where
reference value of the amplitude of the frequency,
-experimental value of the amplitude of the frequency,
-sum of reference values
-sum of experimental values for all *i*, where is not equal to zero.
Noise level was calculated by the formula:

Modeling result

Since transform of different signals was computed during modeling, it is convenient to represent the modeling result in the form of histograms where the horizontal axis represents the calculation error in percentages.
The envelope lines of the histograms are displayed on the graph (Figure) for the convenience of visual display of results.
Envelope histograms for different basis functions
On the graph:
wal-distribution of maximum values of errors (err1) and noise level (err2) for Walsh-Hadamard transform,
trg-for discrete Fourier transform (DFT),
sgn-for generalized DFT, where trigonometric functions are approximated by a meander,
const-for generalized DFT, where trigonometric functions are approximated by piecewise constant functions.
Visual comparison of envelopes for trigonometric functions and their piecewise constant approximation show a more accurate computation of signal parameters among all the bases considered.

Conclusion

Modeling results reveal that piecewise constant functions can be used in spectral analysis of signals.
Application of piecewise constant functions as a basis allows you to obtain a result with the required accuracy and reduce the number of multiplications involved in the analysis:
for one frequency,

multiplications are required,where C is the number of pieces with a constant value for the period.
Reducing the number of multiplications makes getting an end result faster and reduces transform computation time.

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