

Reliability & Availability Analysis of an Anaerobic Batch Reactor Treating Fruit and Vegetable Waste

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Abstract

The paper presents reliability and availability analysis of anaerobic batch reactor treating fruit and vegetable waste. The continuous operation of the reactor is important and therefore its reliability and availability under different failure situations help in understanding the overall performance. Relevant data for 4320 hours on failure and restoration of reactors have been used in this analysis and various rates are estimated from the data. Here, the reactors failure is categorized into six types and is detected by inspection only. Semi-Markov process and regenerative point techniques are used in the entire analysis.

Keywords: wastewater treatment, reliability, semi-Markov, regenerative process.

Notations

0	Operative state
1	Mixing failure
2	Improper influent error
3	Peak load failure
4	Acidified pH < 7
5	Natural failure
6	Forced failure
λ_1	Mixing failure rate
λ_2	improper influent error rate
λ_3	Peak load failure rate
λ_4	Acidifying rate
α_1	Natural failure rate
α_2	Forced failure rate
γ_1	Rate of recovery for mixing failure
γ_2	Rate of recovery for Improper Influent error
γ_3	Rate of recovery for peak load failure
γ_4	Rate of recovery from acidification
γ_5	Rate of recovery from natural failure
γ_6	Rate of recovery from forced failure
©	Symbol for Laplace convolution

Ⓢ	Symbol for Stieltje's convolution
*	Symbol for Laplace transforms
**	Symbol for Laplace Stieltje's transforms
$\varphi_i(t)$	c. d. f. of first passage time from a regenerative state i to a failed state j
$p_{ij}(t), Q_{ij}(t)$	p. d. f. and c. d. f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j in $[0, t]$
$g_m(t), G_m(t)$	p. d. f. and c. d. f. of rate of recovery for mixing failure
$g_i(t), G_i(t)$	p. d. f. and c. d. f. of rate of recovery for improper influent error
$g_p(t), G_p(t)$	p. d. f. and c. d. f. of rate of recovery for peak load failure
$g_a(t), G_a(t)$	p. d. f. and c. d. f. of rate of recovery from acidification
$g_n(t), G_n(t)$	p. d. f. and c. d. f. of rate of recovery from natural failure
$g_f(t), G_f(t)$	p. d. f. and c. d. f. of rate of recovery from forced failure

Introduction

Fruit and vegetable wastes (FVW) are produced in large quantities in markets, local fruit shops, supermarkets, etc. These wastes constitute a source of nuisance in municipal landfills because of their high biodegradability [1], [2]. The organic fraction includes about 75% sugars and hemicellulose, 9% cellulose and 5% lignin. The easy biodegradable organic matter content of FVW (75%) with high moisture facilitates their biological treatment and shows the trend of these wastes for anaerobic digestion [2], [3]. The anaerobic digestion is a process by which almost any organic waste can be biologically converted in the absence of oxygen. This process requires specific environmental conditions and different bacterial populations. Mixed bacterial populations degrade organic compounds and produce as end-product a valuable

high energy mixture of gases (mainly methane-CH₄ and carbon dioxide-CO₂), termed biogas. The anaerobic digestion of FVW is accomplished by a series of biochemical transformations, which can be roughly separated into four metabolic stages. First, particulate organic materials of FVW like cellulose, hemicellulose, pectin, and lignin, must undergo liquefaction by extracellular enzymes before being taken up by acidogenic bacteria [4], [5]. The rate of hydrolysis is a function of factors, such as pH, temperature, composition, and particle size of the substrate and high concentrations of intermediate products. After that, soluble organic components including the products of hydrolysis are converted into organic acids, alcohols, hydrogen, and carbon dioxide by acidogens. The products of the acidogenesis are then converted into acetic acid, hydrogen, and carbon dioxide. Finally, methane is produced by methanogenic bacteria from acetic acid, hydrogen, and carbon dioxide as well as directly from other substrates of which formic acid and methanol are the most important [2], [6].

Experiments were carried out in double-walled glass reactors of 6-l effective volume, maintained at 35 °C by a regulated water bath. Mixing in the reactors was done by a system of magnetic stirring. The biogas production was measured on-line every 2 minutes by Milli gas counter MGC-1 flow meters (Ritter gas meters) fitted with a 4-20 mA output. The methane content in the biogas was measured online using Blusens methane analyser and the data were collected by software supplied by the manufacturer. After seeding and before starting the addition of the substrate, the reactors were fed with 2-4 mL of ethanol, in 4 cycles, as sole carbon and energy source to check the activity of the inoculum [7], [8].

The reactors were fed with mixture of vegetable substrate and fruit substrate. The total organic loading rate (OLR) of mixture fed was 0.5g-5g volatile solids (VS) per litre volume of the reactor. The reactor was operated for duration of 4320 hrs. with an operational cycle (feeding) of once in a week, alternate days and every day for each of the OLR.

The effective operation of such system (or reactors) depends on its reliability which keeps the system performance close to the original expectations. Systems (or reactors) in practice are subject to failure. In order to keep systems, effective and useful in terms of its extensive quality lifetime and preventing the occurrence of system failures; a periodic monitoring is necessary. Reliability models provide the basis of any maintenance quantitative analysis in terms of reliability indices which in turn are helpful in evaluating the overall system performances. Several researchers including [9] and [10] have contributed to this field and have given pathways to meet the real challenges while dealing with the failure analysis. Recently, [11] and [12] have analyzed a desalination plant under different operating conditions and obtained the reliability indices for the plant. Thus, the methodology for system analysis under various failure and repair situations has been widely presented in the literature and the novelty of this work lies in its case study. The numerical results of various reliability indices are extremely useful in understanding the significance of these failures on the system availability and assess the impact of these failures on the overall performance of the system.

Thus, the paper is an attempt to present a case analysis using the failure data of 4320 hrs. of the anaerobic batch reactor treating fruit and vegetable waste. The reliability indices of interest such as mean time to reactor failure and availability have been obtained. During the reactor operation it is important to keep the pH at 7.0-8.0 to have the methanogenic activities. The methanogenic activity is the one which enables the organic matter to disintegrate and further convert to stable products. It is equally important to keep a continuous mixing of the reactor for the biomass to be in complete contact with the organic matter. It has been noted that the reactors were failing due to six reasons during the entire operation i. e., mixing failure, improper influent failure, and peak load failure, failure due to acidification, natural failure, and forced failure. The reactor operation is often influenced by condition of pH and the organic loading rate (OLR). The reactor failure is experienced due to the inconsistency in the substrates and due to mixing inside the reactor. The reactor operation is started with feeble OLR and then slowly increased and during which period, due to the above said reasons the reactor has naturally failed. The reactor is then operated without any feeding to naturally reequip the process. At high OLR, a problem of mixing observed but in course of time the reactor was back to operational condition. The reactor was then loaded with high OLR, to test the peak load conditions to study the operational parameters and the conditions that could influence the fatal failure of the reactor i. e., forced failure. The semi-Markov process and regenerative point techniques are used in the entire analysis.

Using the data, following values of various rates are estimated:

- Estimated rate at which *mixing failure* of the unit occurs: $(\lambda_1) = 0.000490196$ per hour
- Estimated rate at which the *improper influent* error of the unit occurs: $(\lambda_2) = 0.001838235$ per hour
- Estimated rate at which *peak load* failure of the unit occurs: $(\lambda_3) = 0.000570776$ per hour
- Estimated rate at which *acidification* takes place: $(\lambda_4) = 0.000400641$ per hour
- Estimated rate at which *natural failure* of the unit occurs: $(\alpha_1) = 0.00462963$ per hour
- Estimated rate at which *forced failure* of the unit occurs: $(\alpha_2) = 0.006944444$ per hour
- Estimated Rate of *recovery for mixing* failure: $(\gamma_1) = 0.005952381$ per hour
- Estimated Rate of *recovery for improper* influent error: $(\gamma_2) = 0.004464286$ per hour
- Estimated Rate of *recovery for peakload* failure: $(\gamma_3) = 0.002136752$ per hour
- Estimated Rate of *recovery from acidification*: $(\gamma_4) = 0.041666667$ per hour
- Estimated Rate of *recovery from natural* failure: $(\gamma_5) = 0.008333333$ per hour
- Estimated Rate of *recovery from forced* failure: $(\gamma_6) = 0.020833333$ per hour

Model description and assumptions

- There are four batch reactors for operation.

- States 0 is an operative state; states 1 and 4 are the partially operative states whereas all other states are the completely failed states.
- An inspection is carried out to identify the type of failure.
- If a reactor is failed, it gets repaired on priority basis.
- All failure times are assumed to have exponential distribution whereas the restoration times have general distributions.

Transition probabilities and mean sojourn times

A state transition diagram showing the possible states of transition of the reactors is shown in Fig. 1. The epochs of entry into states 0, 1, 2, 3, 4, 5 and 6 are the regeneration points and hence these states are regenerative states. The transition probabilities are given by:

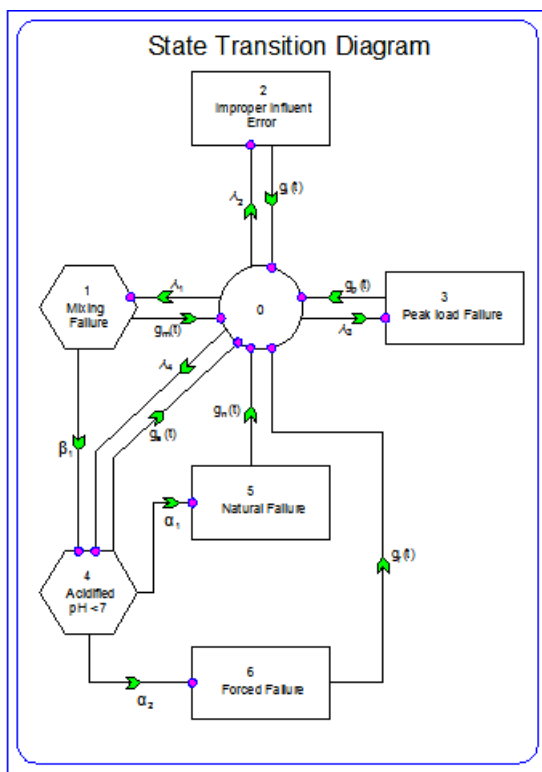


Fig. 1.State Transition Diagram

$$\begin{aligned}
 dQ_{01} &= \lambda_1 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)t} dt & (1) \\
 dQ_{02} &= \lambda_2 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)t} dt & (2) \\
 dQ_{03} &= \lambda_3 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)t} dt & (3) \\
 dQ_{04} &= \lambda_4 e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)t} dt & (4) \\
 dQ_{10} &= g_m(t) e^{-\beta t} dt & (5) \\
 dQ_{14} &= \beta_1 e^{-\beta_1 t} \bar{G}_m(t) dt & (6) \\
 dQ_{20} &= g_i(t) dt & (7) \\
 dQ_{30} &= g_p(t) dt & (8) \\
 dQ_{50} &= g_n(t) dt & (9) \\
 dQ_{60} &= g_f(t) dt & (10) \\
 dQ_{40} &= g_a(t) e^{-(\alpha_1 + \alpha_2)t} dt & (11) \\
 dQ_{45} &= \alpha_1 e^{-(\alpha_1 + \alpha_2)t} \bar{G}_a(t) dt & (12) \\
 dQ_{46} &= \alpha_2 e^{-(\alpha_1 + \alpha_2)t} \bar{G}_a(t) dt & (13)
 \end{aligned}$$

The non-zero element p_{ij} can be obtained by,

$$p_{ij} = \lim_{s \rightarrow 0} \int_0^\infty q_{ij}(t) dt \quad (14)$$

$$p_{01} + p_{02} + p_{03} + p_{04} = 1 \quad (15)$$

$$p_{20} = p_{30} = p_{50} = p_{60} = 1 \quad (16)$$

$$p_{40} + p_{45} + p_{46} = 1 \quad (17)$$

$$p_{10} + p_{14} = 1 \quad (18)$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state 'i', then:

$$\mu_i = E(T) = P(T > t) \quad (19)$$

$$\mu_0 = \frac{1}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)} \quad (20)$$

$$\mu_1 = \frac{1}{(\gamma_1 + \beta_1)} \quad (21)$$

$$\mu_2 = \frac{1}{\gamma_2} \quad (22)$$

$$\mu_3 = \frac{1}{\gamma_3} \quad (23)$$

$$\mu_4 = \frac{1}{(\gamma_4 + \alpha_1 + \alpha_2)} \quad (24)$$

$$\mu_5 = \frac{1}{\gamma_5} \quad (25)$$

$$\mu_6 = \frac{1}{\gamma_6} \quad (26)$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entry into state 'i' is mathematically stated as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^*(0) \quad (27)$$

$$\sum_j m_{ij} = \mu_i \quad (28)$$

$$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0 \quad (29)$$

$$m_{10} + m_{14} = \mu_1 \quad (30)$$

$$m_{20} = \mu_2 \quad (31)$$

$$m_{30} = \mu_3 \quad (32)$$

$$m_{40} + m_{45} + m_{46} = \mu_4 \quad (33)$$

$$m_{50} = \mu_5 \quad (34)$$

$$m_{60} = \mu_6 \quad (35)$$

The mathematical analysis

A. Mean Time to Reactor Failure

Regarding the failed states 2, 3, 5 & 6 as absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\varphi_i(t)$ is obtained:

$$\begin{aligned}
 \varphi_0(t) &= Q_{01}(t) \otimes \varphi_1(t) + Q_{02}(t) \\
 &+ Q_{03}(t) + Q_{04}(t) \otimes \varphi_4(t) \quad (36)
 \end{aligned}$$

$$\varphi_1(t) = Q_{10}(t) \otimes \varphi_0(t) + Q_{14}(t) \otimes \varphi_4(t) \quad (37)$$

$$\varphi_4(t) = Q_{40}(t) \otimes \varphi_0(t) + Q_{45}(t) + Q_{46}(t) \quad (38)$$

Solving the above equation for $\varphi_0^{**}(s)$ by taking Laplace Stieltje's transforms, the mean time to reactor failure (MTRF) when the reactor started at the beginning of state 0, is given by

$$MTRF = \lim_{s \rightarrow 0} \frac{1 - \varphi_0^{**}(s)}{s} = \lim_{s \rightarrow 0} \frac{1 - \frac{N(s)}{D(s)}}{s} \quad (39)$$

where N(s) and D(s) are

$$\begin{aligned}
 N(s) &= Q_{02}^{**}(s) + Q_{03}^{**}(s) + Q_{04}^{**}(s) Q_{45}^{**}(s) + Q_{04}^{**}(s) Q_{46}^{**}(s) \\
 &+ Q_{01}^{**}(s) Q_{14}^{**}(s) Q_{45}^{**}(s) + Q_{01}^{**}(s) Q_{14}^{**}(s) Q_{46}^{**}(s) \quad (40)
 \end{aligned}$$

$$\begin{aligned}
 D(s) &= 1 - Q_{01}^{**}(s) Q_{10}^{**}(s) - Q_{04}^{**}(s) Q_{40}^{**}(s) \\
 &- Q_{01}^{**}(s) Q_{14}^{**}(s) Q_{40}^{**}(s) \quad (41)
 \end{aligned}$$

B. Availability Analysis of the Reactor

Using the probabilistic arguments and defining $A_i(t)$ as the probability of reactor entering into upstate at instant t , given that the unit entered in regenerative state i at $t = 0$, the following recursive relations are obtained for $A_i(t)$:

$$A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t) + q_{04}(t) \odot A_4(t) \quad (42)$$

$$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{14}(t) \odot A_4(t) \quad (43)$$

$$A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) \quad (44)$$

$$A_3(t) = q_{30}(t) \odot A_0(t) \quad (45)$$

$$A_4(t) = M_4(t) + q_{40}(t) \odot A_0(t) + q_{45}(t) \odot A_5(t) + q_{46}(t) \odot A_6(t) \quad (46)$$

$$A_5(t) = q_{50}(t) \odot A_0(t) \quad (47)$$

$$A_6(t) = q_{60}(t) \odot A_0(t) \quad (48)$$

where,

$$M_0(t) = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)t} \quad (49)$$

$$M_1(t) = e^{-\beta_1 t} \bar{G}_m(t) \quad (50)$$

$$M_2(t) = \bar{G}_i(t) \quad (51)$$

$$M_4(t) = e^{-(\alpha_1 + \alpha_2)t} \bar{G}_a(t) \quad (52)$$

On taking Laplace transforms of the above equations and solving them for $A_0^*(s)$, the steady state availability is given by:

$$A_0 = \lim_{s \rightarrow 0} s A_0^*(s) = \lim_{s \rightarrow 0} \frac{s N_1(s)}{D_1(s)} = \frac{N_1(0)}{D_1'(0)} \quad (53)$$

where $N_1(s)$ and $D_1(s)$ are

$$N_1(s) = M_0^*(s) + M_1^*(s) q_{01}^*(s) + M_2^*(s) q_{02}^*(s) + M_4^*(s) q_{04}^*(s) + M_4^*(s) q_{01}^*(s) q_{14}^*(s) \quad (54)$$

$$D_1(s) = 1 - q_{01}^*(s) q_{10}^*(s) - q_{02}^*(s) q_{20}^*(s) - q_{03}^*(s) q_{30}^*(s) - q_{04}^*(s) q_{40}^*(s) - q_{01}^*(s) q_{14}^*(s) q_{40}^*(s) - q_{04}^*(s) q_{45}^*(s) q_{50}^*(s) - q_{04}^*(s) q_{46}^*(s) q_{60}^*(s) - q_{01}^*(s) q_{14}^*(s) q_{45}^*(s) q_{50}^*(s) - q_{01}^*(s) q_{14}^*(s) q_{46}^*(s) q_{60}^*(s) \quad (55)$$

Particular case

For the particular case, it is assumed that the failure rates are exponentially distributed whereas the other rates are general:

$$g_m(t) = \gamma_1 e^{-\gamma_1 t} \quad (56)$$

$$g_i(t) = \gamma_2 e^{-\gamma_2 t} \quad (57)$$

$$g_p(t) = \gamma_3 e^{-\gamma_3 t} \quad (58)$$

$$g_a(t) = \gamma_4 e^{-\gamma_4 t} \quad (59)$$

$$g_n(t) = \gamma_5 e^{-\gamma_5 t} \quad (60)$$

$$g_f(t) = \gamma_6 e^{-\gamma_6 t} \quad (61)$$

The transition probabilities p_{ij} are given below:

$$p_{01} = \frac{\lambda_1}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)} \quad (62)$$

$$p_{02} = \frac{\lambda_2}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)} \quad (63)$$

$$p_{03} = \frac{\lambda_3}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)} \quad (64)$$

$$p_{04} = \frac{\lambda_4}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)} \quad (65)$$

$$p_{10} = \frac{\gamma_1}{(\beta_1 + \gamma_1)} \quad (66)$$

$$p_{14} = \frac{\beta_1}{(\beta_1 + \gamma_1)} \quad (67)$$

$$p_{40} = \frac{\gamma_4}{(\gamma_4 + \alpha_1 + \alpha_2)} \quad (68)$$

$$p_{45} = \frac{\alpha_1}{(\gamma_4 + \alpha_1 + \alpha_2)} \quad (69)$$

$$p_{46} = \frac{\alpha_2}{(\gamma_4 + \alpha_1 + \alpha_2)} \quad (70)$$

Using the data summary and the expressions for MTRF and Availability as in (39) and (53), the following results have been obtained:

- Mean Time to Reactor Failure = **418 hours**
- Availability of the Reactor = **0.977**

Conclusion

Reliability analysis proves to be an effective mathematical tool for analyzing the system performances. Based on the real failure and restoration rates, optimum reliability results are achieved.

In the present analysis, it is worth noting that the value of MTRF is 418 hours which is an average operational time of the batch reactors and availability is 0.977 which shows the expected reactor's availability for operation at any point of time in future is 97.7%.

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