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Generalized Interpolating Polynomial Operator A_n

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Abstract- The article describes the construction of a linear operator which puts into correspondence an arbitrary 2π - periodic function with zero mean trigonometric polynomial. During an operator construction the decomposition in Fourier series, the Weil operator of fractional integration, Lagrange interpolation polynomial, the properties of fractional differentiation and fractional integration are used. An operator type is obtained, the corresponding formula is derived. A formula type is shown taking into account the form of the trigonometric complex numbers. The relationship of the generalized interpolation operator A_n with Fourier operator S_n is cosidered. The approximation of functions by the means of an obtained polynomial operator and the evaluation of error is verified using a computer algebra system Wolfram Mathematica. The approximation of function by a trigonometric polynom obtained by the derived formulas is conducted for different functions at different values of node numbers. The calculations showed that the difference module between the values of 2π - periodic function with a zero mean value and the values of trigonometric polynomial, constructed with the help of an operator $A_n(\varphi;t)$, decrease with the order of α integration (the values $0{,}5<\alpha<1$ were considered). The value of the modulus is less if a midportion of the interval is taken, presumably it is related to the fact that the difference $\varphi - A_n\varphi$ converges on the average. The growth of node number n also make the function approximation better.

Keywords: fractional integration, Weil fractional integral, linear operator, trigonometric polynomial, interpolation polynomial, polynomial operator

1. INTRODUCTION

The issues of integration distribution by fraction order are studied from the very beginning of the integral calculation. Currently, there is an increased interest to the problems of fractional calculation [1]. The fractional integration and differentiation are used during the modeling of the processes and phenomena in various fields [2], [3], [4]. Lagrange interpolation polynomial viewed in this paper, also finds its use in the fields of mathematics and its applications [5].

Let $\varphi(x)$ – 2π -periodic function, which is decomposed into a Fourier series: $\varphi(x) \sim \sum_{k=-\infty}^{\infty} \varphi_k e^{ikx}$, $\varphi_k = \frac{1}{2\pi}\int_0^{2\pi} e^{-ikx}\varphi(x)dx$. The functions with a zero mean value are considered: $\varphi_0 = 0$. As $\varphi^{(n)}(x) \sim \sum_{k=-\infty}^{\infty} (ik)^n \varphi_k e^{ikx}$, the fractional integration by Weil is determined as follows: $I_{\pm}^{\alpha}\varphi \sim \sum_{k=-\infty}^{\infty} \frac{\varphi_k e^{ikx}}{(\pm ik)^{\alpha}}$.

This definition may be interpreted in the following form:

$$I_{\pm}^{\alpha}\varphi = \frac{1}{2\pi}\int_0^{2\pi} \varphi(x-t)\Psi_{\pm}^{\alpha}dt, \quad \alpha > 0,$$ где

$$\Psi_{\pm}^{\alpha}(t) = \sum_{|k|=1}^{\infty} \frac{e^{ikt}}{(\pm ik)^{\alpha}} = 2\sum_{k=1}^{\infty} \frac{\cos\left(kt \mp \frac{\alpha\pi}{2}\right)}{k^{\alpha}}$$ [6].

The paper determines the type of polynomial operator which puts into correspondence the trigonometric polynomial with 2π -periodic function, satisfying the following conditions: $I_{\pm}^{\alpha}(T_n)(t_k) = I_{\pm}^{\alpha}(\varphi)(t_k)$, where t_k – are equidistant nodes at $(-\pi;\pi)$. The check of the obtained formulae is performed using the system Wolfram Mathematica.

2. The construction of generalized polynomial operator *An*

Let's introduce the designation $H_n^T = \left\{ \frac{a_0}{2} + \sum_{k=-n}^{n} a_k \cos kt + b_k \sin kt \right\} = \left\{ \sum_{k=-n}^{n} c_k e^{iks} \right\}$ – the set of all trigonometric polynomials with the order no greater than n [7].

Let's denote a linear operator via A_n , the operator which puts into correspondence the polynomial $\forall \varphi(t) \in C_{2\pi}$ and the polynomial $T_n(t) \in H_n^T$, satisfying the following terms:

$$I_{\pm}^{\alpha}(T_n)(t_k) = I_{\pm}^{\alpha}(\varphi)(t_k),\ k = \overline{-n,n},\ (1)$$

where t_k is presented by equispaced nodes $t_k = \frac{2k\pi}{2n+1}$, $k = \overline{-n,n}$.

Let's consider the class $\varphi(t) \in C(0,2\pi)$, in such units where each free member $c_0 = 0$.

Let's examine A_n.

$I_{\pm}^{\alpha}(\varphi;t) = \sum_{|k|=1}^{\infty} \frac{c_k e^{ikt}}{(\pm ik)^{\alpha}}$ – Weil fractional integration operator [6].

Let's assume that $A_n(\varphi;t) = \varphi_n(t) = \sum_{|k|=1}^{n} c_k(\varphi_n) e^{ikt}$, where $c_k(\varphi_n)$ are complex Fourier ratios $\varphi_n(t)$. Then we obtain the following:

$$\sum_{|k|=1}^{n} \frac{c_k(\varphi_n) e^{ikt}}{(\pm ik)^{\alpha}} = I_{n\pm}^{\alpha}(\varphi)(t_j),\ t_j = \frac{2j\pi}{2n+1},\ j = \overline{-n,n}.$$

Let us consider the "left-hand" fractional integral in detail for certainty $I_{+}^{\alpha}(\varphi;t) = I^{\alpha}(\varphi;t)$ (note that $\Psi_{-}^{\alpha}(t) = \Psi_{+}^{\alpha}(t)$).

Let's prove the following theorem:

Theorem 1

Let's present the polynomial operator $A_n(\varphi;t)$ *in the following form:*

$$A_n(\varphi;t) = \frac{d^{\alpha}}{dt}(L_n(z;t)),\ (2),$$

where $L_n(z;t)$ *– Lagrange interpolation polynomial within the nodes* $t_{-n},...,t_n$ *for the function* $z(t) = I^{\alpha}(\varphi;t)$, $\varphi(t) \in C(0,2\pi)$.

Proof.

Let's represent A_n in the following form (2).

A polynomial operator satisfies the condition (1), therefore,

$$I^{\alpha}\left(\frac{d^{\alpha}}{dt} \left[L_n\left(I^{\alpha}(\varphi;t) \right) \right] \right)(t_k) = I^{\alpha}(\varphi;t_k),$$

$$t_k = \frac{2k\pi}{2n+1},\ k = \overline{-n,n}.$$

Let's consider the presentation $A_n(\varphi;t)$.

At $c_0 = 0$ $T_n(\varphi;t) = \sum_{|k|=1}^{n} c_k e^{ikt}$, then $I^{\alpha}(T_n) = \sum_{|l|=1}^{n} \frac{c_l e^{ilt}}{(il)^{\alpha}}$.

It is clear that $L_n(z;t) = L_n\lfloor I_{+}^{\alpha}(\varphi;t) \rfloor = I^{\alpha}(T_n;t)$.

Then $L_n(z;t) = \sum_{|k|=1}^{n} \frac{c_k e^{ikt}}{(ik)^{\alpha}}$,

According to definition $\frac{d^{\alpha}x}{dt}(t) = D^{\alpha}(x;t) = \sum_{|k|=1}^{\infty} (ik)^{\alpha} c_k(x) e^{ikt}$.

Therefore,

$$D^{\alpha}(L_n(z;t)) = \sum_{|k|=1}^{n} \frac{(ik)^{\alpha} c_k e^{ikt}}{(ik)^{\alpha}} = T_n(\varphi;t).$$

The statement is proved.

Let's obtain the type of polynomial operator A_n.

$$z(t_j) = T_n(z;t_j) = \sum_{|k|=1}^{n} \frac{c_k(\varphi) e^{ikt_j}}{(ik)^{\alpha}},$$

$$j = \overline{-n,n}.$$

$T_n(z;t)$ – the polynomial of the degree n.

$$T_n(z;t) = L_n(z;t),\ (3)$$

where $L_n(z;t)$ – Lagrange interpolation polynomial of n-th degree for the function $z(t)$ along the nodes $t_j = \frac{2j\pi}{2n+1}$, $j = \overline{-n,n}$.

At that there is no c_0 member.

Let's put down the Lagrange polynomial in the complex form

$$L_n(z;t) = \sum_{k=-n}^{n} e^{ikt} \frac{1}{2n+1} \sum_{j=-n}^{n} z(t_j) e^{-ikt_j}$$

,

Let's obtain from (3)

$$\sum_{|k|=1}^{n}\frac{c_k(\varphi)e^{ikt_j}}{(ik)^{\alpha}}=\sum_{k=-n}^{n}e^{ikt}\frac{1}{2n+1}\sum_{j=-n}^{n}z(t_j)e^{-ikt_j}.$$

Here we obtain the coefficients:

$$c_k=\frac{(ik)^{\alpha}}{2n+1}\sum_{j=-n}^{n}z(t_j)e^{-ikt_j},$$

$$k=\overline{-n,n}.$$

By substituting them into a necessary polynomial $T_n(\varphi;t)$, we obtain the following:

$$T_n(\varphi;t)=\sum_{|k|=1}^{n}c_k e^{ikt}=\sum_{|k|=1}^{n}e^{ikt}\frac{(ik)^{\alpha}}{2n+1}\sum_{j=-n}^{n}z(t_j)e^{-ikt_j}=\frac{1}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{|k|=1}^{n}(ik)^{\alpha}e^{ik(t-t_j)}.$$

i.e.

$$A_n(\varphi;t)=\frac{1}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{|k|=1}^{n}(ik)^{\alpha}e^{ik(t-t_j)}\quad(4)$$

So, the theorem is proved.

Theorem 2.

The linear operator that associates $\forall\varphi(t)\in C_{2\pi}$, the polynomial $T_n(t)\in H_n^T$ and satisfying the conditions (1), we obtain the following:

$$A_n(\varphi;t)=\frac{1}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{|k|=1}^{n}(ik)^{\alpha}e^{ik(t-t_j)}.$$

If we take into account that in (4) $(ik)^{\alpha}=|k|^{\alpha}e^{\frac{\alpha\pi i}{2}\operatorname{sgn}k}$, then the polynomial $A_n(\varphi;t)$ may be put down in the following form

$$A_n(\varphi;t)=\frac{1}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{|k|=1}^{n}|k|^{\alpha}e^{i\left[k(t-t_j)+\frac{\pi}{2}\alpha\operatorname{sgn}k\right]}.$$

We obtained the following formula

$$A_n(\varphi;t)=\sum_{|k|=1}^{n}(ik)^{\alpha}c_k^{(n)}\left(I^{\alpha}(\varphi;t)\right)e^{ikt}.$$

While $I^{\alpha}(\varphi;t)=z(t)$,

$$c_m(z)=\frac{c_m\left(D^{\alpha}(z)\right)}{(im)^{\alpha}}=\frac{c_m(\varphi)}{(im)^{\alpha}},$$

$z(const)=0$, $m\neq 0$.

$$c_k^{(n)}(z)=\frac{1}{2n+1}\sum_{j=-n}^{n}e^{-ikt_j}\sum_{|m|=1}^{\infty}c_m(z)e^{-imt_j}=\sum_{|\mu|=1}^{\infty}c_{k+\mu(2n+1)}(z)=\sum_{|\mu|=1}^{\infty}\frac{c_{k+\mu(2n+1)}(\varphi)}{\left(i[k+\mu(2n+1)]\right)^{\alpha}}.$$

Let's introduce $c_k^{(n)}(z)$ in $A_n(\varphi;t)$:

$$A_n(\varphi;t)=\sum_{|k|=1}^{n}(ik)^{\alpha}\sum_{|\mu|=1}^{\infty}\frac{c_{k+\mu(2n+1)}(\varphi)}{\left(i[k+\mu(2n+1)]\right)^{\alpha}}e^{ikt}=\sum_{|k|=1}^{n}e^{ikt}(ik)^{\alpha}\sum_{|\mu|=1}^{\infty}\left[\frac{k}{k+\mu(2n+1)}\right]^{\alpha}c_{k+\mu(2n+1)}(\varphi)$$

Let's offer the other formula (4), considering that the complex numbers may be represented in the trigonometric form.

$$A_n(\varphi;t)=\frac{1}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{|k|=1}^{n}(ik)^{\alpha}e^{ik(t-t_j)}$$

$$(ik)^{\alpha}=|k|^{\alpha}e^{\frac{\alpha\pi i}{2}\operatorname{sgn}k},$$

$e^{\frac{\alpha\pi i}{2}\operatorname{sgn}k}=\cos(\frac{\alpha\pi}{2}\operatorname{sgn}k)+i\sin(\frac{\alpha\pi}{2}\operatorname{sgn}k)$. Let's consider separately

$$\sum_{|k|=1}^{n}(ik)^{\alpha}e^{ik(t-t_j)}=\sum_{|k|=1}^{n}|k|^{\alpha}e^{\frac{\alpha\pi i}{2}\operatorname{sgn}k}e^{ik(t-t_j)}=$$

$$\sum_{|k|=1}^{n}|k|^{\alpha}\left(\cos\frac{\alpha\pi}{2}+i\sin(\frac{\alpha\pi}{2}\operatorname{sgn}k)\right)\left(\cos k(t-t_j)+i\sin k(t-t_j)\right)=$$

$$=\sum_{|k|=1}^{n}|k|^{\alpha}\left(\cos\frac{\alpha\pi}{2}\cos k(t-t_j)+i\cos\frac{\alpha\pi}{2}\sin k(t-t_j)+\right.$$

$$\left.+i\sin\frac{\alpha\pi}{2}\operatorname{sgn}k\cos k(t-t_j)-\sin\frac{\alpha\pi}{2}\operatorname{sgn}k\sin k(t-t_j)\right)$$

$$=\sum_{|k|=1}^{n}|k|^{\alpha}\left(\cos\frac{\alpha\pi}{2}\cos k(t-t_j)-\sin\frac{\alpha\pi}{2}\operatorname{sgn}k\sin k(t-t_j)+\right.$$

$$\left.+i\left(\cos\frac{\alpha\pi}{2}\sin k(t-t_j)+\sin\frac{\alpha\pi}{2}\operatorname{sgn}k\cos k(t-t_j)\right)\right)=$$

$$=\sum_{k=-n}^{-1}|k|^{\alpha}\left(\cos\frac{\alpha\pi}{2}\cos k(t-t_j)+\sin\frac{\alpha\pi}{2}\sin k(t-t_j)+i\left(\cos\frac{\alpha\pi}{2}\sin k(t-t_j)-\sin\frac{\alpha\pi}{2}\cos k(t-t_j)\right)\right)+$$

$$+\sum_{k=1}^{n}k^{\alpha}\left(\cos\frac{\alpha\pi}{2}\cos k(t-t_j)-\sin\frac{\alpha\pi}{2}\sin k(t-t_j)+i\left(\cos\frac{\alpha\pi}{2}\sin k(t-t_j)+\sin\frac{\alpha\pi}{2}\cos k(t-t_j)\right)\right)=$$

$$=\sum_{k=1}^{n}k^{\alpha}\left(\left(\cos\frac{\alpha\pi}{2}\cos k(t-t_j)-\sin\frac{\alpha\pi}{2}\sin k(t-t_j)+i\left(-\cos\frac{\alpha\pi}{2}\sin k(t-t_j)-\sin\frac{\alpha\pi}{2}\cos k(t-t_j)\right)\right)+\right.$$

$$\left.+\left(\cos\frac{\alpha\pi}{2}\cos k(t-t_j)-\sin\frac{\alpha\pi}{2}\sin k(t-t_j)+i\left(\cos\frac{\alpha\pi}{2}\sin k(t-t_j)+\sin\frac{\alpha\pi}{2}\cos k(t-t_j)\right)\right)\right)=$$

$$=\sum_{k=1}^{n}k^{\alpha}\left(2\cos\frac{\alpha\pi}{2}\cos k(t-t_j)-2\sin\frac{\alpha\pi}{2}\sin k(t-t_j)\right)=$$

$$=2\sum_{k=1}^{n}k^{\alpha}\cos\left(\frac{\alpha\pi}{2}+k(t-t_j)\right).$$

Another polynomial representation was obtained $A_n(\varphi;t)$:

$$A_n(\varphi;t)=\frac{1}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{|k|=1}^{n}(ik)^{\alpha}e^{ik(t-t_j)}=$$

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$$=\frac{2}{2n+1}\sum_{j=-n}^{n}I^{\alpha}(\varphi;t_j)\sum_{k=1}^{n}k^{\alpha}\cos\left(\frac{\alpha\pi}{2}+k(t-t_j)\right). \qquad (4')$$

3. The relation of *An* C operator with the Fourier operator and the evaluation of error

Let's consider the relationship of the generalized interpolation operator A_n with Fourier operator S_n.

$$S_n(\varphi;t)=\sum_{k=-n}^{n}c_k(\varphi)e^{ikt}=\frac{1}{\pi}\int_0^{\pi}D_n(t-x)\varphi(x)dx,$$

where $D_n(t)=\dfrac{\sin(2n+1)\frac{t}{2}}{\sin\frac{t}{2}}$ – the Dirichlet kernel.

$$c_k(\varphi)=\frac{1}{2\pi}\int_0^{2\pi}\varphi(x)e^{-ikt}dt.$$

Theorem.

For any function $\varphi(t)\in C_{2\pi}$ the following formula is a true one:

$$A_n(\varphi;t)-S_n(\varphi;t)=\sum_{|k|=1}^{n}e^{ikt}\sum_{|\mu|=1}^{\infty}\left[\frac{k}{k+\mu(2n+1)}\right]^{\alpha}c_{k+\mu(2n+1)}(\varphi)\quad \forall\varphi(t)\in C_{2\pi}$$

,(5)

At that the series (5) is similar on the average.

Proof.

For any function $\varphi(t)\in C_{2\pi}$, that has the order derivative α, the following formulae are the true ones:

$$\frac{d^{\alpha}}{dt}S_n(\varphi(x);t)=S_n\left\{\frac{d^{\alpha}}{dt}\varphi(x);t\right\}.$$

$$S_n\varphi-A_n\varphi=\frac{d^{\alpha}}{dt}\left[S_n(z_0;t)-L_n(z_0;t)\right].$$

$$\varphi_0(t)=\varphi(t)-c_0(\varphi),$$

$$z_0(t)=I^{\alpha}(\varphi_0;t),\ \varphi\in C_{2\pi}.$$

$$S_n(z_0;t)-L_n(z_0;t)=\sum_{k=-n}^{n}\left(c_k(z_0)-c_k^{(n)}(z_0)\right)e^{ikt}.$$

$$S_n(\varphi_0;t)-A_n(\varphi_0;t)=\sum_{k=-n}^{n}(ik)^{\alpha}\left(c_k(z_0)-c_k^{(n)}(z_0)\right)e^{ikt}.$$

$$\varphi_0(t)=\frac{d^{\alpha}}{dt}z_0(t).$$

$$c_k(z_0)-c_k^{(n)}(z_0)=c_k(z_0)-\sum_{\mu=-\infty}^{\infty}c_{k+\mu(2n+1)}(z_0)=-\sum_{|\mu|=1}^{\infty}c_{k+\mu(2n+1)}(z_0)=-\sum_{|\mu|=1}^{\infty}\frac{c_{k+\mu(2n+1)}(\varphi)}{\left(i\left[k+\mu(2n+1)\right]\right)^{\alpha}}.$$

$$A_n(\varphi;t)-S_n(\varphi;t)=\sum_{|k|=1}^{n}(ik)^{\alpha}\sum_{|\mu|=1}^{\infty}\frac{c_{k+\mu(2n+1)}(\varphi)}{\left(i\left[k+\mu(2n+1)\right]\right)^{\alpha}}e^{ikt}=$$

$$=\sum_{|k|=1}^{n}\sum_{|\mu|=1}^{\infty}\left[\frac{k}{\left[k+\mu(2n+1)\right]}\right]^{\alpha}e^{ikt}c_{k+\mu(2n+1)}(\varphi).$$

Theorem.

$A_n(\varphi;t)\ \forall\varphi(t)\in C_{2\pi},\ \frac{1}{2}<\alpha<1$ is similar on the average at the rate of

$$E_n^T(\varphi)_C\le\|\varphi-A_n\varphi\|_C\le\sqrt{1+\sigma_n^2}\,E_n^T(\varphi)_C,$$

$$\sigma_n^2=\max\sum_{\mu=1}^{\infty}\left|\frac{k}{k+\mu(2n+1)}\right|^{2\alpha}.$$

Proof.

Due to (5) and the fact that

$$A_n(\varphi;t)=\sum_{|k|=1}^{n}e^{ikt}(ik)^{\alpha}\sum_{|\mu|=1}^{\infty}\left[\frac{k}{k+\mu(2n+1)}\right]^{\alpha}c_{k+\mu(2n+1)}(\varphi)$$

$$\varphi-A_n\varphi=(\varphi-S_n\varphi)+(S_n\varphi-A_n\varphi)=\sum_{|k|=n+1}^{\infty}c_k(\varphi)e^{ikt}-\sum_{|k|=1}^{n}\sum_{|\mu|=1}^{\infty}\left[\frac{k}{k+\mu(2n+1)}\right]^{\alpha}c_{k+\mu(2n+1)}(\varphi).$$

As $S_n\varphi-A_n\varphi\in H_n^T$, $\varphi-S_n\varphi\notin H_n^T$, then their scalar product makes zero:

$$\left(\varphi-S_n\varphi,S_n\varphi-A_n\varphi\right)=0,\ \varphi\in C_{2\pi}.$$

Therefore,

$$\|\varphi-A_n\varphi\|_C^2=\|\varphi-S_n\varphi\|_C^2+\|S_n\varphi-A_n\varphi\|_C^2=$$

$$=\sum_{|k|=n+1}^{\infty}|c_k(\varphi)|^2+\sum_{|k|=1}^{n}\left|\sum_{|\mu|=1}^{\infty}\left[\frac{k}{k+\mu(2n+1)}\right]^{\alpha}c_{k+\mu(2n+1)}(\varphi)\right|^2\le$$

$$\le\sum_{|k|=n+1}^{\infty}|c_k(\varphi)|^2+\sum_{|k|=1}^{n}\left\{\sum_{|\mu|=1}^{\infty}\left|\frac{k}{k+\mu(2n+1)}\right|^{2\alpha}\sum_{|\mu|=1}^{\infty}\left|c_{k+\mu(2n+1)}(\varphi)\right|^2\right\}\le$$

$$\le\sum_{|k|=n+1}^{\infty}|c_k(\varphi)|^2+\sigma_n^2\sum_{|k|=1}^{n}\sum_{|\mu|=1}^{\infty}\left|c_{k+\mu(2n+1)}(\varphi)\right|^2\le$$

$$(1+\sigma_n^2)\sum_{|k|=n+1}^{\infty}|c_k(\varphi)|^2=(1+\sigma_n^2)\left\{E_n^T(\varphi)_C\right\}^2,$$

$$\varphi\in C_{2\pi},$$

where the following sum is designated by σ_n^2:

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$$\sigma_n^2 = \max \sum_{\mu=1}^{\infty} \left| \frac{k}{k+\mu(2n+1)} \right|^{2\alpha}.$$

4. CALCULATIONS IN WOLFRAM MATHEMATICA SYSTEM

In this section let's present calculation results carried out in the Wolfram Mathematica system. This system contains a number of functions for the interpolation, it is often used in the calculations [8], [9]. In order to test our formulas the following function is taken in the first case: $\varphi(t) = \sin 3{,}6t$. The polynomial $A_n(\varphi;t)$ values are calculated according to the formula (4′) for $n = 8$ and $n = 50$ at $\alpha = 0{,}6$. The figures 1, 2 show the corresponding graphs.

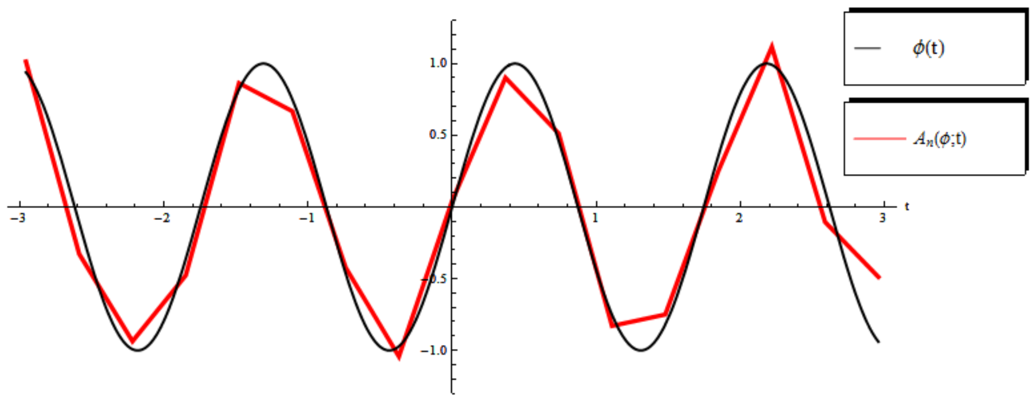
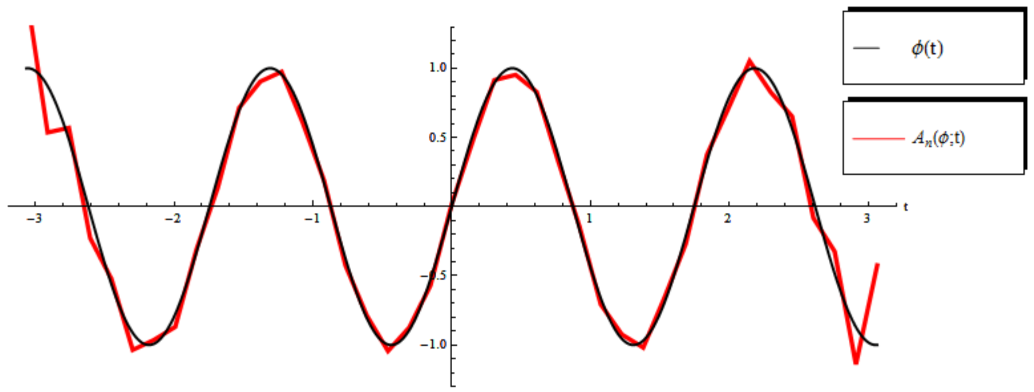
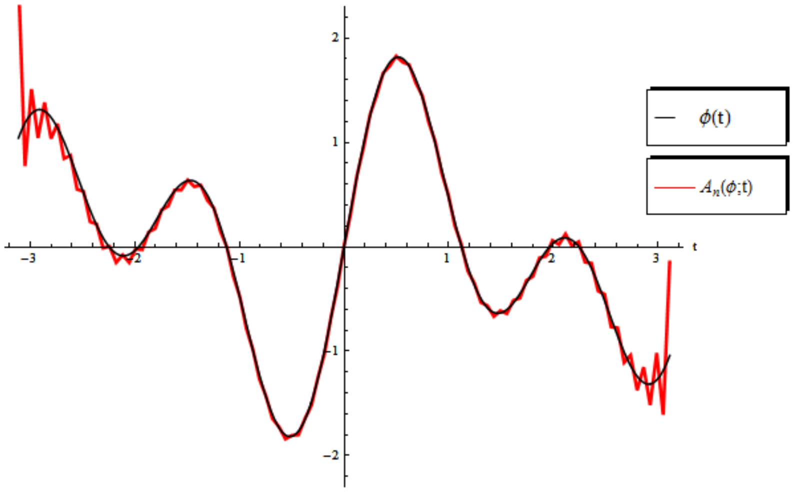


Fig (1). The function $\varphi(t) = \sin 3{,}6t$ approximation by the polynomial $A_n(\varphi;t)$ at $n = 8$

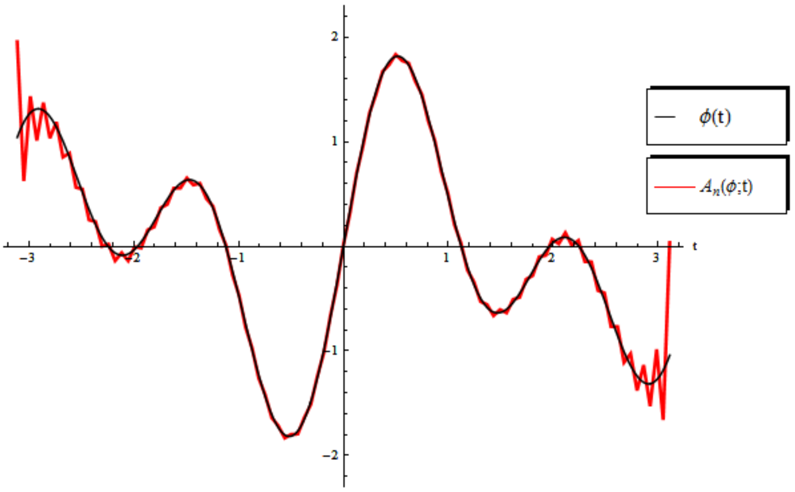


Fig(2). The approximation of the function $\varphi(t) = \sin 3{,}6t$ by polynomial $A_n(\varphi;t)$ for $n = 20$

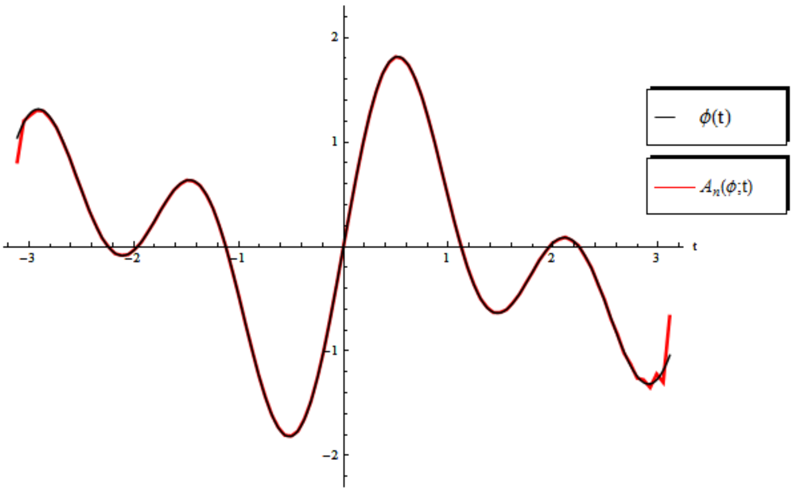
The figures 3, 4, 5 represent the function $\varphi(t) = \sin 3{,}6t + \sin 2t$ approximation by the polynomial $A_n(\varphi;t)$ at $n = 50$ and at different values of α.



Fig(3). The approximation of the function $\varphi(t) = \sin 3{,}6t + \sin 2t$ by polynomial $A_n(\varphi;t)$, $n = 50$, $\alpha = 0{,}6$



Fig(4). The approximation of the function $\varphi(t) = \sin 3{,}6t + \sin 2t$ by polynomial $A_n(\varphi;t)$, $n = 50$, $\alpha = 0{,}8$



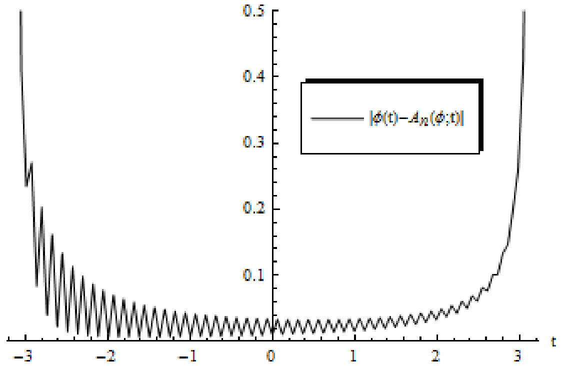
Fig(5). The approximation of the function $\varphi(t) = \sin 3{,}6t + \sin 2t$ by polynomial $A_n(\varphi;t)$, $n = 50$, $\alpha = 0{,}99$

The maximum values of $|\varphi(t) - A_n(\varphi;t)|$ are calculated at different values of α for the function $\varphi(t) = \sin 3{,}6t + \sin 2t$, $n = 50$. Two cases are considered – when the whole interval and the medium part of interval are taken. The results are presented in Table 1.

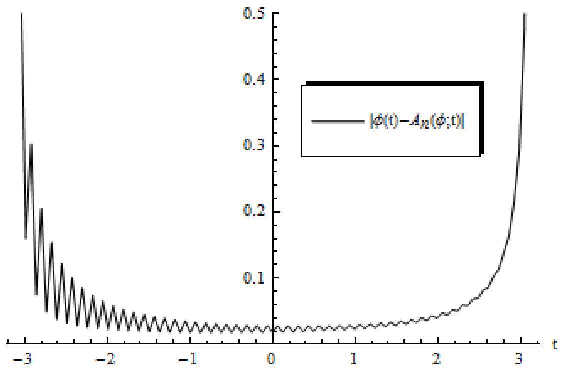
Table 1. Maximum values $|\varphi(t) - A_n(\varphi;t)|$ at $n = 50$, $\varphi(t) = \sin 3{,}6t + \sin 2t$

α	0.6	0.7	0.8	0.9	0.95	0.99
$\max\|\varphi(t_i) - A_n(\varphi;t_i)\|, i = \overline{-n,n}$	1.53177	1.32091	1.07917	0.87513	0.64192	0.37329
$\max\|\varphi(t_i) - A_n(\varphi;t_i)\|\ i = \overline{-\frac{n}{2},\frac{n}{2}}$	0.05513	0.0533	0.04498	0.02799	0.01543	0.00326

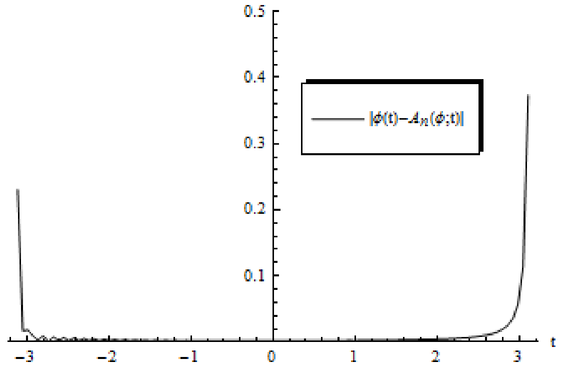
The figures 6, 7, 8 demonstrate the values $|\varphi(t) - A_n(\varphi;t)|$ at different α.



Fig(6). The values $|\varphi(t) - A_n(\varphi;t)|$ at $\alpha = 0{,}6$, $n = 50$



Fig(7). The values $|\varphi(t) - A_n(\varphi;t)|$ at $\alpha = 0{,}8$, $n = 50$



Fig(8). The values $|\varphi(t)-A_n(\varphi;t)|$ at $\alpha=0{,}99$, $n=50$

5. CONCLUSION

The difference module between the values of 2π - periodic function with a zero mean value and trigonometric polynomial values, constructed with the help of the operator $A_n(\varphi;t)$, decreases at α increase (the following values were considered: $0{,}5<\alpha<1$). The value of this module is less if you take the middle part of the interval.

6. SUMMARY

Thus, the formulas are derived for the representation of an operator, which associates trigonometric polynomial and 2π - periodic function with a zero mean value. The approximation of functions by polynomial is checked using computer algebra system Wolfram Mathematica. The work in the system showed that an increase in the number of nodes n improves the function approximation, which is natural for such tasks. We considered the values $0{,}5<\alpha<1$. Also lists the maximum values of the difference module between the values of the initial function and the values of a polynomial are presented. According to these data, one can see the dependence of difference module values from the value α .

7. CONFLICT OF INTEREST

The authors confirm that the presented data do not contain any conflict of interest.

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