

# Calculation of the Elastic Characteristics of Composite Materials with Dispersed Inclusions

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## Abstract

The research is aimed at the development of the technique of evaluating elastic properties of composite materials with dispersed inclusions. The functioning of the matrix and the inclusions is taken into account in a linearly elastic formulation. It is roughly assumed that at the macro level the material is isotropic. Deformation of the objects of such type at the micro level is analyzed in the three-dimensional statement with the use of the finite element method. For the purposes of the research, dispersed particles are represented as spheres arranged in a cubic lattice. Based on the assumption of the approximate circular symmetry of the system, the research of deformations in an area of the composite material in the form of a cube which has one of its corners located in the center of the inclusion is carried out. A scheme of kinematic and force-based boundary conditions for the cube subject to uniform stretching at the macro level is suggested. The adhesion of the dispersed phase with the matrix of the composite was assumed as ideal. The dependence of the elastic properties of cobalt-tungsten carbide composite on the volume content of particles of the disperse phase was determined on the basis of the constructed finite element representation. The comparison of the obtained theoretical results with experimental data showed a rather high accuracy of the proposed approach to the calculation of the modulus of elasticity of the material.

**Keywords:** Composite material, Discrete inclusions, Finite element method, Modulus of elasticity, Poisson's ratio

## Introduction

Issues related to the study of the properties of composites with dispersed inclusions, the practice of their manufacture and use are discussed in a number of papers [1-11]. The development of such materials is an important area intended to meet the growing demands of modern technology to reliability and durability of construction materials.

Elastic characteristics of composite materials are generally determined by mechanical testing [1-4]. Obtaining such data in many cases is often quite a labor-consuming task, which is why it is relevant to develop the methods for the calculation of the elastic properties of composites. Theoretical estimations of the strength and rigidity characteristics of composite materials were examined in a number of works [4-6, 12-15]. The most widely used methods in this field of study are approximate methods (the rule of mixes, power method, Rayleigh's method, etc. ), the accuracy of which is not quite satisfactory for researchers [4, 5].

One of the methods which is now becoming increasingly popular for the theoretical analysis of the materials properties is the finite element method [12-15], which basically allows us to make more precise calculation of the physicomaterial properties of composites. The aim of this study is to develop an efficient scheme of the application of the finite element method in order to determine the elastic properties of composite materials with dispersed inclusions.

The composites of this kind are actually irregular formations which contain randomly arranged inclusions of complex shape [1-4, 16, 17]. However, in the theoretical study of the

mechanical properties of these materials it is acceptable to represent the disperse particles as spheres with the equivalent diameter  $d$  [1-4, 18].

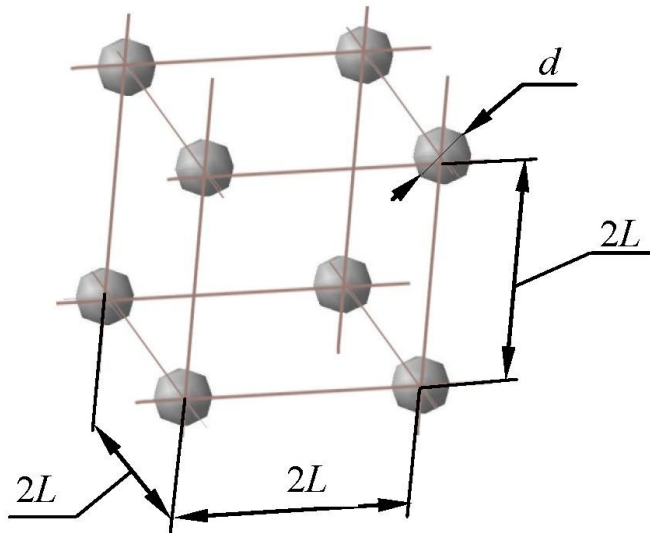
The volume content of the inclusions in the most durable and wear-resistant composites does not exceed 0, 3...0, 4 [1-3, 7], which makes it possible to explore the mechanical properties of composite materials with the use of the simplest model, which consists of monodisperse spheres in a simple cubic lattice and for which the maximum filling of the space with the spheres is 0, 5235. In such a lattice, the percentage of the volume content of the filler will be determined by the ratio  $V_d = (\pi/6)(R/L)^3 \times 100$ , vol % where  $R$  is the equivalent radius of particles and  $2L$  is the distance between the particles. The values of  $V_d$  for the different ratios of  $R/L$  are given in the table.

**Table: Volume content of the inclusions in a cubic lattice**

$R/L$	0, 1	0, 2	0, 3	0, 4	0, 5
$V_d$ , vol %	0, 052	0, 41	0, 45	3, 35	6, 54
$R/L$	0, 6	0, 7	0, 8	0, 9	1
$V_d$ , vol %	11, 31	17, 96	26, 81	52, 35	52, 35

#### Methodology for Constructing Computational Models

It was assumed that at the macro level the material is isotropic and linearly elastic. At the micro level, we examined the matrix and the inclusions of the composite separately, describing them as isotropic linear elastic mediums. It was assumed that the adherence of the dispersed phase to the matrix of the composite is ideal. The dispersed particles were represented in the form of identical spheres arranged in a simple cubic lattice (Fig. 1).



**Fig. 1. Composite structure in the form of a cubic lattice**

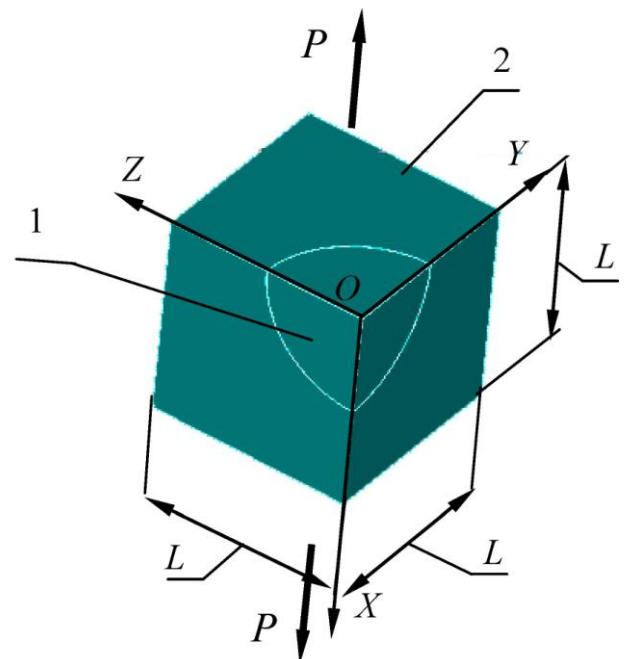
Let a straight rod made of composite material with dispersed inclusions is stretched under the action of two forces, uniformly distributed over its butt ends. We assume that one of the systems of unidirectional edges of the composite lattice

is parallel to the axis of the rod. Due to the assumed cyclic symmetry in the framework of this problem, we limited our calculations to a part of the area of the composite in the form of a cube with side  $L$ , one corner of which is located in the center of the inclusion (Figure 2). The selected part of the composite is subjected to a tensile force  $P$ .

The condition of cyclic symmetry implies that upon deformation the faces of the cube remain flat, and the angles between them do not change. On each face  $X=0, Y=0$  or  $Z=0$  (Fig. 3), zero displacements were assumed along the normal line to the plane. On plane  $X=L$ , the displacement of  $\Delta L_x$  was introduced. Thus, on  $Y=L$  or  $Z=L$  faces the displacements of  $\Delta L_y = \Delta L_z$  must be constant over the entire area. The relationship between the tensile force  $P$  and deformation  $\Delta L_x$  can be represented in the formula  $\Delta L_x = PL / (E_k A)$ , where  $E_k$  is the effective modulus of elasticity of composite, and  $A$  is the surface area of a face of the cube. By setting the displacement  $\Delta L_x$  and calculating force  $P$  for it by using the finite element method, we can find the value of  $E_k$ . Poisson's ratio of the material  $\mu_k$  in such case will be determined by the dependence  $\mu_k = \Delta L_z / \Delta L_x$ .

The creation of solid geometric objects and operations with them was carried out with the help of procedures and solid modeling functions in the free version of MSC/NASTRAN for Windows [4] software package. Fig. 4 shows the finite element model of the composite with volume content of the dispersed phase  $V_d = 6, 2$  vol %.

The adjoining final elements of spherical inclusion and the base of the composite had common nodes at the interface between phases (see Fig. 4). On each of the faces of  $X=L, Y=L$  or  $Z=L$  (Fig. 5) we grouped all the nodes according to the degree of freedom of their movement along the normal line to the surface by using elements of rigid type [4].



**Fig. 2. The considered volume of the composite material: 1 – inclusion; 2 – matrix**

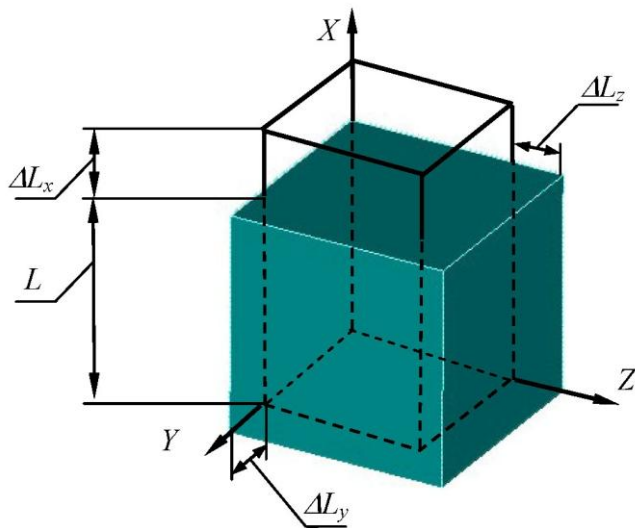


Fig. 3. Deformation of the composite model

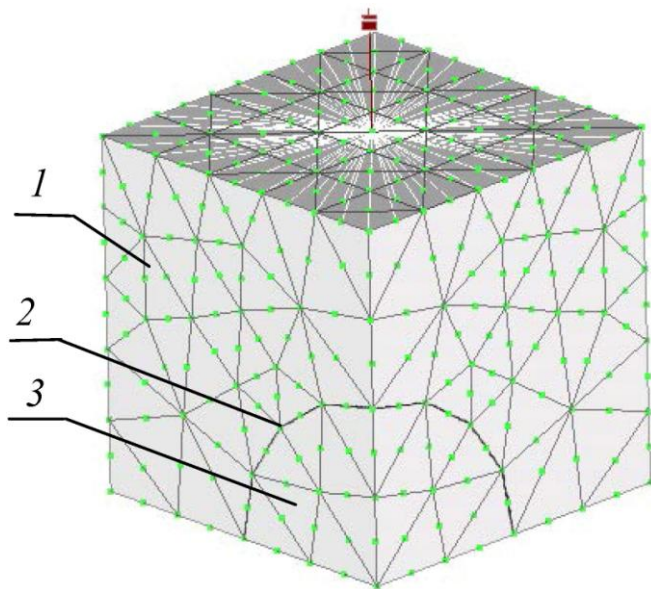


Fig. 4. Finite element model of composite: 1 – matrix; 2 – interface of base and inclusion; 3 – solid inclusion

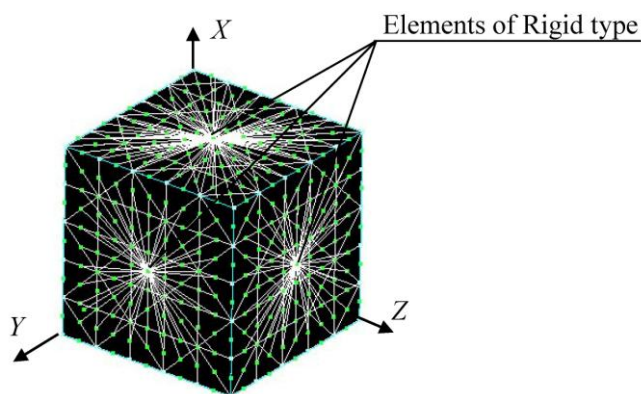


Fig. 5. Application of elements of rigid type for binding nodes on the edge of the model

### The Example of Determining Elastic Parameters of Composite Material

As an example, we provide some of the results of the calculation of the composite of cobalt (matrix) – tungsten carbide (dispersion phase). The modulus of elasticity for the matrix material was taken as  $E_m=211000$  MPa, Poisson's ratio  $\mu_m =0, 25$ ; for the material of the dispersed phase – the modulus of elasticity  $E_d= 430000$  MPa, Poisson's ratio  $\mu_d =0, 3$  [2].

The resulting distribution pattern of the von Mises equivalent stresses [5]  $\sigma_M$  at  $V_d=6, 2$  vol % is shown in the form of stress contour lines (Fig. 6), where tensions are represented by a relative value  $\sigma_o=\sigma_M/\sigma_{av}$ . Here  $\sigma_{av} =P/L^2$  is the mean value of normal stresses  $\sigma_x$ .

To determine the elastic characteristics of the material, a number of models with volume fraction of the dispersed phase from 0, 05 vol % to 52, 35 vol % were examined. The comparison of the results of calculation of the elastic modulus and Poisson's ratio of the composite material with the use of the finite element method with the results established by other theoretical methods and experimental data is shown in Fig. 7.

Fig. 7, a shows that the results of calculation of the elastic modulus of the composite which were the closest to the experimental data were those obtained by the use the finite elements method. The deviation from the experimental values of  $E_k$  at  $V_d=30$  vol % for this method was 5 %, at  $V_d=50$  vol % – 2%. The variance between the values of the Poisson's ratio for the composite material which were obtained by different theoretical methods is insignificant (see Fig. 7, b).

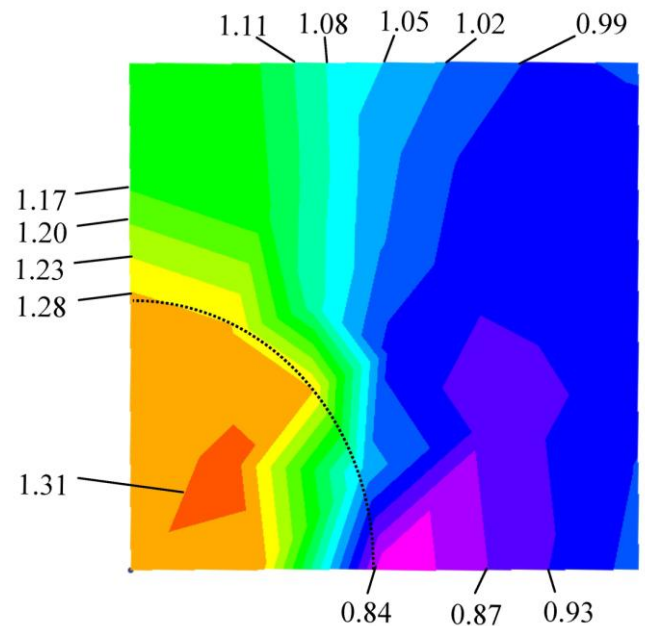
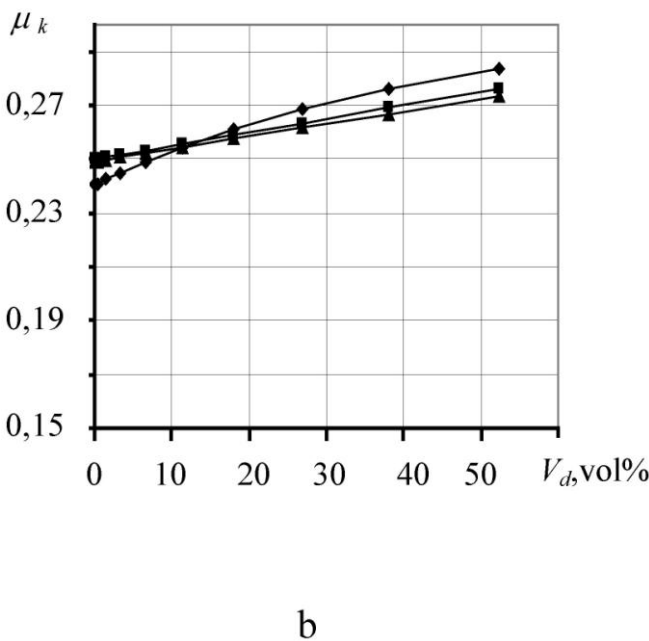
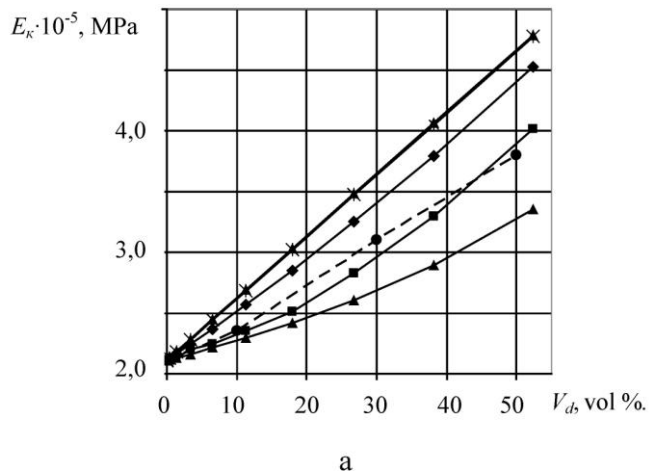


Fig. 6. Contour lines of relative stresses on the face of  $Y=L$  of the composite model



- ◆ - Rayleigh's method;
- ▲ - power method (top and lower borders);
- × - rule of mixes;
- - finite element method;
- - experimental values

**Fig. 7. Values of the modulus of elasticity (a) and Poisson's ratio (b) depending on the volume content of the dispersed phase**

**Conclusion**

A scheme for the formation of finite element models for the calculation of the elastic parameters of composite materials with discrete inclusions is suggested. The sufficiently high precision of the measurements made with the use of the developed technique is confirmed by the comparison of the results of theoretical research with experimental data reported in the literature. The presented approach allowed us to obtain

significantly more accurate results in comparison with Rayleigh's method, power method and the rule of mixes.

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