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Measuring The Position Of Current-Carrying Conductor Based On Mathematical Model Ajusted By The Method Of Full-Scale Model Experiment

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Abstact-The article deals with the issues of measuring the position of plasma piston for controlling its travel and diagnosing magnetoplasma accelerator as complex engineering system. It is solved the inverse problem of determining a position of linearly moving conductor with currentfor any instant time on the known values of pick-off signals reacting to magnetic field of movable conductor. It has been suggested the mathematical model of the sensors allowing tosynthesize special functions of aggregate transformation of values for the purpose of obtaining a resultant function depended only on the dimension of moving conductor with current. For adjusting the model in the process of measuring it is used the method of full-scale model experiment. The conducted experimental research has shown that application of such approach enables to increase accuracy of measuringthe position of moving conductor with current in real time.

Keywords:movingconductor with current, positioncoordinate measuring, approximation offunctional dependence, inverse problem, full-scale model experiment

1. INTRODUCTION

In many technical applications it arises the problem of measuring mechanical position of the source of magnetic fields. The examples may be traverse-metering systemswith the use of magnetometricsensors (magnetoresistorsorHall sensors) andpermanent magnets [1 – 3].

Another example is the problem of measuring the coordinates of position and plasma piston speed in magnetoplasma accelerator [4 – 8]. Plasma piston is, in reality, a moving conductor with current, that is, a moving source of magnetic field. It moves linearly along two conducting slide-rails, and current source is a special high-voltage storage device.High-voltage source, slide-rails and plasma piston make up a closed circuit (figure1).

Topicality of the research described in this article is conditioned by the following. The existing methods of measuring the position of objects using magnetometric sensors presuppose that spatial configuration of the source of magnetic field and its magnetic characteristics are known a priori. At the same time, this condition cannot be fulfilled applicable, for example, to the mentioned problem of movement control of piston in magnetoplasma accelerator, as the plasma piston is a dynamic object with variable and weakly predictable special configuration and, consequently, magnetic characteristics.

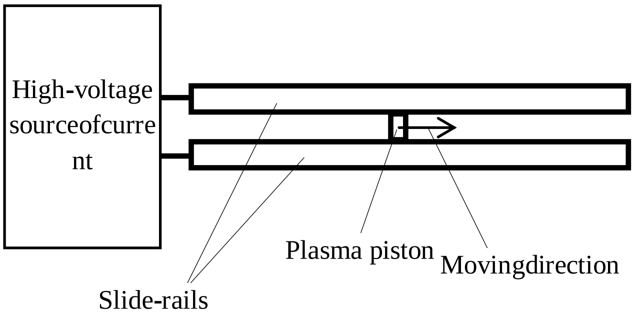

Fig.(1). – Aggregated Structure Chart of MagnetoplasmaAccelerator

The primary necessity of measuring the coordinate position and speed of plasma piston movement is connected with the possibility of the controlled acceleration of plasma, that is, provision ofcommanded speed level of plasma at the output of the accelerator.

The secondary necessity of measuring the coordinate position and speed of plasma piston movement is connected with the possibility of diagnosis oftechnical state of the magnetoplasma accelerator. Time functions of the coordinate position and speed of plasma piston movement in the process of acceleration are its integral diagnostic indicators.

Therefore, the problem of measuring the moving plasma piston (current-carrying conductor) can be considered as subtask of diagnosing magnetoplasma accelerator as a complex technical unit.

2. THE METHOD OF MEASURING THE COORDINATE OF POSITION OF CURRENT-CARRYING CONDUCTOR

It is handled the problem of measuring the coordinate of position of linearly moving conductor with current. The moving conductor is a source of moving magnetic field. If to place Hall sensor at a certain fixed point of the space near the trajectory of movement the conductor, its signal will be dependent on magnetic flux density of thestimulated field, and variation of magnetic induction as a result of movement of the conductor will lead to the change of pickup signal. Therefore, Hall sensor can primary measuring transformer of the coordinate of current-carrying conductor position.

The sources of magnetic field in the accelerator (Figure 1),along with the moving plasma, are slide-rails.For measuring the field created by the current-carrying conductor only, an active surface of Hall sensor is placed perpendicularly to the trajectory of motion.

If to represent the moving conductor with current by rectilinear leg of finite length, then the normal component of magnetic induction $B_n$to active surface of the sensor is described by the expression:

$$B_n(x) = \frac{\mu_0}{4\pi} i \frac{yz}{(x - X_d)^2 + y^2} \frac{1}{\sqrt{(x - X_d)^2 + y^2 + (z/2)^2}}, \quad (1)$$

whereμ_0 – space permeability; i – current strength in moving conductor; x – the coordinate of position of conductor on the axis being parallel to the trajectory of motion; y – perpendicular linear distance to the trajectory from the line to the sensor; z – length of the current-carrying conductor; X_d– dimension of the sensor.

More generally, to have possibility of measuring the coordinate of position of the conductor it is needed for Nsensors, placed in the line of the trajectory of its traveling.

Let us theoretically describe the functionalinterconnection of the coordinate of position of moving conductor and the signals of Hall sensors placed in the line of the trajectory of motion and the conductor reacting to field, in the form of direct and inverse problems.

The direct problem is determination of thesignals of sensors according to the known coordinate of the position of the moving conductor with current.In generalized form with account taken of the expression (1) the signal of Hall sensor represents the function

$$U_{d_j}(x(t)) = \gamma i_d \frac{\mu_0}{4\pi} i \frac{yz}{(x(t) - jX_0)^2 + y^2} \frac{1}{\sqrt{(x(t) - jX_0)^2 + y^2 + (z/2)^2}}, \quad (2)$$

where t –time; γ – the coefficient of sensitivity of Hall sensor; i_d – current strength in controlling circuit of the sensor;j – sensor number, $j = 1,2...,N$; $U_{d_j}(x(t))$– the signal of j-th sensor;X_0 – distance between neighboring sensors.

The inverse problem is determination of

dimension of the moving conductor with current for any moment with accounts taken of the known pick-off signals of the sensors reacting to the field of the moving conductor.

Let us write the analytic representation of the inverse problem in the generalized form:

$$x(t) = \chi(U_1(t), U_2(t), \ldots, U_N(t)),$$

(3)

where χ –a certain functional of the inverse transformation of values of the pick-off signals of the sensor to the dimension of the conductor.

Expansion of the functional $\chi$is, in effect, realization of the method of measuring of the coordinates of position of the moving conductor with current.

Dependence of the signal of Hall sensor on the coordinate of sensor position has the form being symmetrical relative to the axis of ordinates (Figure 2).Peak of the curve corresponds to the coordinate $X_d$of sensor point. Thebranch of this function in sub-range of the values x, exceeding the coordinate X_d,is approximated by exponential expression:

$$U_d^*(x(t))\Big|_{x \geq X_d} = Ae^{-p(x(t)-X_d)},$$

(4)

where $U_d^*(x(t))$–approximable function of Hall sensor; $A$and$p$ –coefficient of approximation, Ahasdimension [B], p – dimension [1/m].

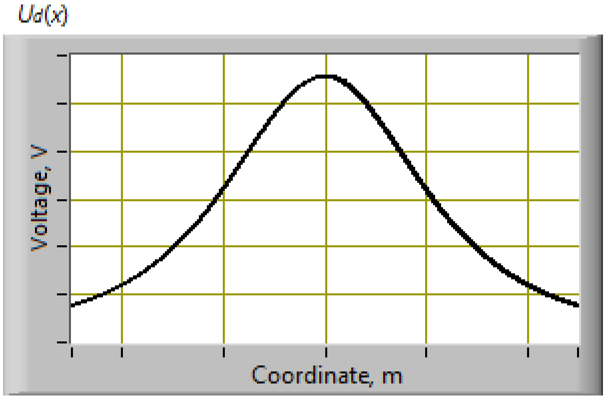

Fig.(2).– The Function of the Signal of Hall Sensor Reacting to the Field of the Moving Conductor

Using approximation (4), in a generalized form let us write the system of equations describing both branches of dependences of the signal of Hall sensor on the coordinate of conductor position:

$$\begin{cases} U_{d_j}^*(x(t))\Big|_{x \geq jX_0} = Ae^{-p(x(t)-jX_0)}; \\ U_{d_j}^*(x(t))\Big|_{x < jX_0} = Ae^{-p(jX_0-x(t))}. \end{cases}$$

(5)

The system of equations (5) is a suggested mathematical model of Hall sensors total distributed in the line of the trajectory of moving conductor with current and reacting to its field.

In the literature [4,9] for measuring the coordinate of position and speed of movement of the current-carrying conductor there the method of coordinate function has been suggested. This generic name is for a group of methods based on different types of pick-off signal approximation. For determination of the coordinate of conductor position on the values of signals of two sensors it is calculated a special function $F(x(t))$ (coordinate function), which depends only on the coordinate of conductor position.

In the works [10,11] it isapplied differential-logarithmic coordinate function connected with approximation of the functions of pick-off signals, with exponential system functions(5). The values of the function of the coordinate $x(t)$ is calculated according to the formula:

$$F(x(t)) = \ln \frac{U_{d_2}(x(t))}{U_{d_1}(x(t))}.$$

(6)

By calculating of the coordinate function in the first sensor, according to the motion of current-carrying conductor, it isused the right branch of the function of a signal value, and in the second sensor – the left branch (Figure 3).

With account taken of the model (5) the signals of the first and second sensors are approximated by the functions:

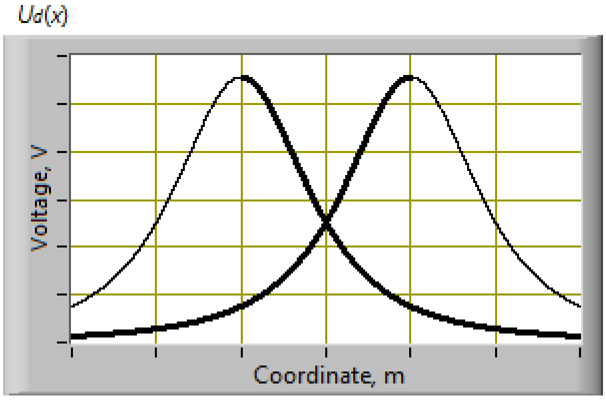

Fig.(3). – The Functions of Two Signals of Hall Sensors Shifted in the Line of the Trajectory of Conductor Motion and Reacting to its Field

$$\begin{cases} U^*_{d_1}(x(t))\Big|_{x\ge jX_0} = Ae^{-p(x(t)-jX_d)}; \\ U^*_{d_2}(x(t))\Big|_{x< jX_0} = Ae^{-p(jX_d-x(t))}. \end{cases} \quad (7)$$

Formula (6) is transformed in

$$F^*(x(t)) = \ln\frac{Ae^{-p(X_d-x(t))}}{Ae^{-p(x(t)-X_d)}} = -p\left(X_{d_1}+X_{d_2}\right)+2px(t),\quad (8)$$

where $F^*(x(t))$–the coordinate function, obtained with using approximating functions (7).

In the equation (8) the first termis a constant, and the second member is a linear function.Using designations: $F_0 = -p\left(X_{d_1}+X_{d_2}\right); k = 2px(t)$, the formula (8) is represented in the form:

$$F^*(x(t)) = F_0 + kx(t). \quad (9)$$

Solving the equation (9), the coordinator of position of the moving conductor with current is calculated by the formula

$$x(t) = \frac{F^*(t)-F_0}{k}. \quad (10)$$

Thus, taking into consideration (6) and (10),for measuring the coordinate of position of the moving conductor with current it is used the mathematical model

$$\begin{cases} F(x(t)) = \ln\frac{U_{d_2}(x(t))}{U_{d_1}(x(t))}; \\ x(t) = \frac{F^*(t)-F_0}{k}, \end{cases} \quad (11)$$

where$F_0$and$k$ – constants,being correspondingly the level of displacement and coefficient of linear function (10). They condition amethod error of measurement of the position of moving conductor with current.

To determine the values $F_0$and$k$it is fulfilled a priori modeling of the moving conductor with current and the sensors reacting to its field.As a result,a method error will be determined by degree of reliability of the mathematical model being used.To solve the mentioned problem it is suggested further using Hall sensors for adjustment of mathematical model in the process of measuring the method of full-scale model experiment [12 – 14].

3. AJUSTMENT OF THE MATHEMATICAL MODEL OF SENSORS

Adjustment of the mathematical model of sensors of the magnetic field by the method of full-scale method experiment consists in ajusting the coefficients of expressions approximating the pick-off signals functions. Such coefficients are the parameters $A$and$p$in the systems (5) and (7). The coefficients of approximation $A$and$p$are adjusted implicitly in consequence ofcalculation of$F_0$and$k$of (10) and (11).

Experimental determination $F_0$and$k$is conditioned by the possibility of discrete determination of the coordinate of position of the conductor regardless of the constant coefficients of approximation. When the conductor is placed strictly between two sensors, their signals will have equal values. The very fact of equation of the values of two sensors is indicative of the conductor in a certain known coordinate (middle between the sensors), but not the value of coordinate which may a priori be unknown.

For measuring the position of moving conductor let us make use ofthe system consisting of three Hall sensors (HS1 – HS3), distributed in the line of the trajectory of motion with equal intervals

$x_0$between the sensors(Figure 4).

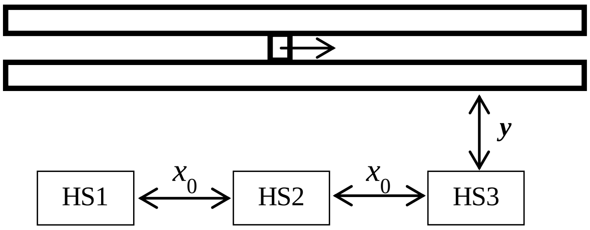

Fig.(4). – Disposition of Three Sensors Relative the Trajectory of Conductor Motion

The sensors HS1 andHS3 are used for measuring the coordinate of position of the moving conductor, and the sensors HS1 andHS2 – for controlling the coordinate of position corresponding to the midpoint between these sensors by the moving conductor.

Thus, differential and logarithmic coordinate function will be calculated from the values of signals of the sensors HS1 andHS3. The formula (6)will take the form:

$$F(t) = \ln \frac{U_{d_3}(t)}{U_{d_1}(t)},$$

(12)

When the signal of the sensorHS1equals the signal of the sensor HS2, the moving conductor with current is at the point of the trajectory corresponding the midpoint between the first and second sensors $X_a = \left(X_{d_1} + X_{d_2}\right)/2$.The value of the calculated coordinate function is fixedat $t(X_a)$according tothe signals of sensorsHS1 andHS3 in accordance to (12), which will beparameter value F_0:

$$F_0 = F\left(t(X_a)\right) = F_{U_{d_1}} = F_{U_{d_2}} .$$

(13)

The parameter*k*is a coefficient of the linear function and calculated from two pairs of the values defining two known function parameters. The first value is $\left(X_a ; F_0\right)$.The second is known a priori.When the current-carrying conductor is at the point of the motion trajectory being equal to the midpoint between the sensors HS1 andHS3, which also corresponds to the coordinate of position of the sensor HS2, the signals U_{d_1} andU_{d_3} will coincide. The values of coordinate function from (12) will be equal to $F\left(t_{X_{d_2}}\right) = 0$, where $t_{X_{d_2}}$ – the moment of time of passing by the current-carrying conductor of the coordinate X_{d_2} of the position of sensor HS2. Therefore, the second value of function has the coordinates $\left(X_{d_2} ; 0\right)$.

Calculation of the slope of coordinate function is done from the formula:

$$k = \frac{F\left(t_{X_{d_2}}\right) - F_0}{\left(X_{d_3} - X_{d_1}\right)/2} = \frac{2F_0}{X_{d_3} - X_{d_1}} .$$

(15)

After determining parameters $F_0$and*k*, in the process of measuring, that is, in real time, the coordinate of the current-carrying conductor is calculated in accordance with the expression (10).

More generally, the mathematical model, adjusted by the method of full-scale model experiment for measuring the coordinate of current-carrying moving conductor position with Hall sensors *N*, distributed in the line of the trajectory of the conductor, takes the form

$$\begin{cases} x(t) = \dfrac{F(t) - F_0}{k}; \\ F(t) = \ln \dfrac{U_{d_{j+2}}(t)}{U_{d_j}(t)}; \\ F_0 = F_{U_{d_j}} = F_{U_{d_{j+1}}}; \\ k = \dfrac{2F_0}{X_{d_j} - X_{d_{j+2}}}. \end{cases}$$

(16)

It should be noted that by increasing the quantity of sensors, in the process of measuring the moving conductor position, it is possible the alteration of using each sensor as the basic measuring or as extra one for adjustment of the parameters of $F_0$and*k*. By using more than three sensors it is necessary to adjust the parameters $F_0$and *k*for each pair of sensors, as these parameters depend: on the physical properties of the moving conductor which in the process of its motion can change; on the characteristics of the measuring transducers. If even all the sensors have been chosen as those with the identical nominal parameters, it is obvious that their value will have a certain scattering.

4. EXPERIMENTAL REASERCH AND DISCUSSION

The study of the suggested approach in using the method of full-scale model experiment for adjustment of mathematical model of the sensors in real time for measuring the coordinate of current-carrying moving conductor position was carried out with the help of computing experiment. It was used the computer programdeveloped in theenvironment of graphic programming LabVIEW[15].

The program includes the elements of control and indication. The first ones are meant for putting by the user of the data intothe program:the total of experimental points, the value of distance $x_0$between two Hall sensorsand the value of transverse distance$y$between the trajectory of motion of the sensor and longitudinal axis of sensor point. The second ones are the graphs of the modeling functional dependences (Figures 5-9).

Figure 5 represents the graph, including the functions of flux density, obtained as a result of modeling, at the points of two Hall sensors, created by a moving sensor. Calculation of the functions was carried out in accordance with (1).

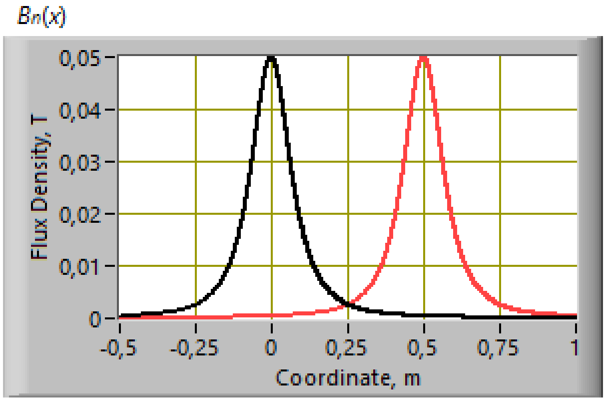

Fig.(5). – The Graph of Functions of Normal Constituents of Flux Density $B_{n1}(x)$, $B_{n2}(x)$ to the Active Surfaces of Hall Sensors

Figure 6 demonstrates the graph of differential and logarithmic coordinate function. The coordinate function was calculated directly from the values of flux density in the places of sensors installation, but not from the values of the pick-off signals. That is why, for determination of a coordinate function the designation$F_B$is used, that is, the coordinatefunction$F$, calculated frommagnetic flux density B:

$$F_B(x(t)) = \ln \frac{B_{n_2}(x(t))}{B_{n_1}(x(t))}. \qquad (17)$$

The expression (17) is analogous to the formula (6), but instead of the signal values of the sensors U_{d_1} andU_{d_2} the values of magnetic flux density $B_{n1}$and$B_{n2}$are used, respectively. Such approach is possible at the first stage of research, when the method error is being analyzed, andinstrumental errors are not taken into account. In this case Hall sensor seems to be an ideal transducer, its signal is strictly proportional to the measured magnetic induction, and the formulas (6) and (17) are equivalent.

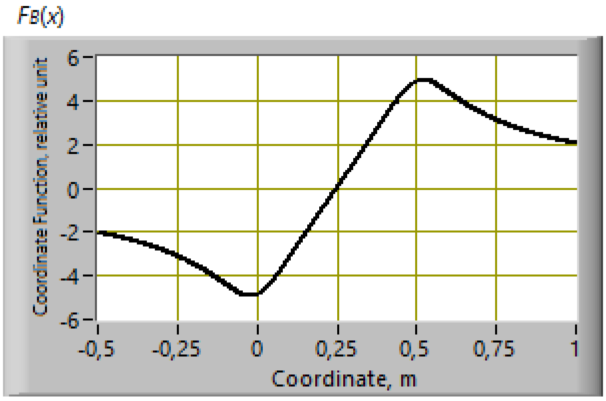

Fig.(6). – The Graph of Differential and Logarithmic Function $F_B(x)$, Computed from the Values of Magnetic Flux Density

It follows from Figure 6 thatthe coordinate function contains the part close to the linear one, there the coordinate of moving conductor position is determined. This part is highlighted in the graph illustrated in Figure 7 (black line). The red line shows the function linearly approximating this part. Thus, it is given a visual estimate of the degree of linearity of measuring part of coordinate function.

After determination of coordinate function from the signals of sensors in the program it is calculated the values of coordinates of position of the model-based moving conductor with current from the formula:

$$x^*(t) = \frac{F_B(t) - F_{B_0}}{k_B},$$

where $x^*(t)$–measured value $x(t)$; F_{B_0}, k_B – parameters analogous to $F_0$and$k$in the formula (10).

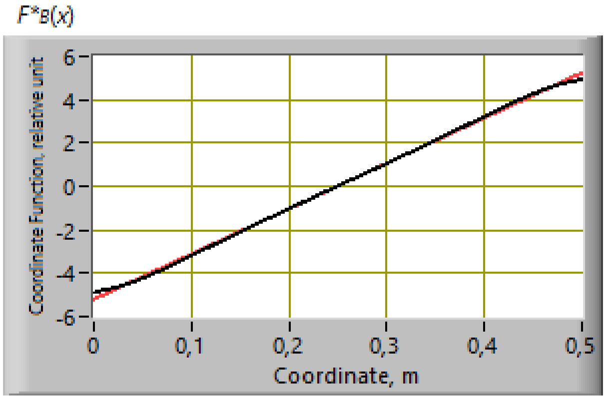

Fig.(7). – The Graph of the Fragment of Coordinate Function $F_B(x)$ (Black Line) and its Linear Approximation $F^*_B(x)$ (Red Line)

Figure 8 shows dependence of the measured values of the coordinates of the conductor positionx^* onreal values of the coordinate$x$ (black line),the red line shows the real values of the coordinate of position.

Further, it is determined in the program the method error $\delta_x$measurement of the coordinate of current-carrying conductor position according to the method of coordinate function:

$$\delta_x(t) = \frac{x^*(t) - x(t)}{x(t)} 100\%$$

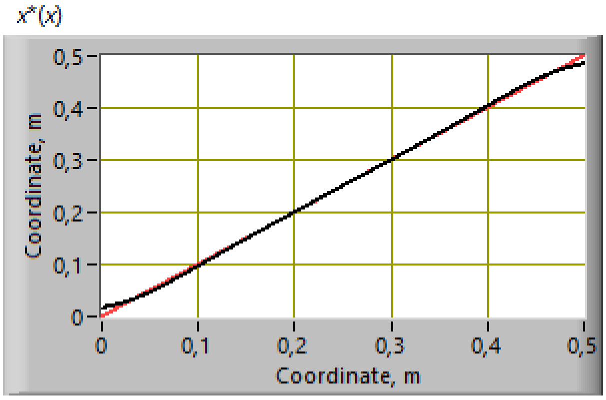

Fig.(8). – The Graph of the Measured Values of Coordinate ofPosition of Conductor *x** on the Real Values of Coordinate*x*

Figure 9 illustrates dependence of values $\delta_x$on real values of the coordinate of position of the conductor *x*(black line).The horizontal red lines in this graph are the cursors, controlled by the user for visual image of the permissible values of error.The points of entry and exit of the error graph in this hall determine the initial and finite points of measuring part of coordinate function.

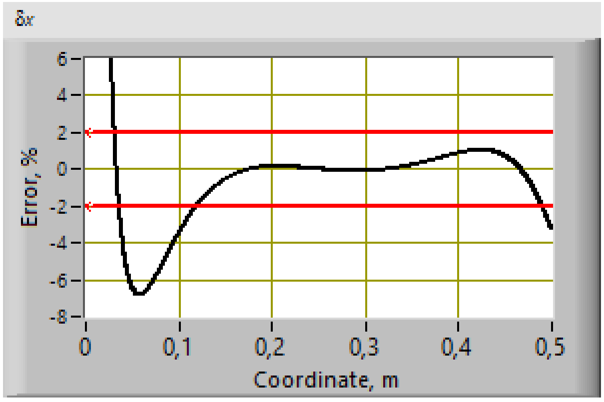

Fig.(9). – The Graph of Dependence of Error of Measurement of the Coordinate of Position of Conductor $\delta_x$onReal Values of Coordinate*x*

As it follows from Figure 9,error of measurement of the coordinate of current-carrying conductor position with the use of full-scale model method of adjustment of the model in real time by realization of the method of differential and logarithmic coordinate function in the range from 0,1to 0,5 m does not exceed 4 %.

The computing experiment has shownthat using the signals of two Hall sensors reacting to magnetic field of the moving conductor with current provides linearized secondary intelligence signal (the graphs in Figures6 and 7)in wide range of change of coordinate function (6).

5. SUMMARY

The suggested realization of the method of coordinate function allows develop apparatus for measuring the coordinate of position of the current-carrying conductor, having high response speed when required the accuracy as a result of simplicity of realization of the computing unit and small quantity of computing operations.

Application of the method of full-scale method experimentfor adjustment of mathematical model of the sensors excludes the necessity of a priori information about special configuration and magnetic characteristics of the source of magnetic field, that is, controlled moving conductor with current. Moreover, it is possible a periodical adjustment of the measuring device, compensating instability of the parameters of the controlled conductor in the process of its moving.

As it is seen from the diagram in Figure 9, there exists the interval on the part of trajectory between the neighboring sensors where it is possible the achievement of insufficient method error of measuring the position of moving conductor with current (not greater than 2 %).

6. CONCLUSION

For measuring the coordinate of position of the moving conductor with current, in particular plasma bunch in the channel of magnetic plasma accelerator, it is reasonable to use the set of magnetometric sensors, distributed in the line of the trajectory of motion of the conductor.

Mathematical modeling of the sensors reacting to the field of moving conductor, allows to synthesize special functions of aggregate value transformation of signals of these sensors, or the purpose of obtaining of the resultant function that depends only on the coordinate of moving conductor position.

Accuracy enhancement (reduction of method error) of measuring the position of moving conductor with current according to the method of coordinate function is possible by virtue of adjustment of mathematical model of sensors by the method of full-scale model experiment in real time.

Further area of the works is upranging of the measured coordinate of position of moving object without additional increase of the sensors in number.

THE CONFLICT OF INTERESTS

The authors confirm that the presented data so not contain the conflict of interests.

COMMENDATIONS

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