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On Intuitionistic Fuzzy Volterra Spaces

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Abstract
The aim of this paper is to study some results concerning intuitionistic fuzzy Volterra spaces and study conditions under which an intuitionistic fuzzy topological space is an intuitionistic fuzzy Volterra space.

Keywords: Intuitionistic fuzzy dense set, intuitionistic fuzzy $G_\delta -$set, intuitionistic fuzzy $F_\sigma -$set, intuitionistic fuzzy first category, intuitionistic fuzzy Volterra space, intuitionistic fuzzy Baire space,
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Introduction
The concepts of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh [16] in his classical paper. Thereafter the paper of C.L.Chang [7] paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalise the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concepts of Volterra spaces have been studied extensively in classical topology in [10,11,12,13]. In this paper we investigate several characterisations of intuitionistic fuzzy Volterra spaces and study under what conditions an intuitionistic fuzzy topological space becomes an intuitionistic fuzzy Volterra space.

Preliminaries
Definition 2.1 [2]. An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$ where the functions $\mu_A : X\to I$ and $\upsilon_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\upsilon_A(x)$) of each element $x\in X$ to the set A on a nonempty set X and $0\le\mu_A(x)+\upsilon_A(x)\le 1$ for each $x\in X$. Obviously every fuzzy set A on a nonempty set X is an IFS's A and B be in the form $A=\{x,\mu_A(x),1-\mu_A(x)/x\in X\}$

Definition 2.2 [2]. Let X be a nonempty set and the IFS's A and B be in the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$, $B=\{x,\mu_B(x),\upsilon_B(x)/x\in X\}$ and let $A=\{A_j : j\in J\}$ be an arbitrary family of IFS's in X. Then we define

(i) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$ and $\upsilon_A(x)\ge\upsilon_B(x)$ for all $x\in X$.

(ii) A=B if and only if $A\subseteq B$ and $B\subseteq A$.

(iii) $\overline{A}=\{x,\upsilon_A(x),\mu_A(x)/x\in X\}$.

(iv) $A\cap B=\{x,\mu_A(x)\cap\mu_B(x),\upsilon_A(x)\cup\upsilon_B(x)/x\in X\}$.

(v) $A\cup B=\{x,\mu_A(x)\cup\mu_B(x),\upsilon_A(x)\cap\upsilon_B(x)/x\in X\}$

(vi) $1_\sim=\{\langle x,1,0\rangle x\in X\}$ and $0_\sim=\{\langle x,0,1\rangle x\in X\}$.

Definition 2.3 [6]. An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of an intuitionistic fuzzy set (IFS, in short) in X satisfying the following axioms:

(i) $0_\sim, 1_\sim\in\tau$.

(ii) $A_1\cap A_2\in\tau$ for any $A_1, A_2\in\tau$.

(iii) $\bigcup A_j\in\tau$ for any $A_j : j\in J\subseteq\tau$.

The complement $\overline{A}$ of intuitionistic fuzzy open set (IFOS, in short) in intuitionistic fuzzy topological space (IFTS, in short) (X,τ) is called an intuitionistic fuzzy closed set (IFCS, in short).

Definition 2.4 [6]. Let (X,τ) be an IFTS and $A=\{x,\mu_A(x),\upsilon_A(x)\}$ be an IFS in X. Then the fuzzy interior and closure of A are denoted by

(i) $cl(A)=\bigcap$ {K: K is an IFCS in X and $A\subseteq K$ }.

(ii) $\text{int}(A)=\bigcup$ {G: G is an IFOS in X and $G\subseteq A$}.

Definition 2.5[6]. Let (X,τ) be any intuitionistic fuzzy topological space(IFTS, in short). Let A be an intuitionistic sets in (X,τ). Then the intuitionistic fuzzy closure operator satisfy the following properties:

(i) $1-IFcl(A)=IF\,\text{int}(1-A)$

(ii) $1-IF\,\text{int}(A)=IFcl(1-A)$

Definition 2.6[8]. An intuitionistic fuzzy set A in intuitionistic fuzzy topological space (X,τ) is called intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X,τ) such that $A \subseteq B \subseteq 1_{\sim}$.

Definition 2.7[8]. If A is an intuitionistic fuzzy nowhere dense set in (X,τ) then $\overline{A}$ is an intuitionistic fuzzy dense set in (X,τ).

Definition 2.8.[8]. Let A be an intuitionistic fuzzy set in (X,τ). If A is an intuitionistic fuzzy closed set in (X,τ) with $IF\,\text{int}(A) = 0_{\sim}$, then A is an intuitionistic fuzzy nowhere dense set in (X,τ).

Definition 2.9.[8]. An intuitionistic fuzzy topological space (X,τ) is called an intuitionistic fuzzy submaximal space if each intuitionistic fuzzy set A in (X,τ) such that $IFcl(A) = 1_{\sim}$ then $A \in \tau$.

Definition 2.10.[8]. Let (X,τ) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in (X,τ) is called intuitionistic fuzzy first category if $A = \bigcup_{i=1}^{\infty} B_i$, where B_i's are intuitionistic fuzzy nowhere dense sets in (X,τ). Any other intuitionistic fuzzy set in (X,τ) is said to be of intuitionistic fuzzy second category.

Definition 2.11.[8]. An intuitionistic fuzzy topological space (X,τ) is called intuitionistic fuzzy first category if the intuitionistic fuzzy $1_{\sim}$ is an intuitionistic fuzzy first category in (X,τ). That is $1_{\sim} = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X,τ). otherwise (X,τ) is said to be of intuitionistic fuzzy second category.

Definition 2.12.[9]. An intuitionistic fuzzy topological space (X,τ) is called an intuitionistic fuzzy Baire space if $IF\,\text{int}(\bigcup_{i=1}^{\infty} A_i) = 0_{\sim}$, where A_i 's are intuitionistic fuzzy nowhere dense sets in (X,τ).

Definition 2.13.15]. An intuitionistic fuzzy topological space (X,τ) is called an intuitionistic fuzzy hyper connected space if every intuitionistic fuzzy open set is intuitionistic fuzzy dense in (X,τ). That is $IFcl(A_i) = 1_{\sim}$ for all $A_i \in \tau$.

Definition 2.14.[15]. Let (X,τ) be an IFTS and be an IFS in X. Then A is called IF G_δ if $A = \bigcap_{i=1}^{\infty} A_i$ where each $A_i \in T$.

Definition.2.15[15]. Let (X,τ) be an IFTS and be an IFS in X. Then A is called IF F_σ if $A = \bigcup_{i=1}^{\infty} A_i$ where each $A_i \in T$.

Definition 2.16. [15]. An intuitionistic fuzzy topological space (X,τ) is called intuitionistic fuzzy P-space, if countable intersection of intuitionistic fuzzy open sets in (X,τ) is intuitionistic fuzzy open. That is, every non-zero intuitionistic fuzzy G_δ −set in (X,τ) is intuitionistic fuzzy open in (X,τ).

Definition.2.17[14]. A fuzzy topological space (X,τ) is called an intuitionistic fuzzy strongly irresolvable space if $IFcl(A) = 1_{\sim}$ for any nonzero fuzzy set A in (X,τ) implies that $IFcl(IF\,\text{int}(A)) = 1_{\sim}$.

Definition 2.18.[15]. An intuitionistic fuzzy topological space (X,τ) is called intuitionistic fuzzy volterra space, if $IFcl(\bigcap_{i=1}^{N} A_i) = 1_{\sim}$ where A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ −sets in (X,τ).

Intuitionistic Fuzzy Volterra Space

Theorem 3.1. If A is an IF dense set and IF G_δ −set in an intuitionistic fuzzy topological space (X,τ), then 1-A is an IF first category set in (X,τ).

Proof. Since A is an IF G_δ −set in (X,τ), $A = \bigcap_{i=1}^{\infty}(A_i)$, where $A_i \in \tau$ and since A is an IF dense set in (X,τ), $IFcl(A) = 1_{\sim}$. Then $IFcl(\bigcap_{i=1}^{\infty}(A_i)) = 1_{\sim}$. But $IFcl(\bigcap_{i=1}^{\infty}(A_i)) \subseteq \bigcap_{i=1}^{\infty}(IFcl(A_i))$. Hence $1_{\sim} \subseteq \bigcap_{i=1}^{\infty}(IFcl(A_i))$. That is, $\bigcap_{i=1}^{\infty}(IFcl(A_i)) = 1_{\sim}$. Then we have $IFcl(A_i) = 1_{\sim}$ for each $A_i \in \tau$ and hence $IFcl(IF\,\text{int}(A_i)) = 1_{\sim}$ which implies that $1 - IFcl(IF\,\text{int}(A_i)) = 0_{\sim}$ and hence $IF\,\text{int}(IFcl(1 - A_i)) = 0_{\sim}$. Therefore 1-$A_i$ is an IF nowhere dense set in (X,τ). Now $1 - A = 1 - \bigcap_{i=1}^{\infty}(A_i) = \bigcup_{i=1}^{\infty}(1 - A_i)$. Therefore

$1-A=\bigcup_{i=1}^{\infty}(1-A_i)$ where (1-A_i)'s are IF nowhere dense sets in (X,τ). Hence 1-A is an IF first category set in (X,τ).

Proposition 3.2. If the IF first category sets B_i are formed from the IF dense and IF G_δ −sets A_i in an IF Volterra space (X,τ), then $IF\,\text{int}(\bigcup_{i=1}^{\infty}(B_i)=0_\sim$.

Proof. Let A_i's (i=1 to N) be IF dense and IF G_δ −sets in an IF Volterra space (X,τ). Then $IFcl(\bigcap_{i=1}^{\infty}(A_i))=1_\sim$. Now $1-IFcl(\bigcap_{i=1}^{N}(A_i))=0_\sim$, implies that $IF\,\text{int}(\bigcup_{i=1}^{N}(1-A_i))=0_\sim$. Since the IFS A_i's (i=1 to N) are IF dense and IF G_δ −sets in (X,τ). By theorem 3.1, (1-A_i)'s are IF first category sets in (X,τ). Let B_i=1-A_i. Hence $IF\,\text{int}(\bigcup_{i=1}^{N}(A_i))=0_\sim$, where B_i's are IF first category sets in (X,τ).

Proposition 3.3. If the IFTS (X,τ) is an IF strongly irresolvable Baire space, then (X,τ) is an IF Volterra space.

Proof. Let A_i's (i=1 to N) be IF dense and IF G_δ −sets in an IF Volterra space (X,τ). Since (X,τ) is an IF strongly irresolvable space, $IFcl(A_i)=1_\sim$, implies that $IFcl(IF\,\text{int}(A_i))=1_\sim$. Then $1-IFcl(IF\,\text{int}(A_i))=0_\sim$. This implies that $IF\,\text{int}(IFcl(1-A_i))=0_\sim$. Hence (1-$A_i$)'s are IF nowhere dense sets (X,τ). Suppose that B_i's are IF nowhere dense sets in (X,τ) in which B_i=1-A_i (i = 1 to N). Now $\bigcup_{i=1}^{N}(1-A_i)\subseteq\bigcup_{i=1}^{\infty}B_i$. Then we have $IF\,\text{int}(\bigcup_{i=1}^{N}(1-A_i))\subseteq IF\,\text{int}(\bigcup_{i=1}^{\infty}B_i)$. Since the IFTS (X,τ) is an IF Baire space, $IF\,\text{int}(\bigcup_{i=1}^{\infty}(B_i)=0_\sim$. This implies that $1-IFcl(\bigcap_{i=1}^{N}(A_i))=0_\sim$. Then we have $IFcl(\bigcap_{i=1}^{N}(A_i))=1_\sim$. Therefore (X,τ) is an IF Volterra space.

Proposition 3.4. If the IFP-space (X,τ) is an IF hyperconnected space, then (X,τ) is an IF Volterra space.

Proof. Let A_i's (i=1 to N) be IF dense and IF G_δ −sets in an IF Volterra space (X,τ). Since (X,τ) is an IF P-space, A_i's are IF G_δ −sets in (X,τ) implies that A_i's are IFOS in (X,τ). Then $\bigcap_{i=1}^{N}(A_i)\in\tau$. Also since (X,τ) is an IF hyperconnected space, $\bigcap_{i=1}^{N}(A_i)\in\tau$ implies that $IFcl(\bigcap_{i=1}^{N}(A_i))=1_\sim$. Therefore (X,τ) is an IF Volterra space.

Proposition 3.5. If the IFTS (X,τ) is an IF submaximal and IF hyperconnected space, then (X,τ) is an IF Volterra space.

Proof. Let A_i's (i=1 to N) be IF dense and IF G_δ −sets in an IF Volterra space (X,τ). Since (X,τ) is an IF submaximal space, $IFcl(A_i)=1_\sim$, implies that $A_i\in\tau$ in (X,τ). Then we have $IF\,\text{int}(A_i)=A_i$. This implies that $IFcl(IF\,\text{int}(A_i))=IFcl(A_i)$. Thus $IFcl(IF\,\text{int}(A_i))=1_\sim$ for the IF dense sets A_i in (X,τ). Thus (X,τ) is IF strongly irresolvable space. Now $A_i\in\tau$ implies that $\bigcap_{i=1}^{N}(A_i)\in\tau$. Also since (X,τ) is IF hyperconnected space, $\bigcap_{i=1}^{N}(A_i)\in\tau$ implies that $IFcl(\bigcap_{i=1}^{N}(A_i))=1_\sim$. Therefore (X,τ) is an IF Volterra space.

Definition 3.6. An IFTS (X,τ) is called IF regular space if each IFOS A in (X,τ) is such that $A=\bigcup_{i=1}^{\infty}A_i$, where $A_i\in\tau$ and $IFcl(A_i)\subseteq A$ for each i.

Definition.3.7. An IFTS (X,τ) is called IF totally second category space if every non-zero IFCS in (X,τ) is an IF second category set in (X,τ).

Theorem 3.8. If (X,τ) is called IF totally second category, IF regular then (X,τ) is an IF Baire space.

Proof. Let (X,τ) is called IF totally second category, IF regular space and A_i be I S and IF dense sets in (X,τ). Let $A=\bigcap_{i=1}^{\infty}A_i$. Then $1-A=1-\bigcap_{i=1}^{\infty}A_i=\bigcup_{i=1}^{\infty}(1-A_i)$. Since A_i

is IFOS FO and IF dense sets in (X,τ), 1-A_i is IF nowhere dense sets in (X,τ) for each $i\in N$. Hence 1-A is IF first category set in (X,τ). Now we claim that $IFcl(A)=1_{\sim}$. Suppose that $IFcl(A)\neq 1_{\sim}$. Then there exists a non-zero IF closed set B in (X,τ) such that $A\subset B\subset 1_{\sim}$. Hence $1-A\supset 1-B\supset 0_{\sim}$. Since B is IFCS in (X,τ), 1-B is IFOS in (X,τ). Since (X,τ) is IF regular and 1-B is IF open there exists C in (X,τ), such that $1-B=\bigcup_{i=1}^{\infty}C$ and $IFcl(C)\subseteq 1-B$. Now $IFcl(\bigcap C)\subseteq \bigcap IFcl(C)\subseteq \bigcap(1-B)$, that is $IFcl(\bigcap C)\subseteq 1-B\subseteq 1-A$, that is $IFcl(\bigcap C)\subset 1-A$. Since 1-A is IF first category set, then $IFcl(\bigcap C)$ is a IF first category set in (X,τ). Now $IFcl(\bigcap C)$ is an IFCS in an IF totally second category space, then $IFcl(\bigcap C)$ is not an IF first category set in (X,τ), which is a contradiction. Hence the assumption that $IFcl(A)\neq 1_{\sim}$ does not hold. Therefore $IFcl(A)=1_{\sim}$. Hence (X,τ) is an IF Baire space.

Proposition 3.9. If (X,τ) is called IF totally second category, IF regular and IF strongly irresolvable space, then (X,τ) is an IF Volterra space.

Proof. Let A_i's (i=1 to N) be IF dense and IF G_δ −sets in an IF Volterra space (X,τ). Since (X,τ) is an IF totally second category, IF regular space, by theorem 3.9, (X,τ) is an IF Baire space. Then (X,τ) is an IF strongly irresolvable Baire space. By proposition 3.3, (X,τ) is an IF Volterra space.

Theorem 3.10. [15] If A is an IF dense and IFOS in (X,τ), then 1-A is an IF nowhere dense set in (X,τ).

Theorem 3.11. Let (X,τ) be an IFTS. Then the following are equivalent:

(i) (X,τ) is an IF Baire space.

(ii) $IF\operatorname{int}(A)=0_{\sim}$ for every IF first category set A in (X,τ).

(iii) $IF\operatorname{int}(B)=1_{\sim}$ for every IF residual set B in (X,τ).

Proof. $(i)\Rightarrow(ii)$. Let A be an IF first category set in (X,τ). Then $A=\bigcup_{i=1}^{\infty}(A_i)$, where A_i 's are IF nowhere dense sets in (X,τ). Now $IF\operatorname{int}(A)=IF\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0_{\sim}$. (Since (X,τ) is an IF Baire space). Therefore $IF\operatorname{int}(A)=0_{\sim}$

$(ii)\Rightarrow(iii)$. Let B be an IF residual set in (X,τ). Then 1-B is an IF first category set in (X,τ). By hypothesis, $IF\operatorname{int}(1-B)=0_{\sim}$ which implies that $1-IFcl(B)=0_{\sim}$. Hence we have, $IFcl(B)=1_{\sim}$.

$(iii)\Rightarrow(i)$. Let A be an IF first category set in (X,τ). Then $A=\bigcup_{i=1}^{\infty}(A_i)$, where A_i 's are IF nowhere dense sets in (X,τ). Since A is an IF first category set in (X,τ), 1-A is an If residual set in (X,τ). By hypothesis, we have $IFcl(1-A)=1_{\sim}$. Then $1-IF\operatorname{int}(A)=1_{\sim}$, which implies that $IF\operatorname{int}(A)=0_{\sim}$. Hence $IF\operatorname{int}(\bigcup_{i=1}^{\infty}(A_i))=0_{\sim}$, where A_i 's are IF nowhere dense sets in (X,τ). Hence (X,τ) is an IF Baire space.

Proposition 3.12. If (X,τ) is called IF totally second category, IF regular and IF P-space, then (X,τ) is an IF Volterra space.

Proof. Let (X,τ) is called IF totally second category, IF regular and IF P-space. Let A_i's (i=1 to N) be IF dense and IF G_δ −sets in an IF Volterra space (X,τ). Since (X,τ) is an IF P-space, A_i's are IF G_δ −sets in (X,τ) implies that A_i's are IFOS in (X,τ). By theorem 3.10 (1-A_i)'s are IF nowhere dense sets in (X,τ). Now the IFS $A=\bigcup_{i=1}^{\infty}(1-A_i)$ is an IF first category set in (X,τ). Since (X,τ) is an IF totally second category, IF regular space, by theorem 3.8 (X,τ) is an IF Baire space. Then by theorem 3.11 $IF\operatorname{int}(A)=0_{\sim}$ in (X,τ). This implies that $IF\operatorname{int}(\bigcup_{i=1}^{\infty}(1-A_i))=0_{\sim}$. Then $IF\operatorname{int}(1-\bigcap_{i=1}^{\infty}(A_i))=1-IFcl(\bigcap_{i=1}^{\infty}(A_i)$ implies that

$1 - IFcl(\bigcap_{i=1}^{\infty}(A_i) = 0_{\sim}$. That is, $IFcl(\bigcap_{i=1}^{\infty}(A_i) = 1_{\sim}$. Then we have $IFcl(\bigcap_{i=1}^{N}(A_i) = 1_{\sim}$. Since $IFcl(\bigcap_{i=1}^{\infty}(A_i)) \subseteq IFcl(\bigcap_{i=1}^{N}(A_i)$. Therefore (X, τ) is an IF Volterra space.

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