International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 21 (2015) pp 42121-42124

A not search robust algorithm to automatic optimization and adaptation of regulatory systems

Shirokov Lev Alexeyevich

Institute of Economics, Management and Information System in Construction and Real Estate, National Research University "Moscow State University of Civil Engineering", 26 Yaroslavskoye shosse, Moscow, Russian Federation, 129337 e-mail: eduarlev@gmail.com

Shirokova Olga Lvovna

Institute of Basic Education, National Research University "Moscow State University of Civil Engineering", 26 Yaroslavskoye shosse, Moscow, Russian Federation, 129337 e-mail: ol.shirokova@gmail.com

Abstract
Formed composite components for a not search algorithm to automatic optimization and adaptation of regulatory systems: standard model, evaluation of optimality, modified Gauss-Newton method, simplified method of forming sensitivity functions with variable parameters for non-linear objects. Greatly simplified implementation, and reduced the need of computing resources for the implementation of the algorithm, create a property robustness for automatic optimization and adaptation of control systems.

Keywords: The control system, optimization, evaluation of optimality, robustness of the algorithm.

Introduction

To intensify the processes, efficient use of raw materials, energy resources in the construction industry, electric power, chemical and petrochemical, light and other industry need optimal and adaptive [1] automatic control system (ACS). However, the actual task of improve the efficiency of the of ACS when variations of characteristics of raw materials, changes in operating points, parameters, and possible structures of objects, changing the characteristics of disturbances.
The article is synthesized parametric optimization and adaptation algorithm for ACS for nonlinear objects, the main feature of which is the robustness [2], ie, the required operation under conditions of uncertainty of changing characteristics of objects.

1. The structure of the a not search algorithm of parametric optimization ACS

Calculation of the vector $\bar{q}$ adjustable controller parameters in the ACP is made from the best performance of the system requirements, given, for example, as a reference model. Quality control can be assessed functional

$$V = LF(\vec{\varepsilon}(t,\vec{q})), t \in [t_0; t_f] \quad (1)$$

where L-linear operator; F-convex positive definite function; $\bar{\varepsilon}$ - n-dimensional vector of the residuals between current and set point the coordinates of the object in time t.

Vector adjustable controller parameters in the ACS will be optimal with a minimum of (1). To minimize the functional (1) is applicable a not search gradient [3] Gauss-Newton method. The algorithm of method for the k-th step can be written as:

$$\begin{cases} \vec{q}(k+1) = \vec{q}(k) + \Delta\vec{q}(k); \\ \Delta\vec{q} = H(k)\nabla V(\vec{\varepsilon})) \\ \nabla V(\vec{\varepsilon}) = \int_{t_0}^{t_f} \Xi^T(t,\vec{q}(k)) \nabla_\varepsilon F(\vec{\varepsilon}(t,\vec{q}(k)))dt, k = 1.2,... \end{cases} \quad (2)$$

where $\Xi = \left\| \xi_{ij}(t) \right\|_{(n\times m)}$ $(n\times m)$ –matrix sensitivity functions $\xi_{ij}(t) = \frac{\partial x_i(t)}{\partial q_j}$ -i-th output coordinates of the object control on the j-th parameter controller settings; T –transposition symbol; ∇ -symbol gradient;

$$\nabla_\varepsilon F(\vec{\varepsilon}(t)) = \frac{\partial F(\vec{\varepsilon}(t))}{\partial \vec{\varepsilon}} = (\frac{\partial F}{\partial \varepsilon_1}, \ldots, \frac{\partial F}{\partial \varepsilon_n}\partial)^T \quad (3)$$

$$H(k) = \left(\frac{1}{t_f} \int_{t_0}^{t_1} \Xi^T(t) \frac{\partial \nabla_\varepsilon F(\vec{\varepsilon}(t))}{\partial \vec{q}} + \sum_{i=1}^{n} \Xi_i^{(2)}(t) \frac{\partial F(\vec{\varepsilon}(t))}{\partial \varepsilon_i})dt \right)^{-1} \quad (4)$$

-Hessian; $\Xi_i^{(2)}(t)$ –matrix sensitivity functions of the second order: $\xi_{ij}^{(2)}(t) = \frac{\partial^2 x(t)}{\partial q_i \partial q_j}$ $\vec{q}$; n –dimension vectors $\vec{x}$ и and $\vec{\varepsilon}$.

For the Gauss-Newton method, taking into account (2), (4) and [4] can be written

$$\Delta\vec{q}(n) = -\Gamma(n)\mathrm{H}(n)\nabla V(\vec{\varepsilon})) \quad (5)$$

where $\Gamma(n)$ -matrix of weighting coefficients.

The complexity of the implementation, effectiveness and robustness of the optimization algorithms are determined: choice of the reference model, optimization criterion, the type of matrix H, the values of the diagonal elements of the matrix T, the matrix calculating method sensitivity functions.
Selecting reference model-a crucial factor in the efficiency optimization. Different approaches to their construction are described in [5-8]. However, for the ACS, operating in non-

stationary parameters and object structures, characteristics of disturbances, decision on the reference model is still relevant. In [8] in order to build a robust optimization algorithm proposed quasiasymptotic approach. To control the quality of transient regulation in the ACS proposed implementation of the integrated management of the regulatory process as a whole by bringing it closer to a quasi-asymptotics configuration, defining the nature of the decay of the transition process in the ACS. This allows for the stabilization systems used null reference model, and for tracking systems-unit, ie virtually eliminate the problem of the synthesis of reference models. Simultaneously, the [8] address the issue of effective criteria for selection of structural and parametric optimization ACS.

2. Modification of algorithm Gauss-Newton

To ensure the robustness of the not search algorithm optimization and simplification of its implementation, consider modifying Gauss-Newton algorithm. Imagine a model regulatory system for scalar functions with zero initial conditions in the form

$$\sum_{i=0}^{n_p} q_i \frac{d^i x(t)}{dt^i} = \sum_{i=0}^{m_p} b_i \frac{d^i z(t-\tau)}{dt^i},\ m_p < n_p \tag{6}$$

where z(t-τ) – some function, offset by the amount of pure delay $\tau \geq 0$, equal to zero for $t < \tau$, such that equation (6) satisfies the conditions of continuity in some open domain D and Lipschitz conditions: b_i-fixed, q_i-varying system parameters.

Consider the Gauss-Newton method for the evaluation of the optimization

$$V = \frac{1}{2t_f} \int_0^{t_f} x^2(t)\,dt$$

where t_f – the duration of the transition of the regulatory process.

Then the corresponding vector-gradient adjustable parameters is written as

$$\nabla V = \frac{1}{t_f} \int_0^{t_f} \overrightarrow{\xi^T}(t)\, x(t)\,dt,$$

where $\vec{\xi}(t) = \left(\xi_1(t), \ldots, \xi_m(t)\right)^T$ –vector sensitivity functions with respect to parameters q_i (i = 1,…,m).

From (4), considered the first term to that indicated by the sign "tilde", we can write:

$$\tilde{H} = \left\| \tilde{h}_{ij} \right\|_{(m\times m)},\quad \tilde{h}_{ij} = \frac{1}{t_f} \int_0^{t_f} \xi_i(t)\,\xi_j(t)\,dt,$$

where $\tilde{h}_{ij} = \tilde{h}_{ji},\quad i, j = \overline{1,m}$.

Consider the elements of the matrix $\tilde{H}$, with the sum of even indices:

$$\tilde{h}_{i1j1} = \frac{1}{t_f} \int_0^{t_f} \xi_{i1}(t)\,\xi_{j1}(t)\,dt,\ i_1, j_1 \in \left[\overline{1,m}\right] \qquad i_1 + j_1 = 2K;\quad K \in \left[\overline{1,m}\right] \tag{7}$$

Integrating (7) by parts, taking into account the relationship sensitivity functions [4] we obtain

$$\tilde{h}_{i1j1} = \xi_{i_1}(t)\,\xi_{j1-1}(t)\big|_0^{t_f} - \int_0^{t_f} \xi_{j1-1}(t)\,d\xi_{i1}(t) \tag{8}$$

In the expression (8) in the case of a stable ACS, taking into account the definition of sensitivity for reduced when $t_f > t_p$, where t_p –control time, we get

$$.\ \xi_{i1}(t)\xi_{j1-1}(t)\big|_0^{t_f} \approx 0\ . \tag{9}$$

Using (9) and taking into account the relationship of sensitivity functions [4] expression (8) takes the form

$$\tilde{h}_{i1j1} = -\int_0^{t_f} \xi_{i1+1}(t)\,\xi_{i1-1}(t)\,dt\ . \tag{10}$$

Obviously, when re-integration (10) likewise obtain

$$\tilde{h}_{i1j1} = -\int_0^{t_f} \xi_{i1+2}(t)\,\xi_{i1-2}(t)\,dt\ .$$

Integrating while the index of sensitivity are equal, we get

$$\tilde{h}_{i1j1} = (-1)^{k_1} \int_0^{t_f} \xi_{i1+k1}(t)\,\xi_{j1-k1}(t)\,dt\ , \tag{11}$$

where $K_1 = (j_1 - i_1)/2$.

Given that

$i_1 + k_1 = j_1 - k_1 = (i_1 + j_1)/2$

expression (11) takes the form

$$\tilde{h}_{i1j1} = (-1)^{k_1} \int_0^{t_f} \xi_{(i1+j1)/2}(t)\,dt\ . \tag{12}$$

Therefore, the elements of the matrix diagonals $\Gamma(n)$ to (5) parallel to the second diagonal, are equal in magnitude and alternating in sign, and the signs of the elements belonging to the main diagonal are positive.

Now consider the elements of the matrix $\tilde{H}$, with the odd sum of the indices:

$$\tilde{h}_{i2j2} = \int_0^{t_f} \xi_{i2}(t)\,\xi_{j2}(t)\,dt;\quad i_2, j_2 \in [\overline{1,m}]\ , \tag{13}$$

where $i_2 + j_2 = 2K + 1;\ K \in [\overline{1,m}]$.

Integrating (13) by parts and taking into account the relationship of sensitivity functions [4], we obtain

$$\tilde{h}_{i2j2} = \xi_{i_2}(t)\,\xi_{j2-1}(t)\big|_0^{t_f} - \int_0^{t_f} \xi_{j2-1}(t)\,d\xi_{i2}(t),$$

where, taking into account (9) and the relationship of sensitivity functions [4]

$$\tilde{h}_{i2j2} = -\int_0^{t_f} \xi_{i2+1}(t)\,\xi_{j2-1}(t)\,dt\ .$$

Similarly, when re-integrating obtain

$$\tilde{h}_{i2j2} = \int_0^{t_f} \xi_{i2+2}(t)\,\xi_{j2-2}(t)\,dt\ .$$

Integration operation will continue as long as the index of sensitivity will not differ by one. Finally, we get

$$\tilde{h}_{i2j2} = (-1)^{k_2} \int_0^{t_f} \xi_{i2+k2}(t)\,\xi_{j2+k2}(t)\,dt\ , \tag{14}$$

where
$K_2 = (j_2 - i_2 - 1)/2$.
Given that $i_2 + K_2 = j_2 + K_2 + 1$, the expression (14) can be written as

$$\tilde{h}_{i2j2} = (-1)^{k_2} \int_0^{t_f} \xi_{i2+k2}(t)\, d\xi_{j2+k2}(t) \qquad (15)$$

or

$$\tilde{h}_{i2j2} = (-1)^{k_2} \frac{K_2 \xi^2_{i2+k2}(t)}{2} \Big|_0^{t_f}.$$

Finally, in accordance with (9) to (15) we obtain

$$\tilde{h}_{i2j2} = 0 \qquad (16)$$

Therefore, the diagonal matrix $\tilde{H}$, the elements of which have odd sum of indexes will contain zero elements.
Matrix composition, inverse to the approximated matrix Hessian $\tilde{H}$, having, for example, for odd number m in view of (7), (12), (13) and (16) following form:

$$\tilde{H}^{-1} = \overline{H} = \begin{pmatrix} \tilde{h}_{11} & 0 & -\tilde{h}_{22} & \dots & \tilde{h}_{kk} \\ 0 & \tilde{h}_{22} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & -\tilde{h}_{kk} & 0 & \dots & 0 \\ \tilde{h}_{kk} & 0 & -\tilde{h}_{k+1,k+1} & \dots & \tilde{h}_{mm} \end{pmatrix}^{-1}, \qquad (17)$$

where $K = (m+1)/2$.

For non-degenerate square matrix $\tilde{H}$ inverse matrix $\tilde{H}^{-1} = \overline{H}$ will only:

$$\tilde{H} = \left\| \frac{H_{ij}}{|\tilde{H}|} \right\|^T = \frac{1}{|\tilde{H}|} \left\| \tilde{H}_{ij} \right\|^T; i, j = \left[\overline{1, m}\right] \; ; \text{i, j} = [1, m],$$

where $\tilde{H}_{ij}$ – algebraic cofactors of the elements $\tilde{h}_{ij}$ in the matrix $\tilde{H}$:

$$\tilde{H}_{ij} = (-1)^{i+j} M_{ij},$$

where M_{ij} – Minor element $\tilde{h}_{ij}$ determinant $|\tilde{H}|$ order $m \ge 2$.
Given the structure of the matrix (17), we can conclude that the cofactors of the elements of the matrix (17) with the odd sum of the indices will be set to zero. Consequently, the matrix $\overline{H}$ will have a structure similar to the structure of matrix $\tilde{H}$ in (17). Matrix $\overline{H}$ gives the best approximation to $\overline{H}^{-1}$ in the sense of minimum standards spherical square

$$N = \left\| \overline{H}^{-1} - H^{-1} \right\|^2,$$

Therefore, in accordance with [9], the resulting matrix $\overline{H}$ be quasi-optimal.
If you are using a diagonal matrix Γ in the algorithm (5) to ensure the sustainability of the adjustment should be [10] heuristically forming elements of the matrix Γ by the algorithm correction vector:

$$\gamma(n) = \gamma(n-1) \times \begin{cases} c_1 \& n_1 = 1 \text{ by } n_1 = 3 \& n > 2 \\ \& V(n) < V(n-1) \& V(n-1) < V(n-2); \\ 1/c_2 \; \& n_1 = 1 \& \text{ return on half-step} \\ \& \; n = const \text{ by } V(n) > V(n-1); \\ \& \; n = const \text{ by } V(n) > V(n-1); \\ 1 \; \& \; n_1 = n_1 + 1 \text{ in other cases,} \end{cases}$$

where $c_1, c_2 > 1 and\, c_1 > c_2$ (e.g., $c_1 = 2;\ c_2 = 1{,}7$) – counter of improving steps.
Final review of the optimization algorithm can be written as

$$\vec{q}(n+1) = \vec{q}(n) - \Gamma(n) \left(\int_0^{t_f} diag\, \Xi^T(t) \Xi(t) dt \right)^{-1} \times \\ \times \int_0^{t_f} \Xi^T(t) \nabla_{\varepsilon} F(t) dt. \qquad (18)$$

Algorithm (18) is characterized by simplicity of implementation. The same sensitivity function matrix is used to calculate the gradient matrix and the extrapolation of acceleration of convergence-approximated Hessian, which saves computing resources.
3. The analyzer the simplified sensitivity for ACS non-linear objects
To optimize of the smooth nonlinear systems with accurate sensitivity functions is quite a difficult task. In order to simplify and to ensure robustness, consider the possibility of using the simplified sensitivity functions. We write the equation with a smooth nonlinearity ψ in the form:

$$\begin{cases} \dfrac{d\vec{x}(t)}{dt} = \psi\left(\vec{z}(t), \vec{x}(t), \vec{x}(t-\tau),\ \vec{q}, \vec{p}\right);\ t \in [t_0; t_1] \\ \vec{y}(t, \vec{q}) = f_y\left(\vec{x}(t, \vec{q}), \vec{x}(t-, \vec{q})\right) \end{cases},$$

where $\vec{y} = (y_1, \dots, y_h)^T$ -h-dimensional vector coordinates of the observed
We expand the right side of the first equation (19) and $\vec{y}(t, \vec{q})$ in a Taylor series in the neighborhood of steady state value vector of phase coordinates $\vec{x} = \vec{x}^0$, defined by a given set of characteristics $\vec{y} = y^0$ process. We introduce the notation:

$$\begin{cases} \dfrac{\partial \psi}{\partial \vec{x}(t)} \Big| \vec{x}(t) = \vec{x}^0 = A_1; \ \dfrac{\partial \psi}{\partial \vec{x}(t-\tau)} \Big| \vec{x}(t-\tau) = \vec{x}^0 = A_2; \\ \dfrac{\partial f_y}{\partial \vec{x}(t)} \Big| \vec{x}(t) = \vec{x}^0 = A_3; \ \dfrac{\partial \psi}{\partial \vec{x}(t-\tau)} \Big| \vec{x}(t-\tau) = \vec{x}^0 = A_4; \end{cases} \qquad (19)$$

where A_i (i = 1,…,4) constant matrices of dimensions, respectively (nxn), (nxn), (pxn) (pxn); $\psi^0 = \psi | \vec{x} = \vec{x}^0$ and $= f^0{}_y = f_y | \vec{x} = \vec{x}^0$.
In view of the first members of decomposition can be written:

$$\begin{cases} \dfrac{d\vec{x}(t)}{dt} = A_1\left(\vec{x}(t) - \vec{x}^0\right) + A_2\left(\vec{x}(t-\tau) - \vec{x}^0\right) + \psi^0 + O_1\left(\vec{x}(t) - \vec{x}^0\right); \\ \vec{y}(t) = A_3\left(\vec{x}(t) - \vec{x}^0\right) + A_4\left(\vec{x}(t-\tau) - \vec{x}^0\right) + f^0{}_y + O_2\left(\vec{x}(t) - \vec{x}^0\right); \end{cases} \qquad (20)$$

where O_1 and O_2 – the remainder in the form of Peano.

The automatic stabilization systems have permanent jobs controlled object coordinates. he automatic control system software tasks vary according to some pre-selected law. This causes a change in time of matrix elements A_i *(i = 1,...,4)* in (20). In addition, changes in elements of A_i also arise when have deviations of output coordinates of the non-linear object from the nominal values. Then

$$\vec{x}^0 = \vec{x}^0(t);\ A_i = A_i(t),\quad i = \overline{1,4} \qquad (21)$$

Taking into account (21), omitting the remaining terms, the system of equations of the object

$$\begin{cases} \frac{d\vec{x}(t)}{dt} = A_1(t)\left(\vec{x}(t) - \vec{x}^0(t)\right) + A_2(t)\left(\vec{x}(t-\tau) - \vec{x}^0(t)\right) + \psi^0; \\ \vec{y}(t) = A_3(t)\left(\vec{x}(t) - \vec{x}^0(t)\right) + A_4(t)\left(\vec{x}(t-\tau) - \vec{x}^0(t)\right) + f^0. \end{cases} \qquad (22)$$

Therefore, the original non-linear system of equations of the controlled system is approximated by a linear system of equations (22) with variable coefficients. Functions of the sensitivity can be determined by the method of [4]. To simplify the analyzer sensitivity can range vector $\vec{x}(t)$ in (22) divided into several intervals $\alpha = \overline{1.N}$ (depending on the degree of nonlinearity of the controlled object and the range of changes in the nominal mode). As a rule, you can virtually take $N = 2 \div 5$ and use the appropriate set of simplified fixed analyzers sensitivity. In the neighboring intervals structure of analyzer sensitivity remains unchanged, and only the differences in the coefficients.

As an example, the optimization of ACS with PID controller of object described by a system of nonlinear equations of the twentieth order. According to the stated technique formed the sensitivity function Studies have confirmed the convergence of the algorithm to optimize its working capacity. In all cases convergence was iterations 8-12.

Conclusions

The retrofit Gauss-Newton algorithm has simplified its implementation by use for the gradient matrix and of the extrapolation of accelerating the convergence of the same sensitivity function matrix, which increases the robustness properties in use and allows to save computational resources. Construction of the simplified analyzer sensitivity functions to optimize and adapt ACS nonlinear objects significantly reduces the computing resources needed for the implementation of systems optimization and adaptation, expands the boundaries of ACS setting, without a priori information. Application of quasi-asymptotic quality control of processes in the ACS significantly expanded the robust properties of optimization algorithms, as virtually eliminate the problem of synthesis and tuning of etalons models and at the same time decided to question the choice for parametric optimization of effective criteria ACS.

References

[1] Aleksandrov A.G. Optimal and adaptive systems // E-book. Moscow, 2003. 278 pp.

[2] Filimonov A.B., Filimonov N.B. Robust correction in control systems with high coefficient of control // Mechatronics, Automation, Control. 2014. № 12. S.3-9.

[3] Chernorutskii I.G. Optimization techniques in control theory.-SPb.: Peter. 2004. 256 p.

[4] Shirokov L.A. Synthesis compact sensitivity to automate parametric design of linear control systems // Mechanical Industry and Engineering Education. 2008. № 3.-S. 22-29.

[5] Aleksandrov A.G. Adaptive management reference model for external perturbations // Automation and Remote Control.-2004.-N 5.-S. 77-90.

[6] AM Bronnikov, Bukov V.N. Terms accurate tracking output of a linear system of reduced order reference model // Automation and Remote Control. 2008.-N 3. pp 60-69

[7] Shchedrin, A.V., Fedenko S.V. Adaptive control system with ID and an implicit reference model // Automation and modern technology. 2006. N 3. S. 8-11.

[8] Shirokov L.A. Quasiasymptotic quality management regulation for automatic adaptation and optimization of control systems. // Mechanical Industry and Engineering Education. 2015. № 3.-S. 2-8.

[9] Tsypkin Ya.Z. Quasioptimal learning algorithms // Automation and Remote Control. 1973. № 6. S.62-73.

[10] Tsypkin Ya.Z. Adaptation and learning in automatic systems. M.: Nauka. 1968. 400c.

[11] Moskovkin, V.M., Merkulov, S.I., Suleiman, B.N.E., Lesovik, R.V. Theorem about the number and structure of the singular points N-dimensional dynamical system of population dynamics Lotka-Volterra in context of informational analysis and modeling (2013) World Applied Sciences Journal, 25 (12), pp. 1751-1753.

[12] Volkov, A. General information models of intelligent building control systems: Basic concepts, determination and the reasoning (2014) Advanced Materials Research, 838-841, pp. 2973-2976.

[13] Volkov, A., Sedov, A., Chelyshkov, P., Doroshenko, A. Using CAD for selecting different ACS engineering systems of buildings and structures in the presence of interference and restrictions (2014) Applied Mechanics and Materials, 580-583, pp. 3231-3233.

[14] Andreev, V.I., Barmenkova, E.V. The modeling of the real building object by using the model of a twolayer beam of variable rigidity on an elastic basis (2012) Applied Mechanics and Materials, 204-208, pp. 3596-3599.