A Three Phase Four Leg Inverter with a Single Vector Quantity as Control Signal

Bhaskar Bhattacharya

Teaching Assistant Department of Electrical Engineering National Institute of Technology, Agartala Tripura, India-799046 bhaskarohmm@gmail.com

Dr. Ajoy Kumar Chakraborty

Associate Professor Department of Electrical Engineering National Institute of Technology, Agartala Tripura, India-799046 akcalll@yahoo.co.in

Abstract

The space vector theory has the unique ability to represent any random but periodic three phase currents at an instant as a single vector quantity. In this paper this ability has been applied to generate control signal for modulating a three phase (3Ph) four leg (4L) inverter to generate a periodic 3Ph output replicating the control signal for any desired shape. Matching the control signal the method for inverter modulation has been three dimensional (3D) Space Vector Pulse Width Modulation (SVPWM) in α - β - γ frame. The time domain output pulses to operate individual switches have been worked out analytically in vector domain by resolving the control signal vector along three appropriate switching state vectors (SSV). A SSV is the vector representation of a valid switching state (SS) of the inverter under modulation. A MATLAB simulation of a four leg inverter working on the proposed method has been presented as an example to validate the proposition.

Keywords: α - β - γ frame; four-leg voltage source inverter; 3D SVPWM; switching state vectors; tetrahedron.

Introduction

Two signals, a control signal and a carrier signal, are generally required to generate pulses of different widths in different pulse width modulation (PWM) methods. These signals are physical quantities and can be measured and displayed. In SVPWM, on the contrary, only one control signal is required instead of two and that is not a physical quantity but is a mathematical entity. It cannot be measured or displayed. The output pulses from the modulated converter determine ON time durations within a sampling time period of all converter switches. The inverter power output follows the control signal by shape. Control signal generation is significant in designing an inverter. In SVPWM it is generated from a three phase signal of the shape that the inverter power output is desired to replicate. The control signal thus generated cannot be compared with any physical line current since it is a single quantity for all three phases and is not any physical current that can be measured or displayed. With a high sampling frequency (10 KHz) compared to power frequency (50 Hz) the input changes from sample to sample and the inverter can be made to work on feed forward method. During each sampling duration all the required activities are

done starting from measuring the sample waveform at that corresponding instant, generating the control signal vector, resolving it along applicable SS vectors, computing duty cycles and finally applying pulses at the gates of the converter switches. The comparison of input and output is done between the wave shapes of the output power and that of the sample waveform from which the control signal has been generated. SVPWM method is a fast computation intensive process and hence a complete mathematical implementation of the method with an appropriate processor can yield best results.

Related Work

The concept of decomposition of the three phase ac to two space vectors is based upon the d-q-0 and α - β - γ transformation theories presented by Park (1929) [1] & Clarke (1951) [2]. Originally it was developed for studies of three phase electrical machines [1]. Since then there have been many applications in power converters and alternating current (AC) drives [3]-[8] where the loads were balanced. As the loads were balanced three-phase without zero sequence components their transformations were confined to the α - β plane i.e. a 2D plane. The space vector modulation (SVM) applied was two dimensional (2D). Three dimensional space vector pulse width modulation (3D SVPWM) in α-β-γ frame for three phase unbalanced systems having non-zero γcomponents, has been first proposed in 1997 by Zhang et al. [9]. Since its introduction 3D SVPWM implementation in α-β-γ frame has been found difficult and was mentioned in [10]. Look up tables were used and few decisions were taken based on voltage polarities of non-zero switching state vectors (NZSV). But for specific advantages of direct access of the fourth wire by the converter's fourth leg and for better utilization of the converter dc voltage, there have been researches for simpler implementations.

Studies reported so far in the allied field have presented different algorithms and their applications in four wire (4W) three phase inverters [10]-[15]. These applications have not applied SVPWM proper. In those reports mapping of active tetrahedron, determination of active vectors, determining the matrix to compute duty cycles have not been decided mathematically but had been done intuitively with the help of look up tables [10],[14]. A recent work on four-leg inverters

has been reported to deliver active power to the balanced or unbalanced and linear or non-linear loads using a cost function to minimize errors between the reference currents and the output currents to generate gate pulses but it has not used SVPWM [13]. The 4L applications with 3D SVPWM and α - β - γ frame have been reported in [16]-[17] but these are on Active Shunt Filter (ASF).

Since introduction by Park and Clarke, basics for converting a balanced three phase system into a two dimensional (2D) vector have been well established. Those basics are: (i) developing an active space in α-β frame by valid switching states (SS) of the converter, (ii) developing a vector for reference signal at an instant, (iii) mapping the reference signal in the active space to find the two adjacent SS vectors, (iv) resolving the control vector upon the two SS vectors found, and (v) to compute from the resolutions the duty cycles of all switches of the converter and to generate desired output pulses applicable for that instant. On the contrary, there is no generally accepted implementation method in 3D SVPWM to apply on four leg converters for unbalanced three phase four wire systems. This has still been in a state of research and different methods have been reported as alternatives. From α-β frame in 2D the logical choice is α-β-γ frame of coordinates in applying 3D SVPWM. But it was found to be the source of difficulties and complexities. To overcome difficulties, the abc frame of coordinates has been proposed for 3D vector analysis in 2003 [18]. Since then a number of literatures using abc frame have been reported [19]-[22]. But as abc frame has all its axes on a common 2D plane, mathematical analysis of 3D vectors is not possible in abc frame. The essential mathematical condition for a frame of three axes to represent a three dimensional vector is, that for any combination of axes, the third axis must lie outside the 2D plane formed by the rest two axes of the frame. Literatures that reported the use of α - β - γ frame in implementing 3D SVPWM on different applications have not explored a total mathematical solution. Instead, one or other method has been applied to bypass or reduce mathematical complicacies and have applied intuitive decision making and have used look up tables to map the active tetrahedron or to determine active vectors or to determine the matrix to compute duty cycles [10],[14]. A generalized 3D SVPWM in α - β - γ frame for 4Leg, 3Level inverter application as power quality compensator has been reported in 2006. It has used four adjacent vectors for duty cycle computations against the use of three vectors as per vector mathematics [17]. Recent works have reported different other techniques & topologies and have not explored a total mathematical solution for 3D SVPWM for a 4L inverter. Those have been: (i) use of effective time concepts presented to avoid high computational burden of conventional SVPWM [23], (ii) application of predictive current control strategy that predicts the switching state in the (k+1)th cycle that minimizes the error between the predicted currents and their references computed at the (k)th cycle [13], (iii) applying per-phase vector control strategy instead of the conventional unified approach for all three phases in SVPWM [24], (iv) using a combination of SVPWM with the DC-side voltage control with a converter having 4 switches in 2 legs instead of six switches in three legs [25], and (v) using a three-leg

inverter for a four-wire active filter [26]. First mathematical implementation of 3D SVPWM has been reported in [27] but the inverter application was not for unbalanced three phase alternating current system.

Proposed System

The objective of this work is to develop a four leg inverter modulated with 3D SVPWM in α - β - γ frame. The modulation would not use any lookup table nor would any decision making be done based on any similarity or dissimilarity. The control signal would be generated following 3D space vector theory, from individual phase currents drawn by an unbalanced three phase four wire load, from a balanced three phase source. The inverter power circuit and the reference load current power circuits would be two different circuits with equal four terminal three phase impedances. Generation of inverter power output equal in magnitude and wave shape with the reference load currents is the desired outcome of the work. This would verify: (1) Whether the reference signal vector, the mathematical entity, generated from measured values of any unbalanced three phase system does truly represent the source; and (2) Whether the 4L bridge circuit output, modulated by the proposed method, actually replicates the system represented by the control signal and thereby validates the mathematical process applied..

A MATLAB/Simulink model has been designed as proposed and has been presented later. The model has broadly two parts. The first part has the arrangement for an unbalanced current to flow from a three phase voltage source connected to an unbalanced three phase four wire load. The second part is the proposed inverter. During each sampling interval a set of activities is executed starting with generation of reference control signal from measured values of phase currents flowing in the first part of the model. This control signal is a 3D vector in α - β - γ frame and it is applied on the controller to implement 3D SVPWM of the inverter to produce output. All these happen in a single sampling period. The inverter output terminals are connected to an unbalanced four wire load which is exactly equal to the load connected in the sample current generating circuit in the first part.

A. 3D SVPWM: a general introduction

In SVPWM a three phase time varying quantity is represented as a single mathematical entity in space domain in α - β - γ frame which is similar to Cartesian frame of coordinates. For balanced three phase quantity the representative vector is a 2D one in α - β frame and for unbalanced three phase system the representative vector is a 3D vector in α - β - γ frame. In 3D SVPWM the reference vector generated is mapped into a predefined 3D space, called the active space. This active space is built around all SSVs which represent all valid switching states (SS) of the converter under modulation. For a 4L inverter as shown in Fig.1 there are fourteen concurrent nonzero SSVs.

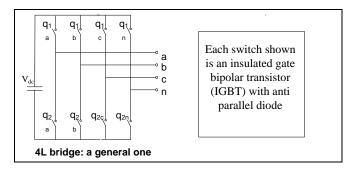


Fig.1: A 4-leg Converter circuit

i. Valid Switching States of a Converter as Mathematical Vectors

A switching state of a four leg bridge can be expressed as a four bit binary word. Such a word completely describes states of all individual switches of a bridge. Each bit from left to right of a SS word represents the ON or OFF state of the upper switch of a leg of the bridge in the same sequential order from left to right. State of the lower switch of each leg is always complementary to that of its upper one. A binary 4-bit word representing a valid switching state of a four leg bridge can be 1000. Fig.2 shows a figure when the SS is 1000. It means that upper switch of leftmost leg is ON and upper switches of rest all legs are OFF. States of lower switches in all legs are complementary to their respective upper switches. Suffixes a,b,c stand for phases while n is for neutral.

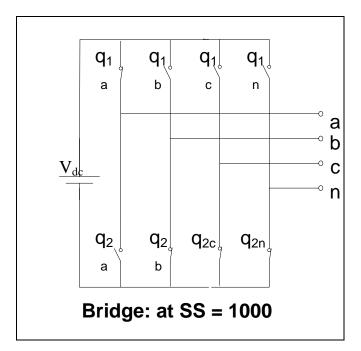


Fig.2: 4-leg Bridge circuit at SS = 1000

The 4-leg converter of Fig.2 has been analyzed as a combination of two parts, a 3-leg one comprising legs a, b & c for three phases and the other part comprises only the neutral leg n. A 3D vector \overline{v}_{3d} to represent a switching state of a 4-leg

converter is obtained as the resultant of a 2D vector \vec{V}_S and a 1D vector \vec{V}_Z . Vector \vec{V}_S is one of the six non-zero switching states corresponding to first three legs of the bridge as shown in Fig. 3. These states are: 100, 110, 010, 011, 001 and 101 as six radial 2D vectors: \vec{V}_1 (\vec{OA}), \vec{V}_2 (\vec{OR}), \vec{V}_3 (\vec{OB}), \vec{V}_4 (\vec{OP}), \vec{V}_5 (\vec{OC}), and \vec{V}_6 (\vec{OQ}) of magnitude V_{dc} (1 per unit) each and at $\pi/3$ radian displacements. Vector \vec{V}_Z corresponds to the γ - component obtained from the appropriate Clarke conversion of 4-bit switching states. Stated mathematically,

$$\overline{\mathbf{V}}_{3d} = \mathbf{\vec{V}}_{S} + \mathbf{\vec{V}}_{Z}. \tag{1}$$

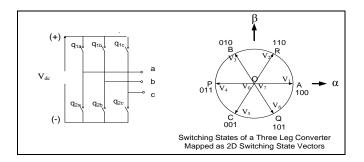


Fig.3: Switching state vectors for first three legs

Vectors \overrightarrow{OC}_1 and \overrightarrow{OC}_2 as shown in Fig. 4 are the two 3D vectors for switching states 0010 and 0011 respectively. These vectors share a common \vec{V}_S vector \overrightarrow{OC} shown in Fig. 3 corresponding to the first three bits 001. But they have different V_z , +1/3 per unit (pu) and -2/3 pu respectively. There are fourteen non-zero SSVs out of sixteen valid switching states of a 4-leg bridge circuit as shown in Fig. 2. The overall active space around all these vectors is a hexagonal right cylinder as shown in Fig. 4. Out of all fourteen SSVs only three SSVs which are immediately adjacent to the control signal are required for necessary computations. To facilitate this task, the overall active space is divided in twenty four tetrahedron spaces, each of which is a space defined by three SSVs. Such tetrahedrons are obtained by dividing the overall cylindrical space into six vertical prisms and then subdividing each prism into four tetrahedrons. This has been illustrated in Fig. 5 by showing all four tetrahedrons (T1, T2, T3, T4) of one vertical prism bound by triangular planes CO₁Q & C'O₂Q' as shown in Fig. 4.

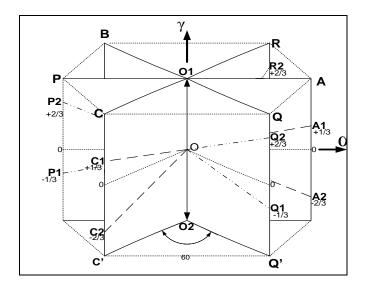


Fig.4: 3D hexagonal right cylindrical active space defined by switching state vectors

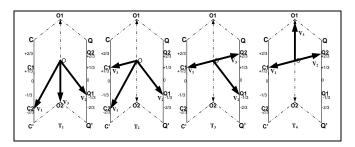


Fig. 5: Four different tetrahedrons in a prism

ii. Basics of Modulation

The control signal vector is mapped in any one of the tetrahedrons. On successful mapping of the control signal it is resolved along the SSVs which form the tetrahedron. Let \vec{i}_{ref} be the reference vector and \vec{i}_1 , \vec{i}_2 & \vec{i}_3 be the SSVs comprising the active tetrahedron i.e. the tetrahedron in which the control signal lies. Then the resolutions of \vec{i}_{ref} along vectors \vec{i}_1 , \vec{i}_2 & \vec{i}_3 can be expressed as (d_1, \vec{i}_1) , (d_2, \vec{i}_2) , & (d_3, \vec{i}_3) , where d_1 , d_2 and d_3 are fractional numbers. Expressed mathematically,

$$\overline{I}_{ref} = (d_1, \overline{I_1}) + (d_2, \overline{I_2}) + (d_3, \overline{I_3})$$

$$\tag{2}$$

The right hand side of (4) implies that during a sampling duration of T_s , the inverter output would be produced by keeping I_1 switching state on for $(d_1 \times T_s)$ time, I_2 switching state on for $(d_2 \times T_s)$ time and I_3 switching state on for $(d_3 \times T_s)$ time. Fractional numbers d_1 , d_2 and d_3 provide the duty cycles (ratio of on-time to the sampling time) of three SS vectors for non-zero outputs. There are occasions to add null value during a sampling period when, $(d_1 + d_2 + d_3) < 1$. Null value is introduced with any one of the null switching states 0000 and 1111. The duration of the null state applied is equal to $(d_Z \times T_s)$, where d_Z is the duty cycle for null state and is obtained from:

$$d_Z = 1 - (d_1 + d_2 + d_3) \tag{3}$$

From inputs of time domain measured values of currents upto computation of all duty cycles are performed in vector domain and finally the gate pulses for individual switches are sent out as output in time domain.

Simulation

A Matlab/Simulink model, developed to validate the proposition made, has been shown in Fig. 6. The block-I of Fig. 6 shows a standard three phase source feeding an unbalanced load with currents of all three phases measured. In block-II of Fig. 6 the measured signals have been processed to intermediate stage to prepare them as input for the generation of the reference signal vector and for further computations. It is apparent that from the initial input of measured values of line currents upto the output stage the entire process is analytical. To obtain the best performance a program code has been developed to translate all of the above steps mathematically and has been applied in the present simulation as an embedded function in the "Interpreted MATLAB Fcn" block shown in block-III of Fig. 6. The block executes the program on the input data and finally sends out 8-bit digital words as output. These output words are fed as gate inputs to the four leg bridge circuit of the inverter shown next in block-IV of Fig. 6. The inverter output is filtered by the filter block and is connected next to an unbalanced load which is the same as in the block-I of Fig. 6. The sampling frequency is 10 KHz.

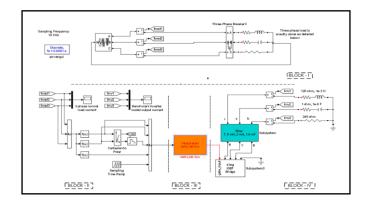


Fig. 6: MATLAB/Simulink model for validating SVM with α-β-γ frame on 4-Leg VSI.

The proposed model is developed in MATLAB-8.1.0.604 (R2013a) language on a personal laptop with an Intel (R) core (TM) 2 Duo CPU with 2.00 GHz with 2-GB RAM. The signal is directly fed to gate drivers of the 4-Leg VSI block ("subsystem3" of the model). The output from the VSI has been shown to pass through a filter block subsystem. It is a T-type filter made up of 5 mH, 2 mH and 1.8 mF. To verify whether the inverter output currents $I_{\rm inv1},\ I_{\rm inv2},\ {\rm and}\ I_{\rm inv3}$ are same as the currents $I_{\rm load1},\ I_{\rm load2},\ {\rm and}\ I_{\rm load3}\ {\rm drawn}$ by the same load from a 3-phase power source two scopes to display these currents are shown in Fig. 6.

Results

Results obtained show that the current delivered by the proposed inverter to a three phase four wire unbalanced load is exactly same by value and shape as the load current supplied by a normal voltage source to an exactly identical three phase four wire unbalanced impedance. Both the currents are shown separately in Fig. 7. There is a minor phase delay in the inverter output currents. This can be attributed to the response time of the inverter as it follows the control signal that is generated from the load currents which are to be replicated. The result obtained verifies that the control signal, a single mathematical entity generated from measured currents of three phases, is a true representative of all three different phase values collectively. The waveforms of output currents also validate the correctness of the mathematical program code applied to modulate the inverter with 3D SVPWM in α - β - γ frame.

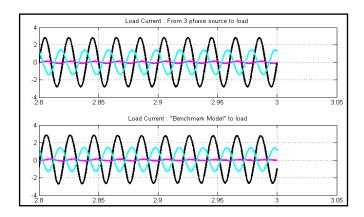


Fig. 7: Known input wave versus inverter output wave

Conclusion

In this work it has been established that the reference space vector in α-β-γ frame 3D SVPWM truly represents the source three phase quantity. Stated differently, a periodically varying three phase quantity can be successfully represented by a single mathematical vector quantity. It has also been established that the applied mathematical program code to implement the modulation is correct. With the verification of the unique ability of 3D SVPWM control signal to represent any random but periodic three phase physical quantity, at any instant, as a single vector quantity would open up multiple FACTS device applications related to unbalanced and distorted three phase 4 wire systems with 3D SVPWM in α - β - γ frame. That the control signal is a mathematical entity and not any physical quantity added with high speed discreet sampling has led to another significant benefit in this work that neither Phase Locked Loop (PLL) nor any feedback controller has been required for generating the control signal.

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