

# Congestion Control in TCP Network Using a Time Delay Model Based Fluid Dynamics

**K. Lefrouni,**

*Ph.D. student, ERMA, Mohammed V University,  
Mohammadia School of Engineers, Morocco Avenue Ibn Sina B.P. 765, Rabat  
lefrouni@emi.ac.ma*

**R. Ellaia,**

*Full Professor, ERMA, Mohammed V University,  
Mohammadia School of Engineers, Morocco Avenue Ibn Sina B.P. 765, Rabat  
ellaia@emi.ac.ma*

## Abstract

In this article, we study the congestion phenomenon in communication networks, initially, we present the control mechanisms currently in use, and show their limitations, secondly, based on the TCP fluid model and a state-feedback AQM mechanism, we develop a controller for regulating TCP traffic. Finally, we validate our study through simulations.

**Keywords:** TCP fluid model, AQM Mechanisms, TCP network, Time-delay systems.

## Introduction

Generally, to determine the mathematical model of a physical process, we assume that its behavior depends only on the states of the system parameters at the present instant, an assumption which is valid for a large class of systems. However, there are situations (transmission of information or material transport) that this assumption does not reflect the real dynamics of the system, and where the use of a conventional model for the analysis and control of such systems can lead to poor performance or even system instability [1]. In such cases, it is essential to consider, in addition to the present state, the past states of the system, this type of system is so called time-delay systems (TDS) [2][3][4]. This is the case of the system studied in this paper, namely, the communication networks. In fact, the transmission of data is never instantaneous, it is always accompanied by a transmission delay [5][6][7], this latter is due to the time required for the propagation of data between the transmitter and the receiver and also the processing time for determining the path to be taken by the data.

Several researchers [8][9][10] have developed algorithms to manage and optimize the transport of data within communication networks, while ensuring a certain level of quality of service (QoS). This has spawned two classes of strategies: queue management policies and scheduling algorithms. During our study, we assumed a FIFO (First In First Out) scheduling and we focused on the first class, namely, queue management mechanisms within routers, also called active queue management (AQM) mechanisms [11][12][13][14].

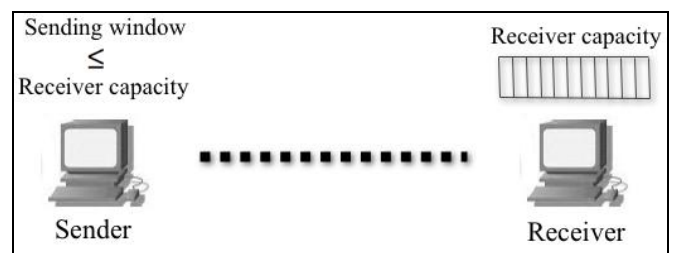
This review paper is organized as follows. In Section 2, we present the network congestion phenomenon and a set of

concepts necessary to understand the issues discussed in this paper. We also present the operating principle of AQM mechanisms. In Section 3, we give an overview on TDS Models, then we introduce a model found by analogy with fluid dynamics, to describe the dynamics of TCP. In Section 4, based on the TCP fluid model and the AQM mechanism, we develop a controller for regulating TCP traffic. Finally, we present our conclusions in Section 5.

## Problem Formulation

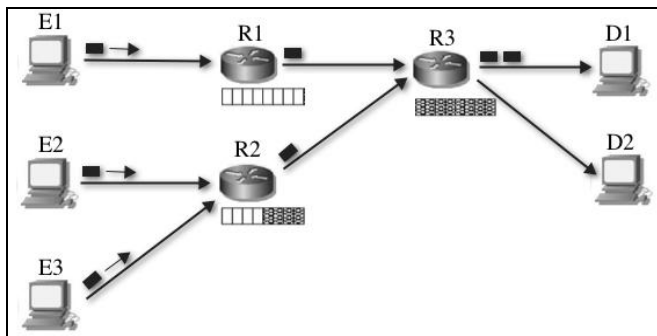
### A. Congestion in TCP Network

From a Sender-Receiver's perspective, the phenomenon of congestion occurs when the amount of data sent by the transmitter is greater than the processing capacity at the receiver. In other words, the incoming data rate is higher than the outgoing data rate. To address this problem, the receiver notifies the sender of the maximum capacity of packets that can be treated, thereafter, the transmitter adjusts its transmission rate so that the amount of data sent is at most equal to the capacity of the receiver.



**Fig.1. Control based on the receiver's capacity**

However, this adjustment takes into consideration only the receiver's capacity, regardless of the network. To illustrate the real congestion problem, consider the network shown in Fig. 2.



**Fig.2. Example of congested network**

In this example, the Sender E1 sends data to the receiver D1, this latter is not saturated, however, the buffer of the router R4 is completely filled due to another transmission to the receiver D2. In this case, neither the sender E1 nor the receiver D1 have information about the network congestion state and the sender E1 continues to send packets, the capacity of the router R4 is exceeded, therefore, any new arriving packets will be dropped. An effective congestion control, therefore, requires both the respect of the receiver capacity and the network capacity.

As the TCP protocol must ensure the reliability of transmissions, it determines, through the acknowledgment mechanism, all lost packets and must therefore be retransmitted. If the problem of congestion is not resolved, all the retransmitted packets will be lost again. Thus, instead of reducing the congestion, this mechanism contributes to the saturation of the network. This phenomenon has appeared at the beginning of the years 1980, in fact, the significant increase of Internet users, has led to a series of network collapses, prompting many researchers to conduct studies to improve the TCP protocol. The first algorithms that have been developed, had as its main objective, the early detection of congestion and the rapid restoration of the network equilibrium. As examples, we cite the Slow Start, the Congestion Avoidance, the Fast Retransmit and the Fast Recovery, for details, see references [15] and [16]. Later, more advanced versions based on the AQM mechanism have emerged.

### B. AQM Mechanism

The first algorithms developed to resolve the congestion problem in the TCP network are control algorithms called End to End, i. e. Sender-Receiver. Indeed, the only information used to determine if there is congestion is the receipt or non-receipt of acknowledgments. Therefore, decisions on increasing or reducing the transmission rate is based on a binary logic, the occurrence or non-occurrence of congestion without taking into consideration, neither the network load rate nor the time required for the readjustment request to be effective.

It is therefore clear that this control technique is not sufficient to obtain interesting performances, especially in terms of quality of service (QoS). Thus, several studies have been conducted [17][18][19], suggesting to involve the network, and in particular the routers, to develop more effective control

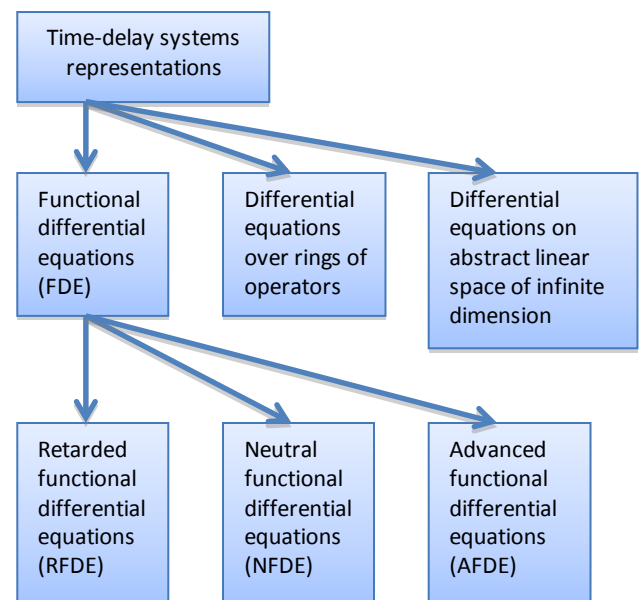
strategies, which gave rise to a new mechanisms called Active Queue Management (AQM) Mechanisms.

Based on the concept of congestion from a Sender-Receiver's perspective, which states that the packet loss is synonymous with congestion, the AQM mechanism force the loss of packets even before there is congestion. The senders consider this loss as a sign of network congestion and reduce accordingly their sending rates. Thus, this mechanism has a closed loop structure, which, depending on changes in network load acts on the packet loss rate in order to regulate the TCP traffic. The question that arises is under what conditions we decide to reject packets and with what rejection rate? This gave rise to several AQM algorithms. The most popular are the Random Early Detection (RED) [11], the Blue [12], the Random Early Marking (REM) [13] and the Adaptive Virtual Queue (AVQ) [14].

## Mathematical Modeling

### A. Time Delay System Models

According to the nature of physical systems, there are several types of delays: constant or variable, known or unknown, deterministic or random. The consideration of these various aspects of delay is absolutely essential when modeling time-delay systems. Indeed, the synthesis of controllers adapted to this class of systems requires a good modeling of their dynamic behavior. By browsing through the different representations used in the literature, we identified three techniques to model time-delay systems (TDS): Differential equations on abstract linear space of infinite-dimension [20][21], Functional differential equations (FDE) [22][23] and Differential equations over rings of operators [2][24]. The figure 3 illustrates the most TDS representations found in the literature.



**Fig.3. Classification of time-delay systems**

The global model of a time delay system, taking into account the impact of the delay on the system state, the control signal and on the output, is as follows:

$$\begin{cases} \dot{x}(t) = A x_t(\theta) + B u_t(\theta), & t \geq t_0, \quad -h \leq \theta \leq 0 \\ y(t) = C x_t(\theta), \end{cases} \quad (1)$$

with

$$x_{t_0}(\theta) = x(t_0 + \theta) = \phi(\theta), \quad \theta \in [-h, 0] \quad (2)$$

$$\begin{aligned} x_t(\theta) &\in C_0([-h, 0]; \mathbb{R}^n) \\ A : C_0([-h, 0]; \mathbb{R}^n) &\rightarrow \mathbb{R}^n \\ B : C_0([-h, 0]; \mathbb{R}^r) &\rightarrow \mathbb{R}^n \\ C : C_0([-h, 0]; \mathbb{R}^n) &\rightarrow \mathbb{R}^p \end{aligned}$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^r$  and  $y(t) \in \mathbb{R}^p$  are respectively the state vector, the control vector and the output vector,  $C_0$  is the set of continuous functions from  $[-h, 0]$  to  $\mathbb{R}^n$  and the functionals  $x_t$  and  $u_t$  are defined by:

$$\begin{aligned} x_t(\theta) &= x(t + \theta) \quad \forall \theta \in [-h, 0], \\ u_t(\theta) &= u(t + \theta) \quad \forall \theta \in [-h, 0]. \end{aligned} \quad (3)$$

#### Remark 1:

The complexity of the synthesis of control laws for this type of system lies in the fact that they are governed by hereditary equations, thus the initial condition which is generally a point is, in this case, a function and therefore the system is infinite dimensional.

#### B. Fluid flow Model of TCP Dynamics

Several models have been developed to describe the dynamics of TCP flows. However, the most popular model, faithfully describing the flow of data within communication networks, is that found by analogy with fluid dynamics, first developed by Misra et al [25], this model is governed by the following equations:

$$\begin{cases} \dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t-R(t))} p(t-R(t)), \\ \dot{q}(t) = \frac{W(t)}{R(t)} N(t) - C, \\ R(t) = T_p + \frac{q(t)}{C}. \end{cases} \quad (4)$$

With  $p$ ,  $R$ ,  $N(t)$ ,  $C$ ,  $q(t)$ ,  $W$  and  $T_p$  represent, respectively, the probability of dropping packets, the Round Trip Time (RTT), the number of TCP sessions, the link capacity, the queue length, the TCP window size and the time taken by the packets to propagate in the communication lines. The linearization around the operating point defined by (5) leads us to the system (6).

$$W_0 = \frac{R_0 C}{N}, \quad p_0 = \frac{2}{W_0^2}, \quad q_0 = (R_0 - T_p)C. \quad (5)$$

$$\begin{cases} \delta \dot{W}(t) = -\frac{2N}{R_0^2 C} \delta W(t) - \frac{R_0 C^2}{2N^2} \delta p(t - \tau(t)), \\ \delta \dot{q}(t) = \frac{N}{R_0} \delta W(t) - \frac{1}{R_0} \delta q(t). \end{cases} \quad (6)$$

In the state-space, the model (6) becomes:

$$\begin{cases} \dot{x}(t) = A x(t) + B u(t - \tau(t)), \\ y(t) = C x(t). \end{cases} \quad (7)$$

where

$$x(t) = [\delta W(t) \quad \delta q(t)]^T, \quad u(t) = p(t),$$

$$A = \begin{bmatrix} a_1 & 0 \\ a_2 & a_3 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

with

$$a_1 = \frac{-2N}{R_0^2 C}, \quad a_2 = \frac{N}{R_0}, \quad a_3 = \frac{-1}{R_0}, \quad b_1 = \frac{-R_0 C^2}{2N^2}.$$

#### Controller based TCP fluid model

We will now synthesize our AQM controller based on the state feedback of the network and defined by the following equation:

$$u(t) = -K x(t), \quad (8)$$

thus depending on the system state  $x(t) = [\delta W(t) \quad \delta q(t)]^T$  and the gain  $K \in \mathbb{R}^{1 \times 2}$ , we determine the control law  $u(t)$  which is nothing else than the packet rejection rate  $p(t)$ .

$$u(t) = p(t) = -K \begin{bmatrix} \delta W(t) \\ \delta q(t) \end{bmatrix} \quad (9)$$

Using (8) the system (7) becomes:

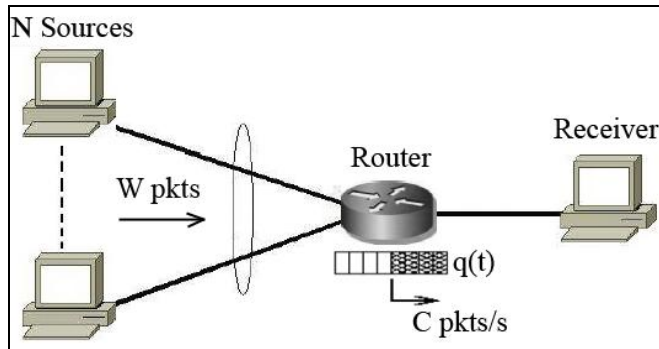
$$\begin{cases} \dot{x}(t) = A x(t) - B K x(t - \tau), \\ y(t) = C x(t). \end{cases} \quad (10)$$

#### Remark 2

: Considering the equation (10), for determining the value of  $\dot{x}(t)$  at  $t=0$ , it is necessary to know the value of  $x(t)$  at the time  $t=0$  and  $t=-\tau$ . In general, to determine  $\dot{x}(t)$  at time  $t'$ ,  $0 \leq t' \leq \tau$ , we need to know  $x(t')$  and  $x(t'-\tau)$ ,  $t'-\tau \in [-\tau, 0]$ , it is therefore necessary to know the values of  $x(t)$  over the interval  $-\tau \leq t \leq 0$ . Thus, unlike the ordinary differential equations when the initial condition is a point, here, it is an infinity of points, in fact, the resolution of the equation (10) requires knowledge of all past values of  $x(t)$  on the interval  $[-\tau, 0]$ .

In order to test the effectiveness of the TCP fluid flow model (4) and its adaptation to the current requirements of the TCP network, especially in terms of quality of service, we consider the network shown in Fig. 4, consisting of  $N$  homogeneous TCP sources (i.e. with the same propagation delay) connected

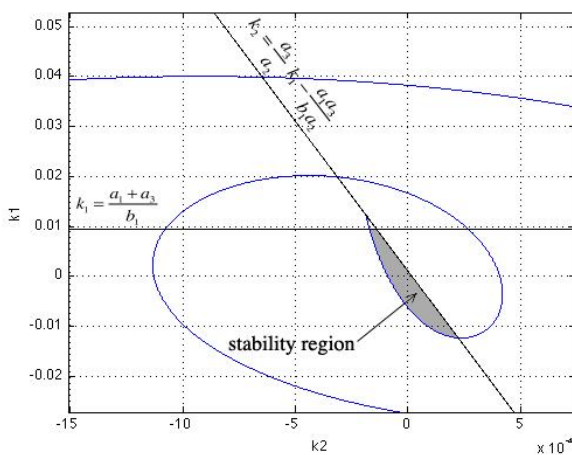
to the receiver via a router characterized by a capacity  $C$  and a queue length  $q(t)$ .



**Fig.4. Network topology with N homogeneous TCP sources.**

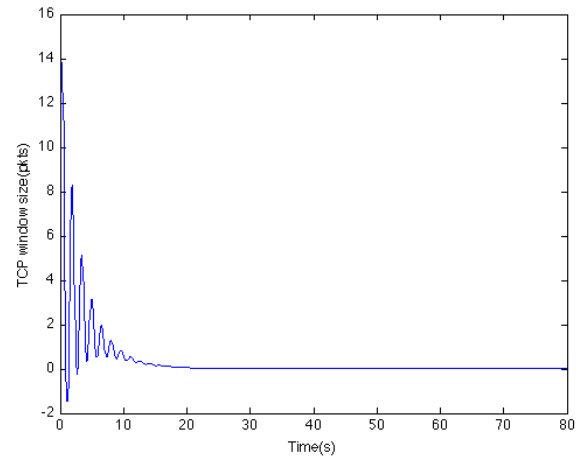
In order to stabilize the size of the router queue to the value  $q_0 = 175$  packets, we consider:  $T_p = 0.2s$ ,  $C = 3750$  packets/s and for  $N = 60$  TCP sessions, we have  $W_0 = 15$  packets,  $p_0 = 0.008$  and  $\tau = 0.6s$ .

By applying the geometric method [26] on the time delay system described by the state space model (10), we find the stability region illustrated in the figure Fig. 5. For more details see [27].



**Fig.5. Stability region for the closed loop system (10)**

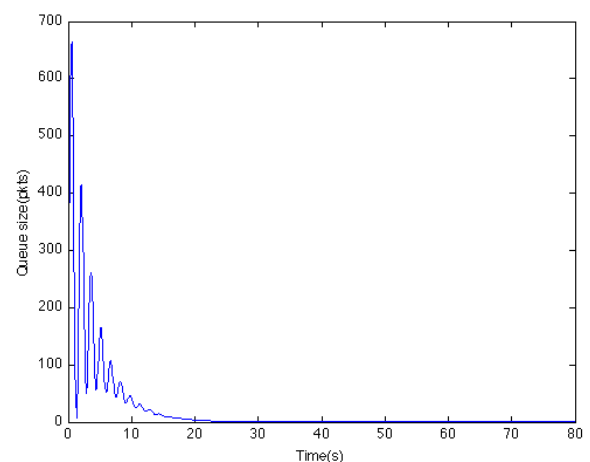
Each point  $(k_1, k_2)$  of the region shown in Fig. 5 defined by a state feedback controller  $u(t) = -Kx(t)$ , such that  $K = [k_1, k_2]$  is the state feedback gain and  $x(t) = [\delta W(t) \ \delta q(t)]^T$  is the state vector. By considering, for example, the pair  $(k_1, k_2) = (-0.009, 0.00015)$  we find the temporal evolution of the TCP window size  $\delta W(t)$  and the queue size  $\delta q(t)$ , illustrated respectively in Fig. 6 and Fig. 7.



**Fig.6. Temporal evolution of the TCP window size with**

## Conclusion

In this paper, we have proposed a state-feedback AQM mechanism for the control of congestion in the communication networks. Considering the communication delays caused by the processing and propagation of packets and using a model derived from the fluid mechanics, we have synthesized a state feedback control law allowing, by acting on the packet loss rate, to involve the network, and in particular the routers, during the process of TCP traffic control. It should also be noted that the studied mechanism forms a delayed closed loop, a structure that has been studied extensively by control engineers. Thus, as future work, we propose to test the different approaches used in the control theory.



**Fig.7. Temporal evolution of the queue size with**

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