

Comparative Analysis of Two Stochastic Models with Varying Demand

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Abstract

The present paper discusses the comparison of two stochastic Models of a cable manufacturing plant with varying demand. Here, two Models- one Model (Model 1) is a single unit system having only repair facility is compared with other Model (Model 2) a single unit system where inspection is carried out. During inspection, three types of failure have been observed which are categorized as repairable failure, replaceable failure and reconditioning/ reinstallation failure. The evaluation of systems is done by means of MTSFs, steady state availabilities, busy period of repairmen, profit functions using Laplace transforms and software package Code-Blocks 13.12. Various graphs have been plotted to provide a better understanding of the behavior of the Models, helps to find which Model is better than the other Model and lead to better estimates of the Model-parameters. System is analyzed by making use of semi-Markov processes and regenerative point technique.

Keywords: Stochastic Model, Cable Manufacturing Plant, Single Unit System, Variation in Demand, Inspection, Semi-Markov Process, Regenerative Point Technique

Introduction

The manufacturing of tools and special equipment is significant part of our modern society. In earlier days faults and accidents were the only way of learning to make safer and more reliable. Inspection is probably the most commonly used approach. When equipment fails, it often leads to downtime in production. In most cases, this is costly business. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. Availability and profit of an industrial system are becoming an increasingly important issue. Obviously, profit increases when the availability of a system increases.

System reliability has been considered as a significant factor in most of the system performance-related studies. Many researchers including [1-3, 16-17] contributed a lot in the field of system reliability modeling by considering various concepts, yet comparative analysis between two types of systems is reported very less in the literature. Mine and Kaiwal [4] analysed repair priority effect on availability of a two-unit system. Pandey and Jacob [5] discussed cost analysis, availability and MTTF of a three state standby complex system under common cause and human failures. Chandrasekhar et al. [6] studied a two-unit standby system

with Erlangian repair time. Madan et al. [7] derived a method for modeling and quantifying the security attributes of intrusion tolerant systems. Gupta et al. [8] analysed reliability and availability of serial processes of plastic-pipe manufacturing plant. Gupta and Tiwari [9] discussed simulation modeling and analysis of a complex system of a thermal power plant. Sharma and Kumar [10] analysed stochastic behavior and performance analysis of an industrial system using GABLT technique. Zhang [11] discussed reliability modelling and maintenance optimization of the diesel system in locomotives.

All these studies have considered the demand as fixed. However, there exist many practical situations where the demand of the units produced is not fixed. Such a situation may be seen in General Cable Energy System [12, 13] and where demand for the Al/Cu wire does not remain constant i.e. it varies and hence sometimes Cable Energy System is put to down mode if demand is lesser than the production.

A Model may be better in some situations and may be worse in some other situations and hence the comparative study becomes more important. Keeping this in view, we, in the present paper, carry out the comparison between two Models – one (Model 1): the reliability modeling of a single unit system with varying demand [14] and the other (Model 2) for analyzing the reliability modeling of a cable manufacturing plant with inspection and varying demand [15] have been compared. Initially, the system is in operative state where demand is not less than the production in each of these two Models. In Model 1, if the operative unit stops working, repairman repairs the failed unit. But in Model 2, firstly inspection is carried out, where three major failures were noted in the system, viz., repairable, replaceable and reconditioning/reinstallation. As variation in demand affects the production of system also, the system is required to be put to down state when the units produced are already in excess. The system in the down state is made operative as soon as the produced units are less in number than those demanded. The comparison is done graphically between the concerned Models considering the particular cases. These graphs are plotted to find the cut-off points for the concerned rates/costs/revenues which will be helpful in taking important decisions so far as the reliability and the profitability of the systems is concerned and can see as to which Model gives more profit as compared to other. The probabilistic analysis of the two Models is analyzed by making use of semi-Markov processes and regenerative point technique.

Notations used for Describing Two Models

Op	Unit is in operative state
$d \geq p, d < p$	Demand is not less than production, demand is less than production
D	Unit is in down unit
F_r	Failed unit under repair
λ, α	Failure rate, repair rate of the operative unit
$\alpha_1 / \alpha_2 / \alpha_3$	Inspection rate/replacement rate/reconditioning/ reinstallation rate
γ_1	Rate of decrease of demand so as to become less than production
γ_2	Rate of increase of demand so as to become not less than production
γ_3	Rate of going from upstate to downstate
γ_4	Rate of change of state from down to up when there is no produce with the system and demand is there
p_1	Probability that during the repair time demand is not less than production
p_2	Probability that during the repair time demand is less than production
p_3, p_4, p_5	Probability of reinstallation/ reconditioning, replacement and repair
AD_i	Availability that the system is in upstate when demand is not less than production for each Model i where $i=1,2$
AP_i	Availability that the system is in upstate when demand is less than production for each Model i where $i=1,2$
Bi	Busy Period analysis of the repairman for each Model i where $i=1,2$
Vi	Expected number of visits of repairman for each Model i where $i=1,2$
DT_i	Expected down time for each Model i where $i=1,2$
$I2, BR2, BRR2$	Busy period analysis of the repairman for inspection, replacement, reconditioning /reinstallation time for Model 2
P_i	Profit incurred to the system for each Model i where $i=1,2$
$g(t), G(t)$	p.d.f. and c.d.f. of repair time for the unit for Model 1
$h(t), g_1(t), g_2(t), g_3(t)$	p.d.f. of inspection, repair, replacement, reconditioning/reinstallation time of the unit for Model 2
$H(t), G_1(t), G_2(t), G_3(t)$	c.d.f. of inspection, repair, replacement, reconditioning/reinstallation time of the unit for Model 2

Comparative Analysis of the Two Models

The transition diagram showing the various states of the system is shown as in Fig. 1. The epochs of entry into states S_0, S_1 and S_3 are regenerative points and thus are regenerative states. States S_2, S_4, S_5, S_6 and S_7 are failed states.

A. Description of Model 1:

The transition diagram showing the various states of the Model 1 is shown as in Fig.1. The epochs of entry into states

S_0, S_1 and S_3 are regeneration points and thus are regenerative states. States S_2 and S_4 are failed states.

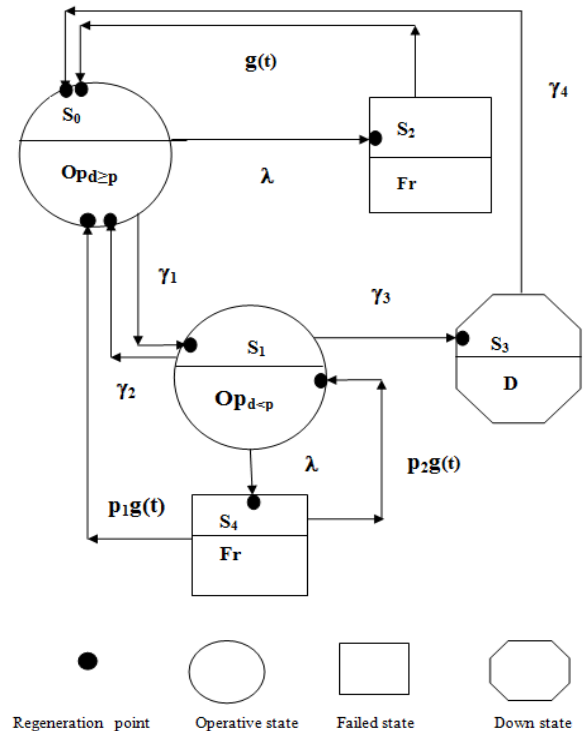


Fig.1: Transition state diagram of Model 1

The transition probabilities (q_{ij}) for Model 1 are given as:

$$\begin{aligned} q_{01}(t) &= \gamma_1 e^{-(\lambda + \gamma_1)t} \\ q_{02}(t) &= \lambda e^{-(\lambda + \gamma_1)t} \\ q_{10}(t) &= \gamma_2 e^{-(\gamma_2 + \gamma_3 + \lambda)t} \\ q_{13}(t) &= \gamma_3 e^{-(\gamma_2 + \gamma_3 + \lambda)t} \\ q_{14}(t) &= \lambda e^{-(\gamma_2 + \gamma_3 + \lambda)t} \\ q_{20}(t) &= g(t) \\ q_{30}(t) &= \gamma_4 e^{-\gamma_4 t} \\ q_{40}(t) &= p_1 g(t) \quad q_{41}(t) = p_2 g(t) \end{aligned}$$

The non-zero elements p_{ij} obtained as $p_{ij} = \lim_{s \rightarrow 0} q_{ij}^*(s)$ and mean sojourn time (μ_i).

$$\begin{aligned} p_{01} &= \frac{\gamma_1}{(\lambda + \gamma_1)}, \quad p_{02} = \frac{\lambda}{(\lambda + \gamma_1)}, \quad p_{10} = \frac{\gamma_2}{(\lambda + \gamma_2 + \gamma_3)}, \quad p_{13} = \frac{\gamma_3}{(\lambda + \gamma_2 + \gamma_3)} \\ p_{14} &= \frac{\lambda}{(\lambda + \gamma_2 + \gamma_3)}, \quad p_{20} = 1, \quad p_{30} = 1, \quad p_{40} = p_1, \quad p_{41} = p_2, \\ \mu_0 &= \frac{1}{(\lambda + \gamma_1)}, \quad \mu_1 = \frac{1}{(\lambda + \gamma_2 + \gamma_3)}, \quad \mu_2 = \int_0^\infty \bar{G}(t) dt = \mu_4, \quad \mu_3 = \frac{1}{\gamma_4} \end{aligned}$$

B. Measures of System Effectiveness of Model 1:

Using probabilistic arguments for regenerative process, various recursive relations for Model 1 solved thoroughly in [14]. Here, only results are shown to shorten the length of

paper. Various measures of system effectiveness for Model 1 are given as:

$$\text{MTSF (M1)} = \frac{\mu_0 + p_{01}\mu_1 + p_{01} p_{13}\mu_3}{p_{01} p_{14} + p_{02}} \quad (1)$$

$$\text{Steady state availability when demand is not less than production (AD1)} = \frac{(1 - p_{14}p_{41})\mu_0}{D_1} \quad (2)$$

$$\text{Steady state availability when demand is less than production (AP1)} = \frac{p_{01}\mu_1}{D_1} \quad (3)$$

$$\text{Busy Period analysis of the repairman (B1)} = \frac{p_{02}(1 - p_{14}p_{41})\mu_2 + p_{01}p_{14}\mu_4}{D_1} \quad (4)$$

$$\text{Expected number of visits of repairman (V1)} = \frac{p_{02}(1 - p_{14}p_{41}) + p_{01}p_{14}}{D_1} \quad (5)$$

$$\text{Expected down time (DT1)} = \frac{p_{01} p_{13}}{D_1} \quad (6)$$

where

$$D_1 = (1 - p_{14}p_{41})\mu_0 + p_{01}\mu_1 + p_{02}(1 - p_{14}p_{41})\mu_2 + p_{01}p_{13}\mu_3 + p_{01}p_{14}\mu_4$$

$$\begin{aligned} \text{Expected total revenue} &= (\text{revenue/time when } d \geq p (C_0)) * \text{steady state availability (AD1) when } d \geq p \\ &+ (\text{revenue/time when } d < p (C_1)) * \text{steady state availability (AP1) when } d < p \\ &= C_0 * \text{AD1} + C_1 * \text{AP1} \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Expected total cost} &= \text{cost per unit time for engaging the repairman } (C_2) * \text{busy Period of the repairman (B1)} \\ &+ (\text{cost per visit of the repairman } (C_3)) * \text{expected number of visits of repairman (V1)} \\ &+ (\text{loss per unit time during the system remains down } (C_4)) * \text{expected down time (DT1)} \end{aligned}$$

$$= C_2 * B1 + C_3 * V1 + C_4 * DT1 \quad (8)$$

$$\begin{aligned} \text{Expected profit} &= \text{Expected total revenue} \\ &- \text{Expected total cost} \\ \text{profit (P1)} &= (C_0 * \text{AD1} + C_1 * \text{AP1}) \\ &- (C_2 * B1 + C_3 * V1 + C_4 * \text{DT1}) \end{aligned} \quad (9)$$

where

$$\text{AD1} = \frac{(1 - p_{14}p_{41})\mu_0}{D_1}, \text{ AP1} = \frac{p_{01}\mu_1}{D_1},$$

$$B1 = \frac{p_{02}(1 - p_{14}p_{41})\mu_2 + p_{01}p_{14}\mu_4}{D_1},$$

$$V1 = \frac{p_{02}(1 - p_{14}p_{41}) + p_{01}p_{14}}{D_1}, \text{ DT1} = \frac{p_{01} p_{13}}{D_1} \text{ and}$$

$$D_1 = (1 - p_{14}p_{41})\mu_0 + p_{01}\mu_1 + p_{02}(1 - p_{14}p_{41})\mu_2 + p_{01}p_{13}\mu_3 + p_{01}p_{14}\mu_4$$

C. Description of Model 2:

The transition diagram showing the various states of the Model 2 is shown as in Fig. 2. The epochs of entry into states S_0, S_1 and S_3 are regenerative points and thus are regenerative states. States S_2, S_4, S_5, S_6 and S_7 are failed states. The transition probabilities are:

$$\begin{aligned} q_{01}(t) &= \gamma_1 e^{-(\lambda + \gamma_1)t} \\ q_{02}(t) &= \lambda e^{-(\lambda + \gamma_1)t} \\ q_{10}(t) &= \gamma_2 e^{-(\gamma_2 + \gamma_3 + \lambda)t} \\ q_{13}(t) &= \lambda_3 e^{-(\gamma_2 + \gamma_3 + \lambda)t} \\ q_{14}(t) &= \lambda e^{-(\gamma_2 + \gamma_3 + \lambda)t} \\ q_{25}(t) &= p_4 h(t) \\ q_{26}(t) &= p_5 h(t) \\ q_{27}(t) &= p_3 h(t) \\ q_{30}(t) &= \gamma_4 e^{-\gamma_4 t} \\ q_{40}(t) &= p_1 g_1(t) \\ q_{41}(t) &= p_2 g_1(t) \\ q_{50}(t) &= g_1(t) \\ q_{60}(t) &= g_2(t) \\ q_{70}(t) &= g_3(t) \end{aligned}$$

The non-zero elements p_{ij} obtained as $p_{ij} = \lim_{s \rightarrow 0} q_{ij}^*(s)$

$$\text{where } p_{01} = \frac{\gamma_1}{(\lambda + \gamma_1)}, p_{02} = \frac{\lambda}{(\lambda + \gamma_1)}, p_{10} = \frac{\gamma_2}{(\lambda + \gamma_2 + \gamma_3)},$$

$$p_{13} = \frac{\gamma_3}{(\lambda + \gamma_2 + \gamma_3)}, p_{14} = \frac{\lambda}{(\lambda + \gamma_2 + \gamma_3)}$$

$$p_{25} = p_1, p_{26} = p_2, p_{27} = p_3, p_{40} = p_4,$$

$$p_{41} = p_5, p_{50} = 1, p_{60} = 1, p_{70} = 1,$$

and hence mean sojourn times (μ_i) are:

$$\mu_0 = \frac{1}{(\lambda + \gamma_1)}, \mu_1 = \frac{1}{(\lambda + \gamma_2 + \gamma_3)}, \mu_2 = \int_0^\infty \bar{H}(t)dt,$$

$$\mu_3 = \frac{1}{\gamma_4}, \mu_4 = \int_0^\infty \bar{G}_1(t)dt = \mu_5, \mu_6 = \int_0^\infty \bar{G}_2(t)dt,$$

$$\mu_7 = \int_0^\infty \bar{G}_3(t)dt$$

D. Measures of System Effectiveness of Model 2:

Using probabilistic arguments for regenerative process, various recursive relations for Model 2 are solved [15]. Various measures of system effectiveness for Model 2 are given as:

MTSF(M2)	$= \frac{\mu_0 + p_{01}\mu_1 + p_{01} p_{13}\mu_3}{p_{01} p_{14} + p_{02}}$	(10)
Steady State Availability when demand is not less than production (AD2)	$= \frac{(1 - p_{14}p_{41})\mu_0}{D_2}$	(11)
Steady State Availability when demand is less than production (AP2)	$= \frac{p_{01}\mu_1}{D_2}$	(12)
Busy Period Analysis of the Repairman (inspection time only) (I2)	$= \frac{p_{02}(1 - p_{14}p_{41})\mu_2}{D_2}$	(13)

Busy Period Analysis of the Repairman (repair time only) (B2)	$= \frac{p_{01}p_{14}\mu_4 + p_{02}p_{25}(1-p_{14}p_{41})\mu_5}{D_2}$ (14)	
Busy Period Analysis of the Repairman (replacement time only) (BR2)	$= \frac{p_{02}p_{26}(1-p_{14}p_{41})\mu_6}{D_2}$ (15)	
Busy Period Analysis of the Repairman (reconditioning/reinstallation time only) (BRR2)	$= \frac{p_{02}p_{27}(1-p_{14}p_{41})\mu_7}{D_2}$ (16)	
Expected number of visits of repairman (V2)	$= \frac{p_{02}p_{25}(1-p_{14}p_{41}) + p_{01}p_{14}}{D_2}$ (17)	
Expected down time (DT2)	$= \frac{p_{01}p_{13}\mu_3}{D_2}$ (18)	

where

$$D_2 = (1-p_{14}p_{41})\mu_0 + p_{01}\mu_1 + p_{02}(1-p_{14}p_{41})\mu_2 + p_{01}p_{13}\mu_3 + p_{01}p_{14}\mu_4 + p_{02}(1-p_{14}p_{41})(p_{25}\mu_5 + p_{26}\mu_6 + p_{27}\mu_7)$$

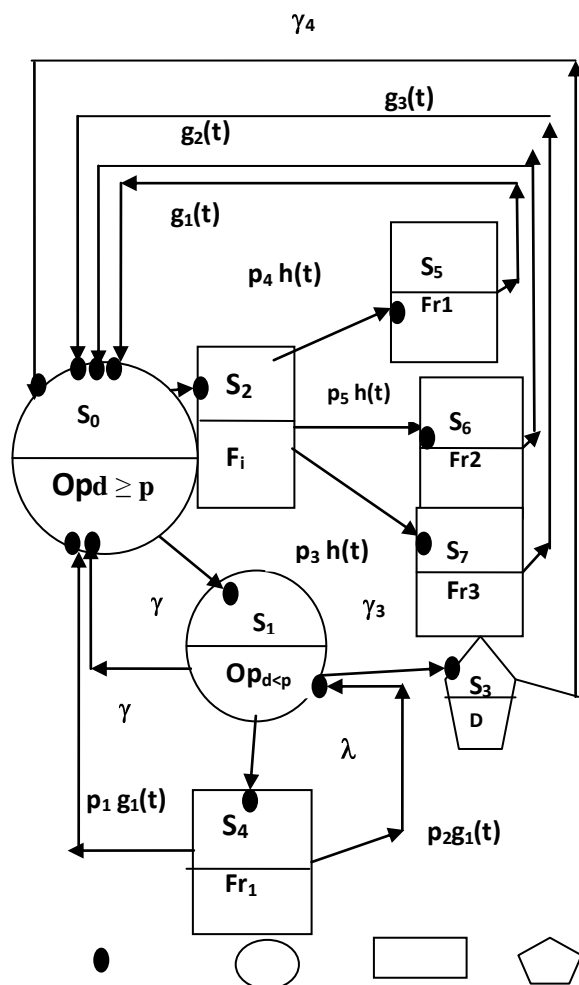


Fig. 2: Transition state diagram of Model 2

Regeneration point Operative state Failed state Downstate

$$\text{Expected total revenue for Model 2} = C_0 \cdot AD2 + C_1 \cdot AP2 \quad (19)$$

$$\begin{aligned} \text{Expected total cost for Model 2} &= C_2 \cdot B2 + C_3 \cdot V2 + C_4 \cdot DT2 \\ &+ (\text{cost } (C_5) \text{ per unit time for which the repairman is busy for inspection}) \cdot I2 \\ &+ (\text{cost } (C_6) \text{ per unit time for which the repairman is busy for replacement}) \cdot BR2 \\ &+ (\text{cost } (C_7) \text{ per unit time for which the repairman is busy for reconditioning /reinstallation}) \cdot BRR2 \\ &+ (\text{cost } (C_8) \text{ per unit replacement}) \cdot RP2 \end{aligned}$$

$$= C_2 \cdot B2 + C_3 \cdot V2 + C_4 \cdot DT2 + C_5 \cdot I2 + C_6 \cdot BR2 + C_7 \cdot BRR2 + C_8 \cdot RP2 \quad (20)$$

Total profit = Expected total revenue - Expected total cost
Using (19) and (20)

$$\begin{aligned} \text{profit (P2)} &= (C_0 \cdot AD2 + C_1 \cdot AP2) \\ &- (C_2 \cdot B2 + C_3 \cdot V2 + C_4 \cdot DT2 + C_5 \cdot I2 \\ &- C_6 \cdot BR2 - C_7 \cdot BRR2 - C_8 \cdot RP2) \end{aligned} \quad (21)$$

$$AD2 = \frac{(1-p_{14}p_{41})\mu_0}{D_2}, \quad AP2 = \frac{p_{01}\mu_1}{D_2},$$

$$B2 = \frac{p_{01}p_{14}\mu_4 + p_{02}p_{25}(1-p_{14}p_{41})\mu_5}{D_2},$$

$$V2 = \frac{p_{02}p_{25}(1-p_{14}p_{41}) + p_{01}p_{14}}{D_2},$$

$$DT2 = \frac{p_{01}p_{13}\mu_3}{D_2}, \quad I2 = \frac{p_{02}(1-p_{14}p_{41})\mu_2}{D_2},$$

$$BR2 = \frac{p_{02}p_{26}(1-p_{14}p_{41})\mu_6}{D_2},$$

$$BRR2 = \frac{p_{02}p_{27}(1-p_{14}p_{41})\mu_7}{D_2},$$

$$RP2 = \frac{p_{02}p_{26}(1-p_{14}p_{41})}{D_2}$$

$$D_2 = (1-p_{14}p_{41})\mu_0 + p_{01}\mu_1 + p_{02}(1-p_{14}p_{41})\mu_2 + p_{01}p_{13}\mu_3 + p_{01}p_{14}\mu_4 + p_{02}(1-p_{14}p_{41})(p_{25}\mu_5 + p_{26}\mu_6 + p_{27}\mu_7)$$

Graphical Analysis

For the particular case, Let us take $g(t) = g_1(t) = \alpha e^{-\alpha t}$, $g_2(t) = \alpha_2 e^{-\alpha_2 t}$, $g_3(t) = \alpha_3 e^{-\alpha_3 t}$, $h(t) = \alpha_1 e^{-\alpha_1 t}$

Different graphs have been plotted for the availabilities and the profit with respect to rates/costs. The values of various parameters are given in respective tables.

Following interpretations can be made from the graphs:

It has been observed that $MTSF = \frac{\mu_0 + p_{01}\mu_1 + p_{01}p_{13}\mu_3}{p_{01}p_{14} + p_{02}}$

for both the Models remain same for change in the values of the inspection rate (α_1). However, the behavior of the availabilities is affected by change in the inspection rate as discussed below:

Fig. 3 depicts the behavior of the availabilities (AD1, AD2) when demand is not less than the production with respect to

the inspection rate (α_1). It is clear from the graph that availability AD2 gets increased with increase in the values of α_1 and availability AD1 remains almost unaffected. It can also be interpreted from the graph that AD2 is $>$ or $=$ or $<$ AD1 according as $\alpha_1 >$ or $=$ or $<$ 0.0763. So, the Model 2 is better than the Model 1 according as $\alpha_1 >$ 0.0763. In case of $\alpha_1 = 0.0763$, both Models are equally good.

Table 1: (AD1, AD2) for different α_1

α_1	AD2	AD1
0.03	0.762733	0.778105
0.13	0.793538	0.778105
0.23	0.79774	0.778105
0.33	0.799408	0.778105
0.43	0.800303	0.778105
0.53	0.800861	0.778105

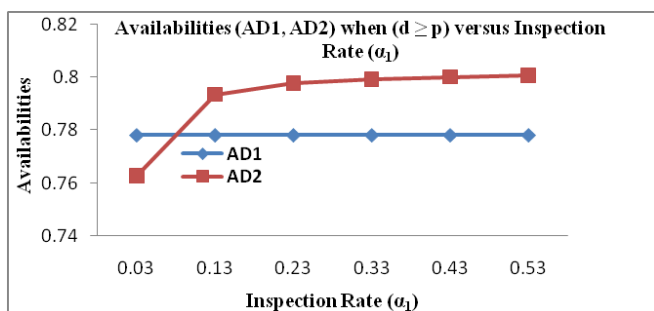


Fig. 3: Availabilities (AD1, AD2) versus α_1

Fig. 4 depicts the behaviour of the Availabilities (AP1, AP2) when demand is less than the production with respect to the inspection rate (α_1). It is clear from the graph that availability AD2 gets increased with increase in the values of α_1 and availability AD1 remains almost unaffected. It can also be interpreted from the graph that AP2 $>$ or $=$ or $<$ AP1 according as $\alpha_1 >$ or $=$ or $<$ 2.531. So, the Model 2 is better or worse than the first Model 1 according as $\alpha_1 >$ or $<$ 2.531. In case of $\alpha_1 = 2.531$, both Models are equally good.

Table 2: (AP1, AP2) for different α_1

α_1	AP1	AP2
2	0.097138	0.097121
2.1	0.097138	0.097125
2.2	0.097138	0.097128
2.3	0.097138	0.097131
2.4	0.097138	0.097134
2.5	0.097138	0.097137
2.6	0.097138	0.097139
2.7	0.097138	0.097142
2.8	0.097138	0.097144
2.9	0.097138	0.097146

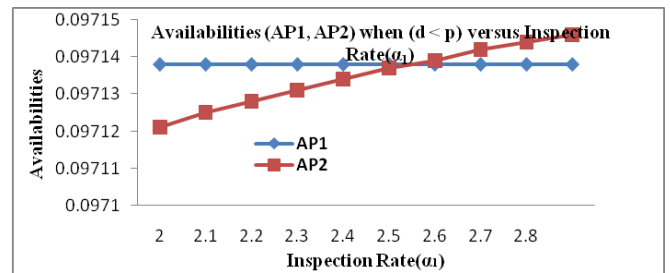


Fig. 4: Availabilities (AP1, AP2) versus α_1

Fig. 5 depicts the behavior of profits (P1, P2) with respect to cost (C_5) per unit time for which the repairman is busy for inspection. It is clear from the graph that profit P2 gets decreased with increase in the values of C_5 and profit P1 remains almost unaffected. Also, $P2 >$ or $=$ or $<$ P1 according as C_5 is $<$ or $=$ or $>$ 602.233. So, if $C_5 <$ 602.233, one should opt for the Model 2. In case of $C_5 = 602.233$, both Models are equally good.

Table 3: (P1, P2) for different C_5

C_5	P1	P2
100	792.303	800.35
300	792.303	797.2417
500	792.303	794.1335
700	792.303	791.0252
900	792.303	787.9169
1100	792.303	784.8087
1300	792.303	781.7004
1500	792.303	778.5922
1700	792.303	775.4839
1900	792.303	772.3757

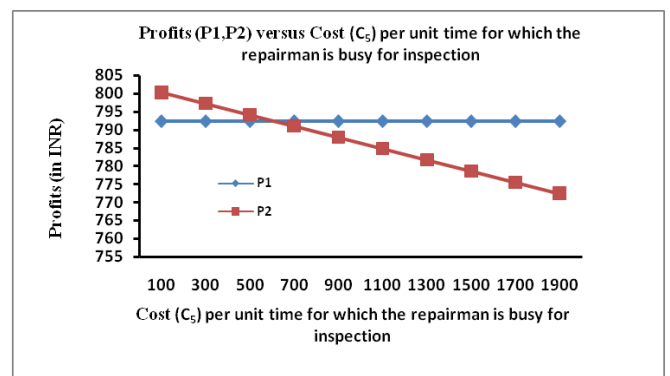


Fig. 5: Profits (P1, P2) versus cost (C_5) per unit time

Fig. 6 depicts the behaviour of profits (P1, P2) with respect to cost (C_2) per unit time for which the repairman is busy for repair. It is clear from the graph that both the profits (P1, P2) get decreased with increase in the values of C_2 . Also, $P2 >$ or $=$ or $<$ P1 according as C_2 is $>$ or $=$ or $<$ 595.187. Hence, Model 2 is better than the Model if $C_2 >$ 595.187. In case of $C_2 = 595.187$, both Models are equally good.

Table 4: (P1, P2) for different C_2

C_2	P1	P2
100	806.303	795.6642
300	795.858	789.4945
500	785.413	783.3248
700	774.968	777.1551
900	764.523	770.9854
1100	754.078	764.8157
1300	743.633	758.646
1500	733.187	752.4763
1700	722.742	746.3066
1900	712.297	740.137

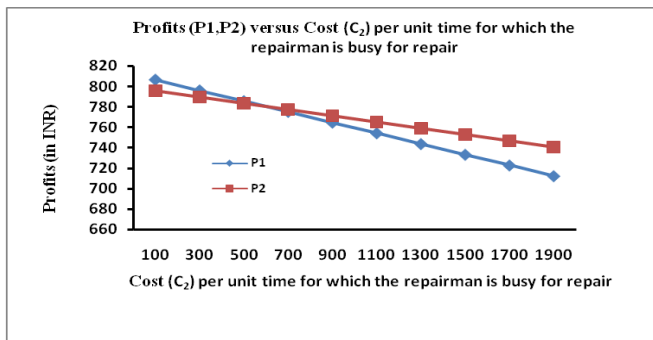


Fig. 6: Profits (P1, P2) versus Cost (C_2) per unit time

Fig. 7 depicts the behaviour of profits (P1, P2) with respect to cost (C_6) per unit time for which the repairman is busy for replacement. It is clear from the graph that profit P2 gets decreased with increase in the values of C_6 and profit P1 remains almost unaffected. Also, $P2 > \text{or} = \text{or} < P1$ according as C_6 is $< \text{or} = \text{or} > 502.123$. Hence, Model 2 is better than the Model 1 if C_6 is < 502.123 .

Table 5: (P1, P2) for different C_6

C_6	P1	P2
100	806.303	807.7839
300	806.303	807.0313
500	806.303	806.2787
700	806.303	805.5261
900	806.303	804.7736
1100	806.303	804.021
1300	806.303	803.2684
1500	806.303	802.5159

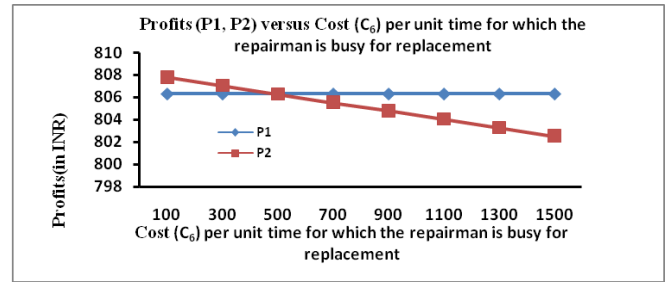


Fig. 7: Profits (P1, P2) versus cost (C_6) per unit time

Fig. 8 depicts the behaviour of profits (P1, P2) with respect to cost (C_7) per unit time for which the repairman is busy for reconditioning /reinstallation. It is clear from the graph that profit P2 gets decreased with increase in the values of C_7 and profit P1 remains almost unaffected. Also, $P2 > \text{or} = \text{or} < P1$ according as C_7 is $< \text{or} = \text{or} > 1424.146$. Hence, Model 2 is better than the Model 1 if C_7 is < 1424.146 .

Table 6: (P1, P2) for different C_7

C_7	P2	P1
300	808.0007	806.303
500	807.6819	806.303
700	807.363	806.303
900	807.0442	806.303
1100	806.7253	806.303
1300	806.4065	806.303
1500	806.0876	806.303
1700	805.7688	806.303

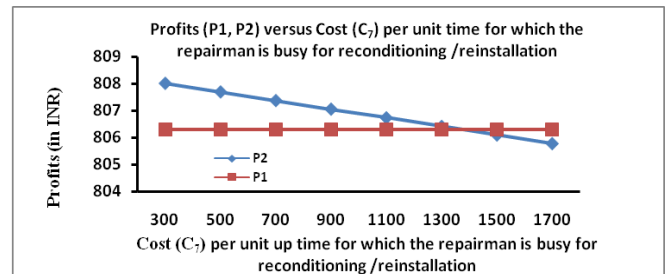


Fig. 8: Profits (P1, P2) versus Cost (C_7) per unit time

Which and when one Model is better than the other has been presented in the following comparison Table 7:

Table 7: Comparison Table

Fig. No. NO.	Fixed Parameter (All rates are taken per hr and all costs are in Indian rupees)	Comparison with respect to	Which Model is better (according to different situations)		
			Model 1 is better if	Model 2 is better if	Both the Models are equally good
3	$\gamma=0.003, \alpha=.05, \alpha_2=.05, \alpha_3=.05, \gamma_1=.07, \gamma_2=.235, \gamma_3=.353, \gamma_4=.4213, p_1 = 0.665, p_2=0.335, p_{25}=0.543, p_{26}=0.321, p_{27}=1-p_{25}-p_{26}, p_4=.665, p_5=1-p_4, C_0=1000, C_1=700, C_6=200, C_3=100, C_4=400, C_2=100, C_7=400, C_8=700$	AD when $d \geq p$	$\alpha_1 < 0.0763$	$\alpha_1 > 0.0763$	$\alpha_1 = 0.0763$
4		AP when $d < p$	$\alpha_1 < 2.531$	$\alpha_1 > 2.531$	$\alpha_1 = 2.531$
5		Profit	$C_5 > 602.233$	$C_5 < 602.233$	$C_5 = 602.233$
6	$C_5 = \text{INR } 600$ Other values are same as mentioned above except C_2	Profit	$C_2 < 595.187$	$C_2 > 595.187$	$C_2 = 595.187$
7	Other values are same as mentioned above except C_6	Profit	$C_6 > 502.123$	$C_6 < 502.123$	$C_6 = 502.123$
8	Other values are same as mentioned above except C_7	Profit	$C_7 > 1413.234$	$C_7 < 1413.234$	$C_7 = 1413.234$

Conclusion

This paper compares two stochastic Models of a cable manufacturing plant with varying demand. The evaluation of systems is done by means of steady state availabilities, busy period of repairmen, profit functions using Laplace transforms and software package Code-Blocks 13.12. Considering particular cases, graphs have been plotted which help to decide which Model is more beneficial as compared to other. It has been observed from the graphs that

- MTSFs for both the Models remain same.
- Availabilities (AD2, AP2) of Model 2 increases as inspection rate increases whereas the availabilities (AD1, AP1) of Model 1 remain almost unaffected. For availabilities ($d \geq p$) and ($d < p$) Model 2 is better than Model 1 if inspection rate > 0.0763 and 2.531 .
- The profit of Model 2 gets decreased with increase in the values of cost of inspection, replacement, reconditioning of system whereas profit of Model 1 remains almost unaffected. Both profits (P1, P2) decreased with increase in the values of cost of repair. Model 2 is better than Model 1 if cost per unit time for which the repairman is busy for inspection, replacement, reconditioning $< 602.233, 502.123, 1413.234$ and cost per unit time for which the repairman is busy for repair > 595.187 .
- Cut-off points for inspection rate and various costs have been obtained which may be quite useful for the system manufacturers, engineers and the system analysts to check which Model is better than the other. Also, one can make such comparative study by taking other parameters such as rates/costs/revenue etc. and can see as to which Model is more beneficial as compared to other.

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