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A Study of the family of Affine Projection Algorithms

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Abstract- The class of affine projection algorithms (APAs) provides faster convergence than LMS-based adaptive filters. Its convergence analysis is extensively studied and still remains an active area of research. Here we consider two different parameters to analyze the performance of APAs. This paper aims to summarize some significant advances in the theory of convergence behavior of APA and its variants forming a family. Several new techniques are emerging in the current literature.

Keywords: Adaptive Filter, Normalized Least Mean Square Algorithm, Affine Projection Algorithm Mean-square error, convergence

Introduction

Background on LMS, NLMS, RLS and APA algorithms

In Fig. 1, we show the prototypical adaptive filter setup, where x(n), d(n) and e(n) are the input, the desired signal and the output error signals, respectively. The **vector** h(n) is the column **vector** of filter coefficient at time n , in such a way that the output of signal, y(n) , is good estimate of the desired signal, d(n).

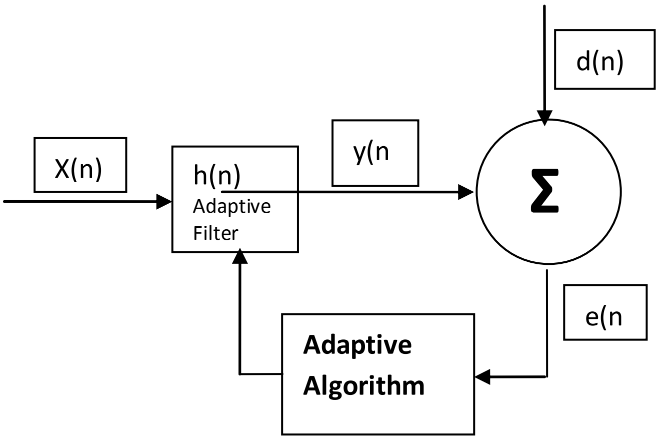

Fig.1. Prototypical adaptive filter setup

The algorithm is the procedure used to adjust the adaptive filter coefficients in order to minimize a prescribed criterion. The algorithm is determined by defining the search method (or minimization algorithm), the objective function, and the error signal nature. The choice of the algorithm determines several crucial aspects of the overall adaptive process, such as existence of sub-optimal solutions, biased optimal solution, and computational complexity.

The basic objective of the adaptive filter is to set its parameters, in such a way that its output tries to minimize a meaningful objective function involving the reference signal. In the final analysis, the choice of one algorithm over another is determined by one or more of the following factors:

Rate of Convergence, Misadjustment, Tracking, Robustness, Stability, Computational Complexity, Computational Cost, Mean Square Error (MSE)[1,2,3]

We use several algorithms for adaptive filter. Some amongst them are Least Mean Square(LMS), Normalized Least Mean Square (NLMS), Recursive Least Square (RLS) and Affine Projection(AP)[3,9]. In this paper we concentrate on different aspects of the Affine Projection Algorithm and its variants. The Affine Projection Algorithm (APA) is an adaptive filtering algorithm family capable of improved convergence with correlated input signals, while providing a computational cost comparable to the LMS algorithm. APA uses the previous input **vectors**, instead of just the current **vector** used by NLMS. Many variants of APA are available such as regularized APA(R-APA), the partial rank algorithm (PRA), the De-correlating algorithm, bi- normalized data-reusing LMS (BNDR-LMS), NLMS with orthogonal correction factor (NLMS-OCF). The APA family encompasses many of its variants, extensions and generalization in its field. During the last three decades after the appearance of the seminal paper on APA by Uzeki et.al[4] a lot of research work has been done in this area.

In the literature review, it has been noticed that all the researchers have obtained unique respective result in their research regarding the affine projection algorithm and verified their proposals with simulated results.

Overview of the Affine Projection Adaptive Filter

The affine projection algorithms (APAs) emerged to improve speed of convergence of gradient based algorithms when the input signals did not exhibit flat spectrum, since the speed of convergence of these algorithms decreased substantially in these cases [5,6,7].

A common feature that includes all APAs is the filter update equation, which uses N (called projection order) **vectors** of the input signal instead of a single **vector** as the NLMS algorithm[8]. Therefore, these algorithms could be assumed as an extension of the algorithm NLMS, or more generally, it can be expressed mathematically as follows.
Let equation (1) be the change of the L adaptive filter coefficients between successive algorithm iterations,

$$\Delta \mathbf{w}_L[n] = \mathbf{w}_L[n] - \mathbf{w}_L[n-1]. \quad (1)$$

To develop the algorithm the expression (2) has to be minimized under N constraints given by (3):

$$\| \Delta \mathbf{w}_L[n] \|^2 = \Delta \mathbf{w}_L^T[n] \Delta \mathbf{w}_L[n] \quad (2)$$

$$\mathbf{w}_L^T[n] \mathbf{x}_L[n-k] = d[n-k] \quad (3)$$

Where k = 0,1, ...,N−1 *and* $\mathbf{x}_L[n]$ is a **vector** that comprises the last L samples of the input **vector** and $d[n]$ represent the desired signal.

The solution of these equations represents the Affine Projection update equation which is given as-

$$\mathbf{w}_L[n] = \mathbf{w}_L[n-1] + \mathbf{A}^T[n](\mathbf{A}[n]\mathbf{A}^T[n])^{-1}\mathbf{e}_N[n] \quad (4)$$

where

$\mathbf{e}_N(n)$ is a **vector** of size $N \times 1$ given by

$$\mathbf{e}_N[n] = \mathbf{d}_N[n] - \mathbf{A}[n]\mathbf{w}_L[n-1] \quad (5)$$

$$\mathbf{A}[n] = (\mathbf{x}_L[n], \mathbf{x}_L[n-1], \ldots, \mathbf{x}_L[n-N+1])^T \quad (6)$$

and $\mathbf{d}_N[n]$ represents the desired signal **vector** of size $N \times 1$

$$\mathbf{d}^T_N[n] = (d[n], d[n-1], \ldots, d[n-N+1]) \quad (7)$$

Equation (4) can be expressed by a general form as below

$$\mathbf{w}_L[n] = \mathbf{w}_L[n-1-\alpha(N-1)] + \mu \mathbf{A}\tau^T[n](\mathbf{A}\tau[n]\mathbf{A}\tau^T[n] + \delta \mathbf{I})^{-1}\mathbf{e}_{N\tau}[n] \quad (8)$$

with $\mathbf{e}_{N\tau}[n] = \mathbf{d}_{N\tau}[n] - \mathbf{A}\tau\mathbf{w}_L[n-1-\alpha(N-1)]$,
$\mathbf{A}\tau[n] = (\mathbf{x}_L[n], \mathbf{x}_L[n-\tau], \ldots, \mathbf{x}_L[n-(N-1)\tau])^T$ *and*
$\mathbf{d}^T_{N\tau}[n] = (d[n], d[n-\tau], \ldots, d[n-(N-1)\tau])$.

The normalized least mean-squares (NLMS) algorithm is most popular adaptive algorithms due to its low computational cost and ease of implementation but the correlated input signals reduce its convergence speed considerably. To overcome this problem, the affine projection algorithm (APA) has been proposed. This algorithm updates the weights based on the last K input **vectors** in order to improve the convergence speed of LMS-type filters for correlated input signals[10]. The APA is also viewed as a generalization of the NLMS because the NLMS is the same as a one-dimensional APA (K=1)[9]. The performance of the APA especially depends on the number of input **vectors**. As more input **vectors** are involved, the convergence speed improves, but the steady-state error gets worse [9].

Affine Projection Algorithm (APA) family

After reviewing the literature in this activated arena of APA we have classified this APA family into subfamilies of its variants on the basis of the role played by some parameters identifying distinctive unique features in the performance of the algorithms.
We have identified some parameters which are involved in Affine Projection Algorithms (APAs). These parameters are:

(a) Step Size(μ)
(b) Projection order(N)
(c) Regularization Factor(δ)
(d) Forgetting Factor (λ)
(e) Smoothing Factor (α)
(f) Time Variable (τ)
(g) Learning Rate (ρ)
(h) Conversion factor (γ)

These parameters are either constant or variable. Concentrating on only two parameters, step size μ and projection order N, APAs are classified as following four subfamilies namely

(a) Classical or Conventional APA
(b) VSS-APA
(c) APA-VPO
(d) VSS-APA-VPO

Fixed step size and fixed projection order(Classical or Conventional APA**):** The most common adaptive algorithms is Least Mean Square (LMS) algorithm, but this algorithm suffers from slow convergence speed if it is driven by colored input signals as is with speech. One method presented to overcome this problem is the Ozeki/ Umeda Affine Projection Algorithm [4]. APA is a useful family of adaptive filters whose main purpose is to accelerate the convergence of LMS-type filters, especially for correlated data at a computational cost that is comparable to that of LMS. While LMS updates the weights based on the current input **vector** and APA updates the weights based on previous input **vectors**. The algorithm applies to update directions that are orthogonal to the last p input **vectors** and thus allows decorrelation of an input process, speeding up the convergence [11]. An APA with a constant step-size parameter has to compromise between the performance's criteria of fast convergence rate, and low misadjustment. The examples of this type of algorithms are : Regularized APA (R-APA), Binormelized Data Reusing LMS (BNDR-LMS, Decorrelating algorithm (D-APA), NLMS with Orthogonal Correction Factor (NLMS- OCF), Partial Rank APA (PR-APA), Periodic APA (P-APA), Generalized Optimal Block Algorithm (GOBA), Selective Regressors APA (SR-APA).

Variable step size and fixed order Affine Projection Algorithms (VSS-APA**):** The step size μ commands the convergence speed and strongly influences the steady state performance of the APA algorithm. As μ increases the convergence is faster but the Mean Squared Error (MSE) in steady state worsens [12,13,14].The variable step size of the algorithm is adjusted recursively from the maximum step size to minimum value based on rough estimation of the performance surface gradient square. An appropriate time varying value of the maximum step size is chosen to be inversely proportional to the instantaneous energy of the input signal **vector**. Then this time varying upper bound value of the step size is used to stability of the algorithm [15,16,17]. Variable step size algorithms allow adjusting the step size to the algorithm needs within both transient and steady states.
The theoretical MSE for the APA can be approximated by [18]

$$\text{MSE} \approx \frac{\mu N_v^2}{(2-\mu)} + \sigma_v^2 \qquad (9)$$

where σ_v^2 represents the measurement noise variance. The MSE can be approximated
when $\mu \approx 0$ by

$$\text{MSE} \approx \frac{\mu\sigma_v^2}{(2-\mu)} + \sigma_v^2 \qquad (10)$$

In concise, the APAs with low step size exhibit optimal behavior in steady state. However low values of μ, slow down their convergence speed. Several algorithms dynamically adjust the step size [19], all of them improve the MSE at steady state and do not worsen the convergence speed, but their computational cost is similar to the original APA. The examples of this type of algorithms are: Variable Step-Size APA with Forgetting Factor (VSS-APA-FF), Variable Step-Size APA with Forgetting Factor and its regularized version (VSS-APA-FF-REGU) Variable Step Size Selective Partial Update APA (VSS-SPU-APA)[11], Variable Step Size Selective Regressors APA (VSS-SR-APA)[20]. Fig.2 shows the effect of forgetting factor in variable step size APAs.

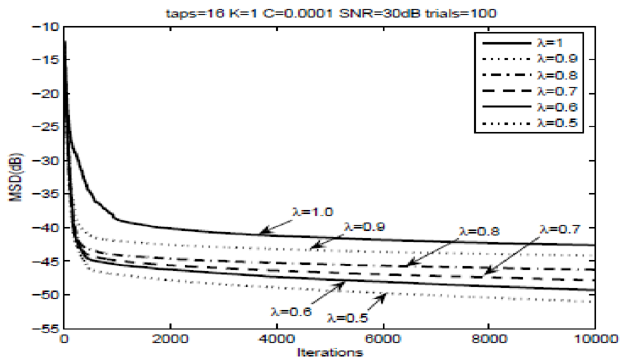

Fig.2. Effects of λ in VSS-APA-FF (VSS-APA when λ = 1.0), G2 colorization. VSS-NLMS-FF K=1,taps=16,C=0.0001, SNR=30dB[20]

Variable projection order and fixed step size Affine Projection Algorithms (APA-VPO**):** The Affine Projection Algorithm (APA) is a stochastic-gradient based adaptive filtering algorithm. Some approaches focus on controlling the step size or regularization parameter of the algorithm while others aim to control its projection order or to select optimal input regressors[21]. The variable projection order affine projection algorithms adjust its projection order to their convergence needs and therefore decrease their computational cost. Furthermore they reach a good MSE at steady state as it can be expected from[5]. These algorithms decrease the projection order when they reach the steady state or lower convergence speed is allowed. A first version of these algorithms, where the number of input data **vectors** to update the filter coefficients are selected within each algorithm iteration. All these strategies guarantee a good behavior at both steady and transient states, but mainly they try to optimize the computational cost when the algorithm does not need to work with high projection orders. The examples of this type of algorithms are: APA with Variable Projection Order (APA-VPO),APA with Orthogonal Correction Factor (APA-OCF), Set Membership APA with Variable Data Reuse –Factor (SM-APA-VDR), Dynamic Selection of input vectors APA (DS-APA). Fig. 3 show the MSE curve of APAs with variable projection order ranging from 2 to 16 and fig. 4 shows Normalized misalignment of various algorithms.

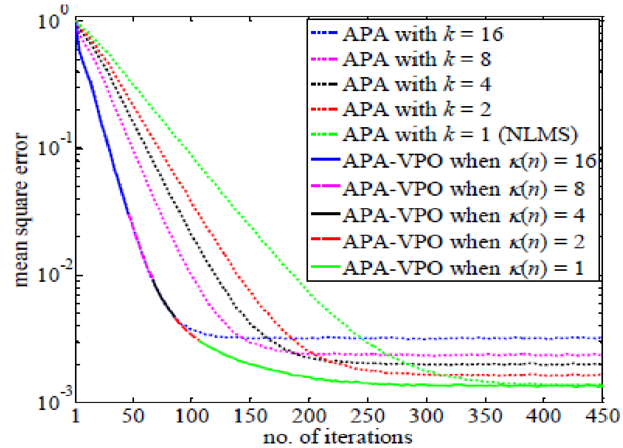

Fig. 3. MSE curves of APA-VPO, NLMS, and APA with different projection orders ranging from 2 to 16 . Input is Gaussian and μ=0.5[22]

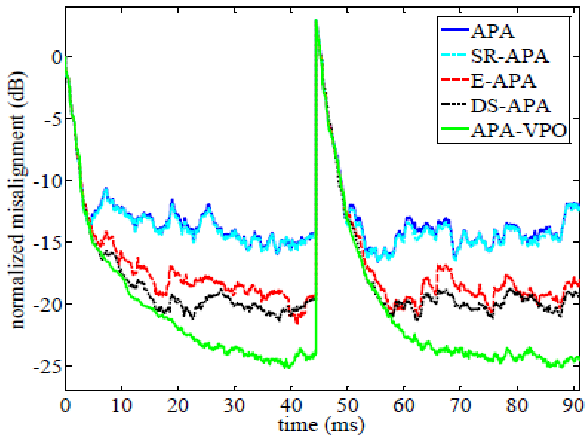

Fig. 4. Normalized misalignment of different algorithms for network echo path identification using voice input[22].

Variable projection order and Variable step size Affine Projection Algorithms (VSS-APA-VPO**):** Variable step size algorithms exhibit good steady state performance and the algorithms that adjust their projection orders reduce the computational cost when the convergence speed can be decreased. The APA with N = 1 and low step size would exhibit the lower cost and better steady state [22]. On the other hand high projection order and step size values improve the convergence properties of the algorithm. Thus algorithms that suitably adapt both parameters would eventually improve algorithm performance in almost any issue. An example of this kind of algorithm was proposed in [22].Here we emphasize an alternative algorithm that dynamically and simultaneously adjusts N and μ. We have used some of the AP variants which mention earlier. We start from the variable step size APA (VSSAPA) proposed in [5]. The step size follows an update rule that maximizes the change of the coefficients between iterations. We start from the variable step size APA (VSSAPA) proposed in [5]. The step size follows an update rule that maximizes the change of the coefficients between iterations as

$$\mu[n] = \mu_{max} \frac{\|P[n]\|^2}{\|P[n]\|^2 + C'} \qquad (11)$$

where $\mathbf{p}[n]$ represents an estimation of the mean value of $\varepsilon_N[n]$, and it is recursively obtained from $\mathbf{p}[n] = \alpha\mathbf{p}[n-1]+(1-\alpha)\varepsilon\ [n]$, with $0 < \alpha < 1$ and $C \approx \frac{N}{SNR}$, where SNR is the signal to noise ratio. Following the rules given above the algorithm would reach the steady state for a given projection order N, then it would be convenient to decrease the projection order to get lower cost and better MSE. In order to know when the algorithm should change its projection order we propose to use the condition presented in [23], where it is stated that an APA of order N has reached its steady state when

$$\gamma = \frac{R}{N} \leq 0.32 \qquad (12)$$

Where R *is* the number of elements of **vector** $\mathbf{e}_N[n] = (e1[n], e2[n] \ldots eN[n])^T$ in equation (6) that fulfils

$$e_i^2[n] \geq \frac{\mu[n] N \sigma_v^2}{2-\mu[n]} + \sigma_v^2 \qquad (13)$$

When the projection order decreases it is advisable to adjust the step size as well, in order to get a meaningful convergence speed. The new step size should fulfill

$$\mu[n] = \mu_{max} \frac{2\mu[n-1]N[n-1]}{\mu[n-1](N[n-1]-N[n])+2N[n]} \qquad (14)$$

Since the new projection order in time n is $N-1$ (it decrements a unit) when (12) is fulfilled, the proposed algorithm would carry out the step size readjustment following

$$\mu = \frac{2\mu N}{\mu+2N-2} \qquad (15)$$

Thus the algorithm would change to the following projection order until $N = 1$, at this stage it would work as the variable step size NLMS. The examples of this type of algorithms are: Exponentially Weighted APA (EW-APA), Switching Exponentially Weighted APA (SEW-APA), APA with Evolving order (E-APA). Fig. 5 shows the comparative performance the different APAs.

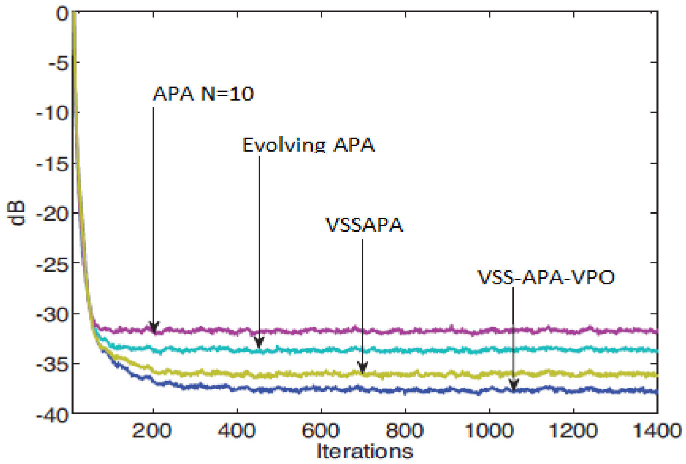

Figure 5: Comparative performance of different APAs[23].

Beside above four subfamilies of APA, we can involve two other sub families of APA, such as Fast Affine Projection Algorithm and Pseudo Affine Projection Algorithm. In Fast Affine Projection Algorithm the computational complexities have been reduced hence no delay between input and output. All these sub families form the APAs family. A lot of research is being done continuously on various aspects on these subfamilies.

Conclusion

In this paper we have described the different parameters involved in APA and considering only two parameters for classifying the family of APAs. Here we also consider impact of these families of APAs on convergence behavior, which show that we can optimize the convergence and MSE by handling the projection order and step size properly.

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