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BER analysis of OFDM system over Nonlinear fading channels

V. Jagan Naveen
Associate Professor ECE Department GMR Institute of Technology Rajam, Andhra Pradesh India -532127
jagannaveen801@gmail.com

K. Krishna kishore
Asst Professor ECE Department GMR Institute of Technology Rajam, Andhra Pradesh India -532127

P.Rajesh kumar
Professor ECE Department,AUCE Andhra University Vishakhapatnam AndhraPradesh, India -530003

Abstract
In the recent years, wireless communications have made use of orthogonal frequency division multiplexing (OFDM) technique to allow for high data rate transmission in multi path fading channels. This growing interest is due to realization that OFDM is an efficient scheme to convey information in frequency selective fading and flat fading channels without requiring complex time domain equalization techniques. In this paper the performance of OFDM with M-ary PSK system is measured over fading distribution. The non-linearity present in the propagation medium is utilized in terms of α-μ variants. In flat fading environment, channel estimation is done with the help of the trained symbols. Analysis is carried out in terms of Bit Error Rate (BER) versus Channel SNR.

Keywords— OFDM, M-ary PSK, Fading distribution, Bit error rate (BER), channel estimation.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation (MCM) technique which seems to be an attractive candidate for fourth generation (4G) wireless communication systems. OFDM offers high spectral efficiency, immune to the multipath delay, low inter-symbol interference (ISI), immunity to frequency selective fading and high power efficiency [1]. Due to these merits OFDM is chosen as high data rate communication systems such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for micro wave access (mobile Wi-MAX)[2].

OFDM system is capable of mitigating a frequency selective channel to a set of parallel fading channels, which need relatively simple processes for channel equalization. A key assumption in the theoretical explanation of the Rayleigh, Rician, Nakagami and Weibull distribution was that the statistics of the channel do not change over the local area under consideration[3]. However, to describe the long-term signal variations lognormal distribution is being used [4]. These distributions are helpful in designing of wireless systems to make the systems more robust to noise. There are many other fading models available in which we need to explore the non-linear fading environment[5].The interesting model that we could find in literature is α-μ distribution [6], which provides the generalized model for fading distribution. Depending upon the value of α and μ, this model can be utilized for the generation of Nakagami-m and Weibull variants. However, it can be treated as One-Sided Gaussian, Rayleigh and Negative Exponential distributions as its special cases. The generalized fading model using three parameter generalized gamma distribution describing all forms of multipath fading and shadowing in wireless systems are analyzed in [7].

This paper is organized as follows: In Section 2, Statistical characterization of fading channel is discussed. Section 3, describes the fading channel model to use in OFDM system. In Section 4, analysis of OFDM system with M-ary PSK has been done over different fading parameters and Section 5 concludes the paper.

II. Statistical Characterization Of Fading Channel

A. *Characterization of Fading Channel*

Statistical model of the fading channel is to Clarke's credit that he statistically characterized the electromagnetic field of the received signal at a moving terminal through a scattering process[8]. In Clarke's proposed model, there are N plane waves with arbitrary carrier phases, each coming from an arbitrary direction under the assumption that each plane wave has the same average power. A plane wave arriving from angle u with respect to the direction of a terminal movement with a speed of v, where all waves are arriving from a horizontal direction on x-y plane. As a mobile station moves, all plane waves arriving at the receiver undergo the Doppler shift. Let x(t)be a baseband transmit signal[9]. Then, the corresponding pass band transmit signal is given as

$$\hat{x}(t) = \mathrm{Re}\left[x(t)e^{j2\pi f_c t}\right] \quad (1)$$

Where Re[x(t)] denotes a real component of x(t), passing through a scattered channel of I different propagation paths with different Doppler shifts, the pass band received signal can be represented as

$$\hat{y}(t) = \mathrm{Re}\left[c_i(t)e^{j2\pi(f_c+f_i)(t-\tau_i)}x(t-\tau_i)\right] = \mathrm{Re}\left[y(t)e^{j2\pi f_c t}\right] \quad (2)$$

Where c_i, τ_i and f_i denote the channel gain, delay, and Doppler shift for the i^{th} propagation path, respectively. For the mobile speed of v and the wavelength, Doppler shift is given as

$$f_i = f_m \cos\theta_i = \frac{V}{\lambda}\cos\theta_i \quad (3)$$

Where f_m is the maximum Doppler shift and θ_i is the angle of arrival for the i^{th} plane wave. Note that the baseband received signal is given as

$$y(t) = \sum_{i=1}^{I} C_i e^{-j\phi(t)} x(t - \tau_i) \quad (4)$$

Where $\phi_i(t) = 2\pi\{(f_c + f_i)t_i - f_i t_i\}$. According to equation (1), the corresponding channel can be modelled as a linear time-varying filter with the following complex base band impulse response:

$$h(t,\tau) = \sum_{i=1}^{I} C_i e^{j\phi_i(t)} \delta(t - \tau_i) \quad (5)$$

Where $\delta(i)$ is a Dirac delta function. As long as difference in the path delay is much less than the sampling period T_s, path delay t_i can be approximated as $\hat{t}$, then equation (5) can be represented as

$$h(t,\tau) = h(t)\delta(t - \hat{\tau}) \quad (6)$$

Where, $h(t) = \sum_{i=1}^{I} C_i e^{-j\phi_i(t)}$

Assuming that x(t)=1, the received pass band signal $\hat{y}(t)$ can be expressed as

$$\hat{y}(t) = \mathrm{Re}[y(t)e^{j2\pi f_c t}]$$
$$= \mathrm{Re}[\{h_I(t) + jh_Q(t)\}e^{j2\pi f_c t}]$$
$$= h_I(t)\cos 2\pi f_c t - h_Q(t)\sin 2\pi f_c t \quad (7)$$

Where $h_I(t)$ and $h_Q(t)$ are in-phase and quadrature components of h(t), respectively given as

$$h_I(t) = \sum_{i=1}^{I} C_i \cos\phi_i(t)$$

$$h_Q(t) = \sum_{i=1}^{I} C_i \sin\phi_i(t)$$

Assuming that I is large enough, $h_I(t)$ and $h_0(t)$ in above equations can be approximated as Gaussian random variables by the central limit theorem. Therefore amplitude of the received signal over the multipath channel subject to numerous scattering components follows the Rayleigh distribution [10].

$$\tilde{y}(t) = \sqrt{h_I^2(t) + h_Q^2(t)}$$

The power spectrum density (PSD) of the fading process is found by the Fourier transform of the autocorrelation function of $\hat{y}(t)$ and is given

Where, $\Omega p = E[h_I^2(t)] + E[h_Q^2(t)] = \sum_{i=2}^{I} C_i^2$ (8)

The power spectrum density in equation (8) is often referred to as the classical Doppler spectrum. Meanwhile, if some of the scattering components are much stronger than most of the components, the fading process no longer follows the Rayleigh distribution. In this case, the amplitude of the received signal follows the Rician distribution and thus, this fading process is referred to as Rician fading. The strongest scattering component usually corresponds to the line-of-sight (LOS) component (also referred to as specular components). Other than the LOS component, all the other components are non-line of sight (NLOS) components (referred to as scattering components) [11].

$y(t) = (h_I^2(t) + h_Q^2(t))^{1/2}$

Let $\hat{p}(\theta)$ denote a probability density function (PDF) of angle of arrival (AoA) for the scattering components and μ_0 denote AoA for the specular component. Then, the PDF of AoA for all components given as

$$p(\theta) = \frac{1}{k+1}\tilde{p}(\theta) + \frac{k}{k+1}\delta(\theta - \theta_0)$$

Where K is the Rician factor, defined as a ratio of the specular component power c^2 and scattering component power $2\sigma^2$, shown as

$$K = \frac{c^2}{2\sigma^2} \quad (9)$$

B. *Generation of Fading Channels*

In general, the propagation environment for any wireless channel in either indoor or outdoor may be subject to LOS (Line-of-sight) or NLOS (Non Line-of-Sight). A probability density function of the signal received in the LOS environment follows the Rician distribution. Any received signal in the propagation environment for a wireless channel can be considered as the sum of the received signals from an infinite number of scatters. By the central limit theorem, the received signal can be represented by a Gaussian random variable. In other words, a wireless channel subject to the fading environments can be represented by a complex Gaussian random variable, $W_1 + jW_2$ where W_1 and W_2 are the independent and identically-distributed (i.i.d.) Gaussian random variables with a zero mean and variance of σ^2. Let X denote the amplitude of the complex Gaussian random variable $W_1 + jW_2$, such that $X = \sqrt{W_1^2 + W_2^2}$.

Then, note that X is a Rayleigh random variable with the following probability density function (PDF):

$$f_x(x) = \frac{x}{\sigma^2} e^{\frac{x^2}{2\sigma^2}} \quad (10)$$

Where $2\sigma^2 = E\{X^2\}$. Furthermore X^2 is known as a chi-square (χ^2) random variable.

Generate two independent Gaussian random variables with a zero mean and unit variance Z_1 and Z_2, Rayleigh random variable X with the PDF can be represented by

$$X = \sigma\sqrt{(Z_1^2 + Z_2^2)} \quad (11)$$

The Rayleigh random variable X with the average power of $E\{X^2\} = 2\sigma^2$ can be generated by Equation (11). In the LOS environment, there exists a strong path which is not subject to any loss due to reflection, diffraction, and scattering, the amplitude of the received signal can be expressed as $X = c + W_1 + JW_2$ where c represents the LOS component while W1 and W2 are the independent Gaussian random variables with a zero mean and variance of σ^2 as in the non-LOS environment. It has been known that X is the Rician random variable with the following PDF[12].

$$f_y\ x = \frac{x}{\sigma^2} e^{\frac{x^2+c^2}{2\sigma^2}} I_0\left(\frac{x_c}{\sigma^2}\right) \quad (12)$$

Where $I_0(.)$ is the modified zeroth-order Bessel function of the first kind. Note that equation can be represented in terms of the Rician K-factor defined in Equation (9).

C. *The α-μ Distribution*

The α--μ distribution is a general fading distribution that can be used to represent the small- scale variation of the fading signal. For a fading signal with envelope r, an arbitrary parameter α >0, and a -root mean value $\hat{r}=\sqrt[\alpha]{E(r^\alpha)}$the α--μ probability density function p(r) of r is written as

$$P(r)=\frac{\alpha\mu^{\mu}r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)}\exp(-\mu\frac{r^\alpha}{\hat{r}^\alpha}) \quad (13)$$

Where μ ≥ ½ is the inverse of the normalized variance of r^α

$$\mu = \frac{E^2(r^\alpha)}{E(r^{2\alpha})-E^2(r^\alpha)} \quad (14)$$

and $\Gamma(z)=\int_0^\infty t^{z-1}exp(-t)\,dt$ is the Gamma function

The k-th moment $E(r^k)$ is obtained as

$$E(r^k)=\frac{\hat{r}^k\Gamma(\mu+k/\alpha)}{\mu^{k/\alpha}\Gamma(\mu)} \quad (15)$$

The fading model for the α-μ distribution considers a signal composed of clusters of multipath waves propagating in a non-homogeneous environment. Within any one cluster, the phases of the scattered waves are random and have similar delay times with delay-time spreads of different clusters being relatively large. The clusters of multipath waves are assumed to have the scattered waves with identical powers. The resulting envelope is obtained as a non-linear function of the modulus of the sum of the multipath components. The non-linearity is manifested in terms of a power parameter α > 0, such that the resulting signal intensity is obtained not simply as the modulus of the sum of the multipath components, but as this modulus to a certain given power. Assume that at a certain given point the received signal encompasses an arbitrary number n of multipath components, and the propagation environment is such that the resulting signal is observed as a non-linear function of the modulus of the sum of these components. Suppose that such a non-linearity is in the form of a power so that the resulting envelope is observed as the modulus of the sum of the multipath components to the power of 2 α, α > 0.Hence, for the α-μ Distribution the envelope r can be written as a function of the in-phase and quadrature elements of the multipath components so that

$$r = \sqrt[\alpha]{\sum_{i=1}^{N}({x_i}^2 + {y_i}^2)} \quad (16)$$

Where x_i, y_i are in-phase and quadrature elements of multipath components represented by symbol i., i.e.. where x_i, y_i are mutually independent Gaussian processes, with $E(x_i)=E(y_i)=0$.

The α-μ Distribution is a general fading distribution that includes the Weibull and the Nakagami-m distribution as special cases. Weibull includes the Rayleigh and the Negative Exponential distributions whereas Nakagami-m includes the Rayleigh and the One-Sided Gaussian distributions. The Lognormal distribution may also be well approximated by the α-μ Distribution[13-14].

The Weibull distribution can be obtained from the α-μ Distribution by setting μ=1 in (13)

$$P(r)=\alpha\beta\gamma^{\alpha-1}e^{-\beta\gamma^\alpha} \text{ where } \beta = \hat{r}^{-\alpha} \quad (17)$$

Rayleigh distribution is obtained by substituting $\alpha = 2$ in (13)

$$P(r)=\frac{r}{\gamma^2}e^{\frac{-r^2}{2\gamma^2}} \text{where} \gamma^2 = \frac{\hat{r}^2}{2} \quad (18)$$

Negative exponential distribution is obtained by substituting $\alpha = 1$ in (17)

$$P(r)=\delta e^{-\delta r} \text{where} \delta = \hat{r}^{-1} \quad (19)$$

Nakagami-m, Rayleigh, and One-Sided Gaussian The Nakagami-m distribution can be obtained from the α-μ Distribution by setting α=2 in In such a case

$$P(r) = \frac{2r^{2\mu-1}\mu^\mu}{\Omega^\mu\Gamma(\mu)}e^{(-\mu\frac{r^2}{\Omega})} \quad (20)$$

One sided Gaussian distribution can be obtained by setting $\mu = 1/2$ in equation (20)

$$P(r)=\sqrt{\frac{2}{\pi\hat{r}^2}}e^{\frac{-r^2}{2\hat{r}^2}} \quad (21)$$

Here, non-linearity is introduced propagation medium. However, at different values of α different fades can be generated.

III. Channel Model

The sub-channel spacing is equal to inverse of time period, so that the produced parallel fading sub- channels have flat fading characteristics. Here α -μ distribution has been utilized for generation of Weibull distribution by setting μ=1 and varying the value of α.

In flat fading environment, the base-band signal at the input of receiver y (t) is as described as follows:

Y (t) =x (t)*r (t) +n (t) (22)

Where x (t) denotes the base-band transmitted signal, r (t) is the Weibull distributed channel envelope and n (t) is the additive white Gaussian noise with zero mean.

The envelope r(t) can be written as a function of the in-phase and quadrature elements of the multipath components so that

$$r = \sqrt[\alpha]{\sum_{i=1}^{N}({x_i}^2 + {y_i}^2)} \quad (23)$$

Wherex_i, y_i are in-phase and quadrature elements of multipath components represented by symbol i.e,

$$x_i(t) = r_0.\cos\,(2\pi f_m cos\theta_0 t + \emptyset_0) \quad (24)$$

$$y_i(t) = r_0.\sin\,(2\pi f_m cos\theta_0 t + \emptyset_0(25)$$

where x_i, y_i are mutually independent Gaussian processes, with $E(x_i)=E(y_i)=0$.

The random nature of noise in time domain will, on occasions, cause a transmitted symbol to be distorted such that the receiver interprets it as a different symbol in the modulation scheme alphabet. Under these circumstances, a given average level of AWGN introduces an average number of symbol errors where each symbol error causes one or more bit errors at output of the receiver. The errors may be characterized by a bit error rate (BER).

Channel estimators will help in equalization at the receiver end. These usually need some kind of information as a reference and may be transmitted purposely or based on channel statistics. Channel estimation allows the receiver to approximate the effect of the channel on the signal to eliminate it. It is essential for removing inter symbol interference and noise, and is used in diversity combining, maximum likelihood (ML) detection, angle of arrival estimation, pilot is basically a reference carrier/tone or reference signal/symbol which is known at the receiver end in terms of positioner sequence/pattern and used for the channel estimation because it provide channel state information as it is under gone the most recent channel behavior.

There are two main problems in designing channel estimators for wireless OFDM systems:

1. The arrangement of pilot information, where pilot means the reference signal used by both transmitters and receivers.
2. The design of estimator with low complexity and good channel tracking ability.

The nature of the OFDM enables powerful estimation and equalization techniques. In communication system, the estimation is generally performed using received signal and applying some knowledge about the transmitted sequence/known symbols or channel statistics.

IV. RESULTS

A. Results without Estimation of Channel

1) OFDM-BPSK, QPSK, 8PSK and 16PSK signal is simulated in MATLAB environment by choosing total number of sub-carriers 400,IFFT length 1024 by using guard interval of length 256. The results presented in figures 1, 2, 3 and 4 are simulated by varying the value of α and keeping μ=1. Here the BER values have been obtained by varying α over a range of 1 to 7, however, improvement in BER was not significant for higher values of α, so range has been kept from 1 to 7, for M-ary PSK OFDM system.From the simulations, it has been verified that the results obtained for α=2 are same as that of results obtained by using Rayleigh fading distribution. With different values of α BER is estimated. It has been observed that changing the value of μ there is no change in these BER curves. Here the variable parameter is α which varies the fading frequently and μ has no role to change this fading envelope.

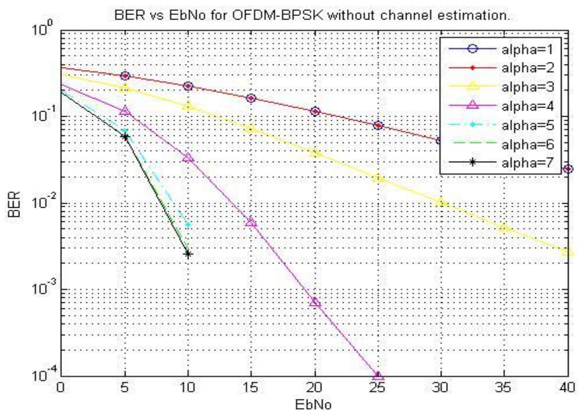

Fig 1: BER vs EbNo for OFDM-BPSK without channel estimation

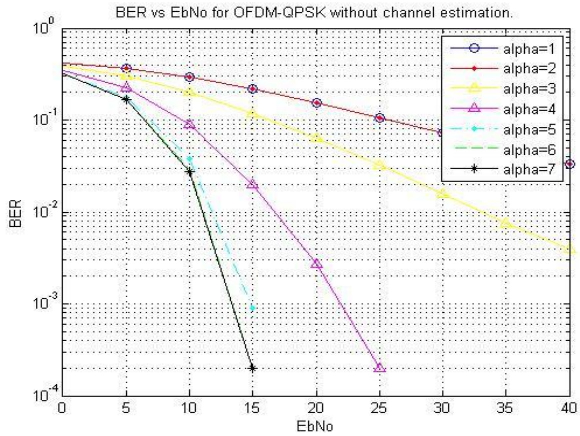

Fig 2: BER vs EbNo for OFDM-QPSK without channel estimation.

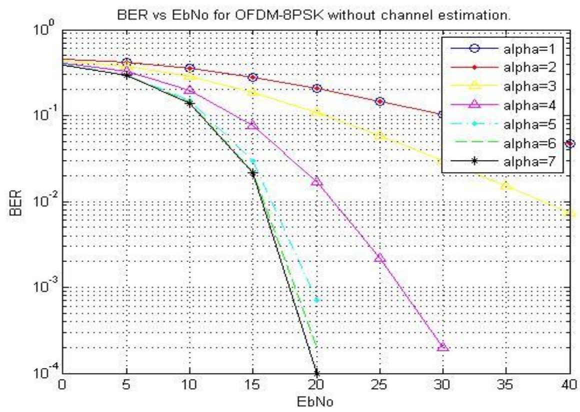

Fig 3: BER vs EbNo for OFDM-8PSK without channel estimation.

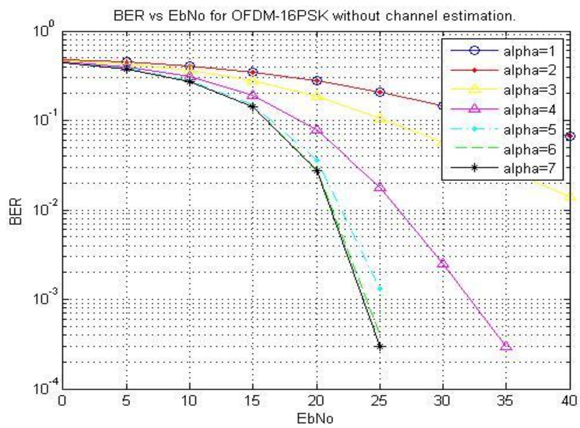

Fig 4: BER vs EbNo for OFDM-16PSK without channel estimation.

BER varies in the range of 10^{-1} to 10^{-4} and SNR varies from 0 to 35 dB for OFDM-BPSK,-QPSK,-8PSK and -16PSK. In case of OFDM-BPSK the BER value of 10^{-4} is obtained at SNR of 20dB, for OFDM-QPSK the BER value of 10^{-4} is obtained at SNR of 25dB,for OFDM-8PSK the BER value of 10^{-4} is obtained at SNR of 30dB, for OFDM-16PSK the BER value of 10^{-4} is obtained at SNR of 35dB.

TABLE 1: shows the comparing different modulation techniques for α=7, μ=1 and at SNR value of 25 dB without channel estimation.

Parameters	BPSK	QPSK	8-PSK	16-PSK
No. of Symbols	2	4	8	16
Phase Difference	180	90	45	22.5
BER	0.0015	0.02	0.15	0.2

B. Results with Estimation of Channel:

Trained symbols are added to source signal as discussed in the channel estimation. The percentage of such symbol may be varied depending upon the system response to the trained sequence. By varying the percentage length of trained sequence from 10% to 50% and maintaining α=7, OFDM-BPSK,QPSK,8PSK and 16PSK plots are obtained and shown in figure 5, 6, 7 and 8 respectively.

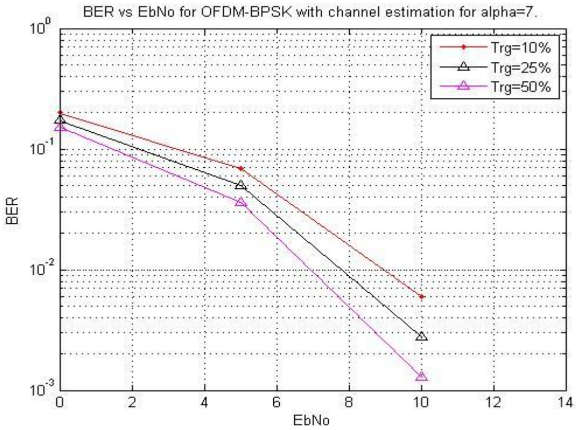

Fig 5: BER vs EbNo for OFDM-BPSK with channel estimation.

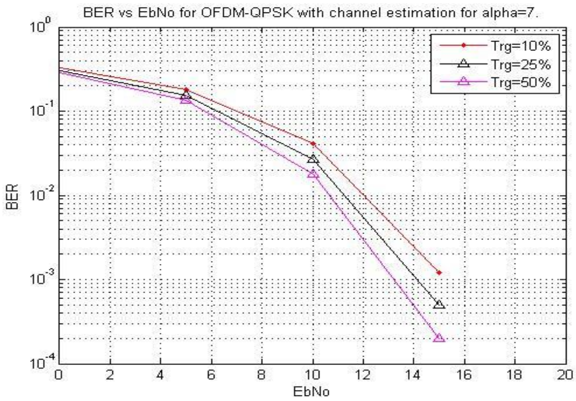

Fig 6: BER vs EbNo for OFDM-QPSK with channel estimation.

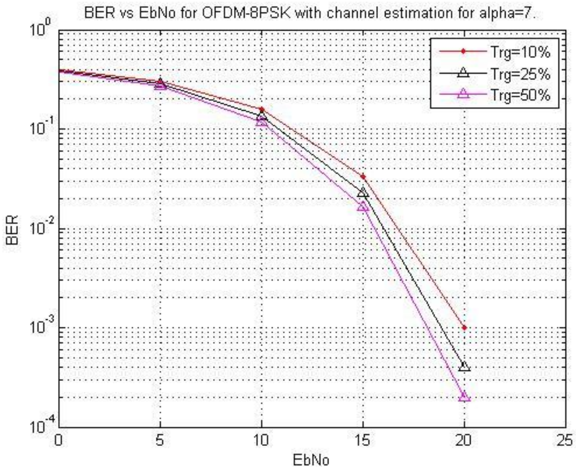

Fig 7: BER vs EbNo for OFDM-8PSK with channel estimation.

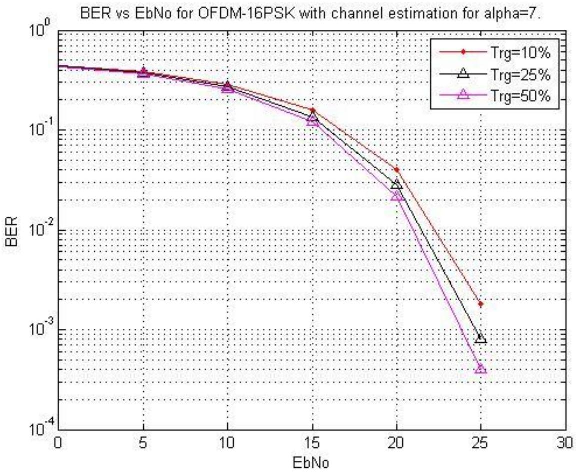

Fig 8: BER vs EbNo for OFDM-16PSK with channel estimation.

Comparison of different modulation scheme for α=1 and 3 without channel estimation and for α=7 with channel estimation are shown in figure 9, 10 and 11 respectively.

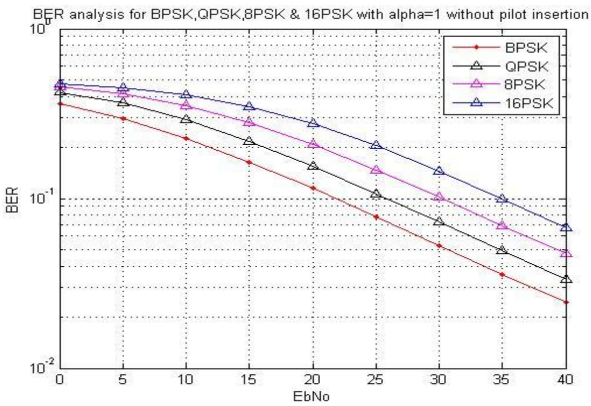

Fig 9: BER analysis for different OFDM system for α=1without channel estimation.

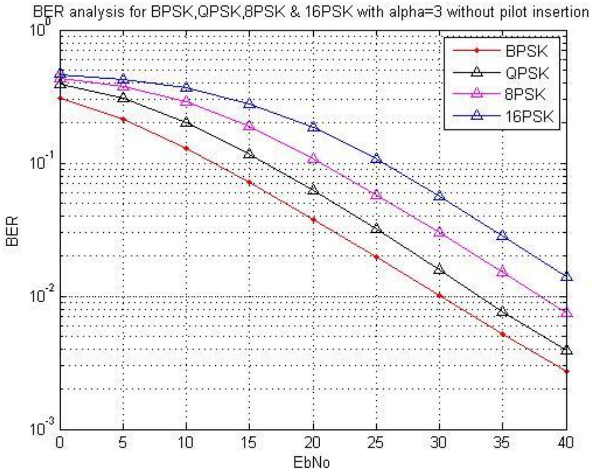

Fig 10: BER analysis for different OFDM system for α=3 without channel estimation.

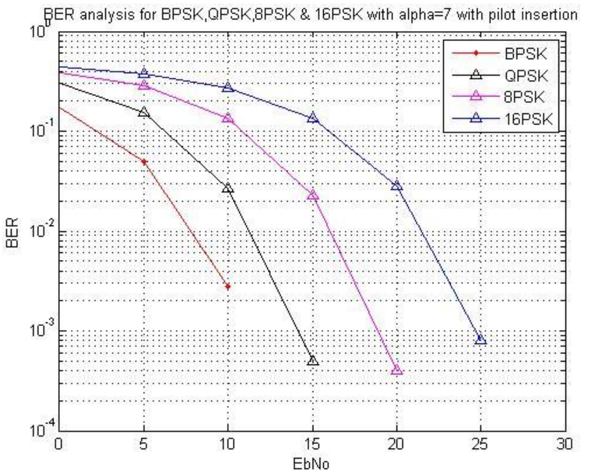

Fig 11: BER analysis for different OFDM system for α=7 with channel estimation.

TABLE 2: Shows the comparison of results with estimation for different modulation techniques for α=7, μ=1 and at SNR value of 25 dB with channel estimation

Pilot Length	10%	25%	50%
BPSK	0.006	0.003	0.0015
QPSK	0.04	0.03	0.02
8-PSK	0.17	0.15	0.12
16-PSK	0.29	0.27	0.26

V. Conclusion

Channel is considered as flat fading channel. So, all the signal components will experience same amount of fading as the channel response is flat. From the channel envelope equation, the envelope of the channel(r) depends on α value that is envelope decreases with increase in α value. This results in less interference to the signal and leads to low BER. From the obtained results it can be concluded that for higher values of α, BER decreases. But when α reaches 7 and beyond that value the reduction in BER is too small which is negligible. So α = 7 is the optimum value for proper communication. At transmitter different types of M-ary phase shift keying modulation technique is used to represent bits. Among these modulation techniques BPSK gives better reconstruction of signal at receiver, because it provides large phase offset value as it has 180 degree differ in its symbols representation.

If channel behavior is not estimate then information related to phase and frequency rotation effected by channel cannot be determined. So it leads to errors at receiver. In order to reduce the channel effect pilot data is added along with transmitting data which gives better plot and reduction in BER. From the plot it is understood that as pilot length increases BER decreases.

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