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Vendor Selection in Intuitionistic Fuzzy Sets Environment: A Comparative Study by MADM methods

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Abstract
Now-a-days in world competitive market Multi Criteria Decision Making (MCDM) method are getting more and more popular for selecting the best alternatives among the available alternative used in our problem definition. Various companies are required some core competences in order to be able to compete with numerous competitors in industries and sustain their situation in such a market. It is impossible for a company to successfully produce low-cost, high-quality products and maximize company revenue without satisfactory supplier. Companies achieve this target are those which their processes perform great and exploit from competitive price, quality and guarantee etc. Since some parameters such as price, delivery time and quality are so dependent on the performance of company supply chain management. So the result can highly impress the final price and quality of products. One of the main process of supply chain management is supplier selection process which its accurate implementation can dramatically increase company competitiveness. This process is an unstructured and complicated, involving qualitative and quantitative multi-criteria, problem of purchasing management in supply chain. In present article five different MCDM method including ELECTRE, Simple Additive Weighting (SAW), TOPSIS, Weight Product Matrix (WPM) and Lexicographic are applied on a specific problem to find out potential supplier among the numerous conflicting suppliers. Finally this article compared the result to determine the best MCDM method in our present proposed work for supplier selection problem.

IndexTerms— Vendor Selection, IFS, MCDM.

1. INTRODUCTION

Multiple criteria decision making (MCDM) method is one of the most widely used decision methods in different fields, such as sciences, engineering, business, government management, Industrial Process, commercial etc. Here supplier selection problem is one of the most multi-criteria decision making problem which is affected by several conflicting factors. The typical MCDM problem is concerned with the ranking order of decision alternatives. Different MCDM methods may have different ranking of alternatives on the same problem because, many criteria have been considered during the process of supplier selection. There are several MCDM model has been proposed for finding out best supplier either in crisp form, fuzzy form and intuitionistic fuzzy form.

Electre model approach to Intuitionistic fuzzy set is quite capable to find out potential supplier among the numerous suppliers, in this model concordance and discordance operator has been used for outranking relationship. The relative power of each concordance set is measured by means of the concordance index on the other hand discordance set measure the power of the discordance index. Marbini and Tavana [1] proposed the fuzzy ELECTRE I method to take into account the uncertain, imprecise and linguistic assessments provided by a group of decision makers. Vahdani and Hadipour [2] proposed an interval-valued fuzzy ELECTRE method to solve a maintenance strategy selection problem with imprecise information, and the criteria weights are linguistic terms which be expressed in interval valued fuzzy numbers, and those weights are unequal. The proposed method can also for solving MCDM problems. Vahdani, Jabbari, Roshanaei and Zandieh [3] proposed an ELECTRE method with interval weights and data to solve MCDM problems, and applied the proposed method to select the best supplier. The ELECTRE II and III methods enjoy a wide acceptance in solving multi-criteria decision-making (MCDM) problems. Another family of MCDM models uses what is known as "outranking relations" to rank a set of alternatives. A prominent role in this group is played by the ELECTRE method and its derivatives. The ELECTRE approach was first introduced in [4]. The main idea is the proper utilization of what is called "Outranking relations". Soon after the introduction of the first version known as ELECTRE I [5], this approach has evolved into a number of other variants. Today the most widely used versions are known as ELECTRE II [6, 7] and ELECTRE III [8]. In this paper I have proposed the electre method approach to intuitionistic fuzzy set. Intuitionistic fuzzy set (IFS) was first introduced by Atanassov in 1986 [9], which is characterized by a membership function and a non-membership function. The IFS generalizes the fuzzy set and introduced by Zadeh in 1965[10], and has been found to be highly useful to deal with vagueness. This method only consider the weak concordance set and worse discordance set for outranking relationship of different alternative, and finally compared the ranking of the alternative to other models.

SAW (Simple Additive Weighting) is one of the oldest, most widely known and practically used methods (Hwang, Yoon 1981 [11]; Chu,Shyu,Hshiung, Khosala [12]) for supplier selection among the available supplier. Intuitionistic Fuzzy Simple Additive Weighting (IFSAW) is a method for selecting the more appropriate vendor (basically applicable for more than one decision maker). In this method, concentration and dilation operator has been applied for cost minimization;

Intuitionistic Fuzzy Weighted Averaging (IFWA) operator is utilized to aggregate individual opinions of decision makers for rating the importance of criteria and alternatives.

The most important function of the purchasing function is the selection of appropriate vendor, since it brings significant savings of time and cost for the organizations. It is a MADM problem, there are several method has been proposed for solving this problem using classical set theory or fuzzy set theory. TOPSIS was first developed by Hwang and Yoon (1981) for solving a MADM problem. Data consideration in this method is crisp. Chen, Lin and Huang [13] extended the concept of TOPSIS method to develop a methodology for solving supplier selection problems in fuzzy environment. TOPSIS method combined with intuitionistic fuzzy set is proposed to select appropriate vendor in group decision making environment. IFWA operator is utilized to aggregate individual opinions of decision makers for rating the importance of criteria and alternatives. TOPSIS simultaneously considers the distances to the ideal solution and negative ideal solution regarding each alternative and selects the most relative closeness to the ideal solution as the best alternative. Finally, a numerical example for vendor selection is given to illustrate application of intuitionistic fuzzy TOPSIS method.

IFWPM is one of the most famous multi-criteria decision making approaches have been proposed for vendor selection. These approaches enable us to deal with evaluation, selecting and ranking vendors in a fuzzy or crisp environment. IFWPM method is used to select an appropriate vendor. We have used intuitionistic fuzzy multiplication operation for calculating the aggregation score for each vendor. Finally a score function is used to rank the vendor with largest score. A numerical example illustrated our proposed approach. The model is explained by an illustrative example.

In Lexicographic scoring method criteria are ordered by ranking or priority (preference, significance). In this method alternative with the highest value is the winner in lexicographic procedure and will be eliminated from the list of alternatives. Then next higher value of alternatives will be considered for elimination and the procedure will be repeated till the getting of best alternatives. By meaning, lexicographic method consists in obtaining an arbitrarily small improvement for the most important criterion through any loss in other less important criteria [14].

The organization of the paper is as follows: Section one introduces the various MADM methods with review of literature. Section two states the various MADM methodologies and their IFS approaches. Section three gives the application of IFS methodology with a numerical example. Section four the final conclusions of the results from the numerical examples.

2. METHODOLOGY

2.1. ELECTRE

Let $A = \{A_1, A_2, \dots A_n\}$ be a set of alternatives called vendors and $C = \{c_1, c_2, \dots c_m\}$ be a set of criteria, the procedure for intuitionistic fuzzy Electre method has been given as follows:

Step 1: Intuitionistic fuzzy Decision matrix is given as folows:

$$\begin{bmatrix} [\mu A_1(c1), VA_1(c1), \pi A_1(c1)] & \dots & [\mu A_1(cm), VA_1(cm), \pi A_1(cm)] \\ [\mu A_2(c1), VA_2(c1), \pi A_2(c1)] & \dots & [\mu A_2(cm), VA_2(cm), \pi A_2(cm)] \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ [\mu A_n(c1), VA_n(c1), \pi A_n(c1)] & \dots & [\mu A_n(cm), VA_n(cm), \pi An(cm)] \end{bmatrix}$$

Step 2: The weight of Criteria in intuitionistic fuzzy Set is as follows:

$$[\{\mu_w(c_1), v_w(c_1), \pi_w(c_1)\}, \{\mu_w(c_2), v_w(c_2), \pi_w(c_2)\} \dots \{\mu_w(c_m), v_w(c_m), \pi_w(c_m)\}]^T$$

Step 3: The Intuitionistic Fuzzy Weighted decision matrix is calculated and given as follows:

$$R' = \begin{bmatrix} [\mu A_1W(c1), VA_1W(c1), \pi A_1W(c1)] & \dots & [\mu A_1W(cm), VA_1W(cm), \pi A_1W(cm)] \\ [\mu A_2W(c1), VA_2W(c1), \pi A_2W(c1)] & \dots & [\mu A_2W(cm), VA_2W(cm), \pi A_2W(cm)] \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ [\mu A_nW(c1), VA_nW(c1), \pi A_nW(c1)] & \dots & [\mu A_nW(cm), VA_nW(cm), \pi A_nW(cm)] \end{bmatrix}$$

Step 4: The Concordance and Discordance set is determined.The concordance set is determined. If alternative $A_p$is preferred to alternative A_q for all criteria, the concordance set is composed. This can be written as:

$$C(p,q) = C_{pq} = \{J \mid \mu_{pj} > \mu_{qj} \; and \; v_{pj} < \mu_{qj}$$

$C(p,q)$ is the collection of attributes where A_p is better than or equal to A_p. Then concordance indexes are calculated. The concordance index of $C(p,q)$ is defined as;

$$C_{pq} = \sum_{j^*} w_j$$

j* are criteria contained in the concordance set $C(p,q)$.The concordance set is determined. If alternative $A_p$is worse to alternative A_q for all criteria, the concordance set is composed. This can be written as:

$$D(p,q) = C_{pq} = \{J \mid \mu_{pj} < \mu_{qj} \; and \; v_{pj} > \mu_{qj}$$

$D(p,q)$ is the collection of attributes where A_p is worse than or equal to A_p. Then discordance indexes are calculated. The discordance index of $D(p,q)$ is defined as:

$$D_{pq} = \left[\frac{\sum_{j^*} \{\mu_{pj^*} - \mu_{pj^*}\}}{\sum_{j=1}^{n} \{\mu_{pj} - \mu_{qj}\}}, \frac{\sum_{j^*} \{v_{pj^*} - v_{pj^*}\}}{\sum_{j=1}^{n} \{v_{pj} - v_{qj}\}} \right]$$

j* are criteria contained in the discordance set $D(p,q)$.

Step 5: Accuracy Function[17] for IFS $\{\mu_A(x), v_A(x)\}$ is

$$Accuracy\ Function = \mu_A(x) + v_A(x)$$

Step 6: Outranking Relationship

$$C_{pq} \geq \bar{C} \; and \; D_{pq} \leq \bar{D}$$

2.2. IF SAW

Step 1: Determine the weights of decision makers:

Assume that decision group contains decision makers. The importance of the decision makers are considered as linguistic terms expressed in intuitionistic fuzzy numbers. Let $d_k = [l_k, m_k, p_k]$ be an intuitionistic fuzzy number for rating of kth decision maker. Then the weight of kth decision maker can be obtained as:

$$\lambda_K = \frac{(\mu_K + \pi_K(\frac{\mu_K}{\mu_K + V_K}))}{\sum_{K=1}^{L}(\mu_K + \pi_K(\frac{\mu_K}{\mu_K + V_K}))}$$

$$\sum_{K=1}^{l}\lambda_K = 1 \qquad k = 1,2,3............l \qquad ...(1)$$

Step 2: Construct the aggregated intuitionistic fuzzy decision matrix of benefit criteria based on the opinions of decision makers. in this matrix cost criteria are not include.

Let $R^{(k)} = (r_{ij}{}^{(k)})_{m\times n}$ is an intuitionistic fuzzy decision matrix of each decision maker. $\lambda = \{ \lambda_1, \lambda_2, \lambda_3,... \lambda_l \}$ is the weight of each decision maker and. In group decision making process, all the individual decision opinions need to be fused into a group opinion to construct aggregated intuitionistic fuzzy decision matrix. in order to do that, IFWA operator proposed by **Xu(2007)[15]** is used $R = (r_{ij})_{m\times n}$ where

$$r_{ij} = IFWA_\lambda(r_{ij}{}^{(1)}, r_{ij}{}^{(2)}, r_{ij}{}^{(3)}........r_{ij}{}^{(l)})$$
$$= \lambda_1 r_{ij}{}^{(1)} \oplus \lambda_2 r_{ij}{}^{(2)} \oplus \lambda_3 r_{ij}{}^{(3)} \oplus ....\lambda_l r_{ij}{}^{(l)} \quad ...(2)$$
$$= [1 - \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k},]$$

Here $r_{ij} = [\mu_{A_i}(c_j), V_{A_i}(c_j), \pi_{A_i}(c_j)] \quad i = 1,2,3....m \quad j = 1,2,3.....p-1$

Step 3: Construct aggregated intuitionistic fuzzy decision matrix of cost criteria.IFWA operator proposed by Xu

$$r_{ij} = IFWA_\lambda(r_{ij}{}^{(1)}, r_{ij}{}^{(2)}, r_{ij}{}^{(3)}........r_{ij}{}^{(l)})$$
$$= \lambda_1 r_{ij}{}^{(1)} \oplus \lambda_2 r_{ij}{}^{(2)} \oplus \lambda_3 r_{ij}{}^{(3)} \oplus ....\lambda_l r_{ij}{}^{(l)}$$
$$= [1 - \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k},] \quad ...(3)$$

Step 4: Use concentration and dilation in the cost criteria for cost minimizing

Step 5: Compute weight of criteria

All the criteria may not be assumed equal importance. We represent a set of grades of importance.in order to obtain W all the individuals decision maker opinion for the importance of each criteria need to be fused.

Let $W_j{}^k = [\mu_j{}^k, V_j{}^k, \pi_j{}^k]$ be an intuitionistic fuzzy number assigned to criteria X_j by the kth decision maker. Then the weight of criteria can be calculated by using IFWA operator:

$$r_{ij} = IFWA_\lambda(r_{ij}{}^{(1)}, r_{ij}{}^{(2)}, r_{ij}{}^{(3)}........r_{ij}{}^{(l)}) \quad ...(3)$$
$$= \lambda_1 r_{ij}{}^{(1)} \oplus \lambda_2 r_{ij}{}^{(2)} \oplus \lambda_3 r_{ij}{}^{(3)} \oplus ....\lambda_l r_{ij}{}^{(l)}$$
$$= [1 - \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k}] \quad ...(4)$$

$$w = [w_1, w_2, w_3..........w_l]^T$$

Step 6: Construct final aggregated intuitionistic fuzzy decision matrix:

It can be determined after combining the benefit (step 3) and cost (step 4) aggregated intuitionistic fuzzy decision matrix.

$$R = \begin{bmatrix} [\mu_{A_1}(c_1), V_{A_1}(c_1), \pi_{A_1}(c_1)] & ... & [\mu_{A_1}(c_m), V_{A_1}(c_m), \pi_{A_1}(c_m)] \\ [\mu_{A_2}(c_1), V_{A_2}(c_1), \pi_{A_2}(c_1)] & ... & [\mu_{A_2}(c_m), V_{A_2}(c_m), \pi_{A_2}(c_m)] \\ ... & ... & ... \\ ... & ... & ... \\ ... & ... & ... \\ [\mu_{A_n}(c_1), V_{A_n}(c_1), \pi_{A_n}(c_1)] & ... & [\mu_{A_n}(c_m), V_{A_n}(c_m), \pi_n(c_m)] \end{bmatrix} ...(6)$$

Step 7: Derive total intuitionistic fuzzy score for individual alternatives by multiplying by final aggregated intuitionistic fuzzy decision matrix by their respective weight (i.e. weight of criteria).

$$C(A_i) = \sum_{j=1}^{m} W \otimes R \qquad ...(7)$$

Step 8: Rank of alternative determined by using score function [17].

$$S(Ai) = [\mu_{A_1}(c_1) - V_{A_1}(c_1)] \qquad ...(8)$$

2.3. IF TOPSIS

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives and $X = \{c_1, c_2, \ldots, c_m\}$be a set of criteria, the procedure for intuitionistic fuzzy saw method has been given as follows:

Step 1: Determine the weights of decision makers:

Assume that decision group contains l decision makers. The importance of the decision makers are considered as linguistic terms expressed in intuitionistic fuzzy numbers. Let $d_k = [l_k, m_k, p_k]$ be an intuitionistic fuzzy number for rating of the kth decision maker. Then the weight of kth decision maker can be obtained as:

$$\lambda_K = \frac{(\mu_K + \pi_K(\frac{\mu_K}{\mu_K + V_K}))}{\sum_{K=1}^{L}(\mu_K + \pi_K(\frac{\mu_K}{\mu_K + V_K}))} \qquad ... (1)$$

$$\sum_{K=1}^{l}\lambda_K = 1 \;, k = 1,2,3....l$$

Step 2: Construct the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers. One thing is very important that is in this matrix cost criteria are not included.

Let $R^{(k)} = (r_{ij}{}^{(k)})_{m\times n}$ is an intuitionistic fuzzy decision matrix of each decision maker. $\lambda = \{\lambda_1, \lambda_2, \lambda_3,... \lambda_l\}$ is the weight of each decision maker and. In group decision making process, all the individual decision opinions need to be fused into a group opinion to construct aggregated intuitionistic fuzzy decision matrix. In order to do that, IFWA operator proposed by **Xu(2007)** is used $R = (r_{ij})_{m\times n}$ where

$$r_{ij} = IFWA_\lambda(r_{ij}{}^{(1)}, r_{ij}{}^{(2)}, r_{ij}{}^{(3)}........r_{ij}{}^{(l)})$$
$$= \lambda_1 r_{ij}{}^{(1)} \oplus \lambda_2 r_{ij}{}^{(2)} \oplus \lambda_3 r_{ij}{}^{(3)} \oplus ....\lambda_l r_{ij}{}^{(l)}$$
$$= [1 - \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(1-\mu_{ij}{}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}{}^{(k)})^{\lambda_k},] \quad ...(2)$$

Here $r_{ij} = [\mu_{A_i}(c_j), V_{A_i}(c_j), \pi_{A_i}(c_j)] \quad i = 1,2,3....m \quad j = 1,2,3.....n$

Step 3: Compute weight of criteria

All the criteria may not be assumed of equal importance.W represent a set of grades of importance.In order to obtain W all the individuals decision makers opinion for the importance of each criteria need to be fused.

Let $W_j^{\,k} = [\mu_j^{\,k}, V_j^{\,k}, \pi_j^{\,k}]$ be an intuitionistic fuzzy number assigned to criteria X_j by the kth decision maker. Then the weight of criteria can be calculated by using IFWA operator:

$$r_{ij} = IFWA_\lambda(r_{ij}^{(1)}, r_{ij}^{(2)}, r_{ij}^{(3)} \ldots\ldots r_{ij}^{(l)})$$
$$= \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \lambda_3 r_{ij}^{(3)} \oplus \ldots\ldots \lambda_l r_{ij}^{(l)}$$
$$= [1 - \prod_{k=1}^{l}(1-\mu_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(1-\mu_{ij}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}^{(k)})^{\lambda_k},] \quad \ldots(3)$$
$$w = [w_1, w_2, w_3 \ldots\ldots w_l]^T \qquad w_j = [\mu_j, v_j, \pi_j] \text{ where } j = 1,2,3\ldots\ldots n$$

Step 4: Construct aggregated weighted intuitionistic fuzzy decision matrix:

All the information of alternative are known, and also weight of criteria are known. We find out aggregated intuitionistic fuzzy decision matrix with the following formula:

$$R \otimes w = [\mu_{A_iW}(c_j), V_{A_iW}(c_j), \pi_{A_iW}(c_j)]$$
$$\text{where} \quad \mu_{A_iW}(c_j) = \mu_{A_i}(c_j) \times \mu_W(c_j);$$
$$V_{A_iW}(c_j) = V_{A_i}(c_j) + V_W(c_j) - V_{A_i}(c_j) \times V_W(c_j)$$
$$\pi_{A_iW}(c_j) = 1 - \{\mu_{A_i}(c_j) \times \mu_W(c_j) + V_{A_i}(c_j) + V_W(c_j) - V_{A_i}(c_j) \times V_W(c_j)\}$$

i=1, 2, 3…n j=1, 2, 3…m

Hence aggregated intuitionistic fuzzy decision matrix are as follows:

$$R' = \begin{bmatrix} [\mu_{A_1}W(c_1), V_{A_1}W(c_1), \pi_{A_1}W(c_1)] & .. & [\mu_{A_1}W(c_m), V_{A_1}W(c_m), \pi_{A_1}W(c_m)] \\ [\mu_{A_2}W(c_1), V_{A_2}W(c_1), \pi_{A_2}W(c_1)] & \ldots & [\mu_{A_2}W(c_m), V_{A_2}W(c_m), \pi_{A_2}W(c_m)] \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ [\mu_{A_n}W(c_1), V_{A_n}W(c_1), \pi_{A_n}W(c_1)] & \ldots & [\mu_{A_n}W(c_m), V_{A_n}W(c_m), \pi_{A_n}W(c_m)] \end{bmatrix}$$

Step 5: Construct the intuitionistic fuzzy positive ideal solution and fuzzy negative idea solution. Let j_1 and j_2 are the benefit criteria and cost criteria. General form of positive and negative ideal solution is as follows:

$$A^+ = [(\mu_{A^+W}(c_j), V_{A^+W}(c_j), \pi_{A^+W}(c_j)]$$
$$A^- = [\mu_{A^-W}(c_j), V_{A^-W}(c_j), \pi_{A^-W}(c_j)]$$
$$\mu_{A^+W}(c_j) = (\underset{i}{Max}(\mu_{A_iW}(c_j)|j \in j_1), (\underset{i}{Min}(\mu_{A_iW}(c_j)|j \in j_2)$$
$$V_{A^+W}(c_j) = (\underset{i}{Min}(V_{A_iW}(c_j)|j \in j_1), (\underset{i}{Max}(V_{A_iW}(c_j)|j \in j_2) \quad \ldots(4)$$
$$\mu_{-W}(c_j) = (\underset{i}{Min}(\mu_{A_iW}(c_j)|j \in j_1), (\underset{i}{Max}(\mu_{A_iW}(c_j)|j \in j_2)$$
$$V_{A^-W}(c_j) = (\underset{i}{Max}(V_{A_iW}(c_j)|j \in j_1), (\underset{i}{Min}(V_{A_iW}(c_j)|j \in j_2) \ i = 1,2,3\ldots\ldots n, j = 1,2,3\ldots m$$

Step 6: Calculate the separation of measure

$$S^+ = \sqrt{\sum_{j=1}^{n}[\frac{1}{2n}(\mu_{A_iW}(c_j) - \mu_{A^+W}(c_j))^2 + (V_{A_iW}(c_j) - V_{A^+W}(c_j))^2 + (\pi_{A_iW}(c_j) - \pi_{A^+W}(c_j))^2]}$$
$$S^- = \sqrt{\sum_{j=1}^{n}[\frac{1}{2n}(\mu_{A_iW}(c_j) - \mu_{A^-W}(c_j))^2 + (V_{A_iW}(c_j) - V_{A^-W}(c_j))^2 + (\pi_{A_iW}(c_j) - \pi_{A^-W}(c_j))^2]}$$
$$i = 1,2,3\ldots.n \quad j = 1,2,3\ldots.m \qquad \ldots(5)$$

Step 7: Calculate the rank of alternative.

$$CA_i = \frac{S_i^-}{S_i^- + S_i^+} \quad i = 1,2,3\ldots\ldots\ldots n \qquad \ldots(6)$$

2.4. IF WPM

Let A={ $A_1, A_2, A_3 \ldots\ldots A_n$ } be a set of alternative and X={ $c_1, c_2, c_3 \ldots. c_m$ } be a set of criteria, hence intuitionistic fuzzy WPM had been suitable for getting the most appropriate vendor. Procedure for intuitionistic fuzzy WPM method has been given as follows. Rating of alternative with respect to criteria is as follows:

i=1, 2, 3 …n j=1, 2, 3 …m

$$\mathbf{r} = \begin{bmatrix} [\mu_{A_1}(c_1), V_{A_1}(c_1), \pi_{A_1}(c_1)] & & [\mu_{A_1}(c_m), V_{A_1}(c_m), \pi_{A_1}(c_m)] \\ [\mu_{A_2}(c_1), V_{A_2}(c_1), \pi_{A_2}(c_1)] & \ldots & [\mu_{A_2}(c_m), V_{A_2}(c_m), \pi_{A_2}(c_m)] \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ [\mu_{A_n}(c_1), V_{A_n}(c_1), \pi_{A_n}(c_1)] & \ldots & [\mu_{A_n}(c_m), V_{A_n}(c_m), \pi_{n}(c_m)] \end{bmatrix}$$

The weight of criteria is as follows:

$$\mathbf{w} = [(\mu_w(c_j), v_w(c_j), \pi_w(x_j)), (\mu_w(c_j), v_w(c_j), \pi_w(c_j)) \ldots\ldots (\mu_w(c_j), v_w(c_j), \pi_w(c_j))]^T$$

ALGORITHM OF STEPS:

Step 1: The defuzzification of weight of criteria are calculated as follows:

$$W_w(c_j) = \frac{\mu_w(c_j) + (1 - V_w(c_j))}{\sum_{j=1}^{n}\mu_w(c_j) + (n - \sum_{j=1}^{n}V_w(c_j))} \quad . \qquad \ldots\ldots.(1)$$

Step 2: Derive total intuitionistic fuzzy score for individual alternatives by multiplying by final aggregated intuitionistic fuzzy decision matrix by their respective weight (i.e weight of criteria)

$$A_i = \prod_{j=1}^{m}(\mu_{A_i}(c_j), v_W(c_i), \pi_W(c_i))^{W_j} \qquad \ldots\ldots(2)$$

Step 3: Using score function find out the rank of alternative[17]

$$S(A_1) = [\mu_{A_1}(c_1) - V_{A_1}(c_1)] \qquad \ldots(3)$$

2.5. IF LEXICOGRAPHIC

Step1: Criteria are ranked based on assigned weights(IFS form) by decision makers or calculated weights.

Step2: The alternative with maximum value(membership function) and minimum value(non-membership function) under the most important criteria will be considered as a winner in lexicographic procedure.

Step3: If two or more alternative has the equal value then comparison of their payoff under the next most important criteria will be consider as the winner.

Step4: Finally to rank the alternatives winner will be eliminated from the list of alternatives and the procedure will be repeated for the remaining alternatives.

3. NUMERICAL EXAMPLE 16]

A company wanted to select the most appropriate supplier for one of the key element in its manufacturing process. Four suppliers have remained as alternative for the further evaluation. In order to evaluate alternative supplier we use data from only one decision maker.In this problem we have considered four alternative A_1, A_2, A_3 and A_4 respectively. Also we have considered six criteria gradually C_1, C_2, C_3, C_4, C_5 and C_6 respectively denoted by:

C_1=Quality, C_2=Cost, C_3=Technical capability, C_4=delivery, C_5=Service, C_6=Flexibility.

Table 1.Intuitionistic Fuzzy Decision Matrix of alternative vs. criteria

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(.6,.3,.1)	(.5,.3,.2)	(.4,.3,.3)	(.5,.4,.1)	(.9,0,.1)	(.2,.5,.3)
A_2	(.6,.3,.1)	(.7,.1,.2)	(.4,.1,.5)	(.5,.3,.2)	(.7,.2,.1)	(.2,.4,.4)
A3	(.4,.2,.4)	(.4,.3,.3)	(.4,.3,.3)	(.6,.2,.2)	(.8,.1,.1)	(.2,.5,.3)
A4	(.6,.3,.1)	(.3,.1,.6)	(.1,.4,.5)	(.7,.2,.1)	(.5,.2,.3)	(.3,.3,.4)

Table 2:Weight of criteria

Weights	C_1	C_2	C_3	C_4	C_5	C_6
W	(.2,.4,.4)	(.2,.2,.6)	(.1,.5,.4)	(.15,.5,.35)	(.25,.3,4)	(.1,.3,.6)

3.1. Application with Electre Method

Step: 1 Intuitionistic Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(.6,.3)	(.5,.3)	(.4,.3)	(.5,.4)	(.9, 0)	(.2,.5)
A_2	(.6,.3)	(.7,.1)	(.4,.1)	(.5,.3)	(.7,.2)	(.2,.4)
A_3	(.4,.2)	(.4,.3)	(.4,.3)	(.6,.2)	(.8,.1)	(.2,.5)
A_4	(.6,.1)	(.3,.1)	(.1,.4)	(.7,.2)	(.5,.2)	(.3,.3)

Step: 2 Weight of Criteria in the form of Intuitionistic Fuzzy Set

	C_1	C_2	C_3	C_4	C_5	C_6
W_i	(.2,.4)	(.2,.2)	(.1,.5)	(.15,.5)	(.25,.3)	(.1,.3)

Step: 3 Intuitionistic Fuzzy Weighted Decision matrixes

	C_1	C_2	C_3	C_4	C_5	C_6
S_1	(.12,.58)	(.10,.44)	(.075,.65)	(.22,.7)	(.22,.3)	(.02,.65)
S_2	(.12,.58)	(.14,.28)	(.075,.55)	(.17,.65)	(.17,.44)	(.02,.58)
S_3	(.05,.46)	(.08,.44)	(.09,.65)	(.12,.6)	(.12,.37)	(.02,.65)
S_4	(.12,.46)	(.06,.28)	(.01,.7o)	(.12,.6)	(.12,.44)	(.03,.51)

Step: 4 Concordances and Discordances Sets

Concordance Set	
$C(1,2) = \{1,5\}$	$(.2,.4) + (.25,.3) = (.4,.12) = .52$
$C(1,3) = \{2,5,6\}$	$(.2,.2) + (.25,.3) + (.1,.3) = (.46,.018) = .47$
C(1, 4)= {3, 5, 6}	$(.1,.5) + (.25,.3) + (.1,.3) = (.392,.045) = .43$
C(2, 1)= {1, 2, 3, 6}	$(.2,.4) + (.2,.2) + (.1,.5) + (.1,.3) = (.48,.012) = .492$
C(2, 3)= {2, 6}	$(.2,.2) + (.1,.3) = (.19,.15) = .34$
C(2, 4)= {2, 3, 5}	$(.2,.3) + (.1,.5) + (.25,.3) = (.46,.045) = .505$
C(3, 1)= {3, 6}	$(.1,.5) + (.1,.3) = (.19,.15) = .34$
C(3, 2)= { }	{ } = 0
C(3, 4)= {3, 4, 5 }	$(.1,.5) + (.15,.5) + (.25,.3) = (.42,.075) = .495$
C(4, 1)= {1, 6}	$(.2,.4) + (.1,.3) = (.28,.12) = .4$
C(4, 2)= {1}	$(.2,.4) = .6$
C(4, 1)= {1, 4, 6}	$(.2,.4) + (.15,.5) + (.25,.3) = (.49,.06) = .55$

$$Average = \bar{C} = \frac{5.14}{12} = .42$$

Discordance Set	
D(1, 2)= {2, 3, 6}	.72
D(1, 3)= {3}	.51
D(1, 4)= {1, 6}	.35
D(2, 1)= {5}	.18
D(2, 3)= {}	.0
D(2, 4)= {1, 6}	.40
D(3, 1)= {2, 5}	.22
D(3, 2)= {1, 2, 6}	.54
D(3, 4)= {1, 6}	.33
D(4, 1)= {3, 5}	.17
D(4, 2)= {3, 5}	.24
D(4, 3)= {3, 4, 5}	.17

$$Average = \bar{C} = \frac{3.29}{12} = .27$$

Determination of outranking Relationship:

C_{pq}	$C_{pq} \geq \bar{C} = .42$	D_{pq}	$D_{pq} \leq \bar{C} = .27$
$C_{12} = .52$		$D_{12} = .72$	
$C_{13} = .47$		$D_{13} = .51$	
$C_{14} = .43$		$D_{14} = .35$	
$C_{21} = .492$	Yes	$D_{21} = .18$	Yes
$C_{23} = .34$		$D_{23} = 0$	
$C_{24} = .505$		$D_{24} = .4$	
$C_{31} = .34$		$D_{31} = .22$	
$C_{32} = 0$		$D_{32} = .54$	
$C_{34} = .495$		$D_{34} = .33$	
$C_{41} = .4$		$D_{41} = .17$	
$C_{42} = .6$	Yes	$D_{42} = .24$	Yes
$C_{43} = .55$	Yes	$D_{43} = .17$	Yes

$A_4 > A_2$ $A_4 > A_3$ $A_2 > A_1$
$A_4 > A_2, A_{3,} A_1$

3.2. APPLICATION WITH IFSAW METHOD

Step 1: Aggregated Intuitionistic Fuzzy Decision matrix of Benefit Criteria

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(.6,.3)	(.5,.3)	(.4,.3)	(.5,.4)	(.9,0)	(.2,.5)
A_2	(.6,.3)	(.7,.1)	(.4,.1)	(.5,.3)	(.7,.2)	(.2,.4)
A_3	(.4,.2)	(.4,.3)	(.4,.3)	(.6,.2)	(.8,.1)	(.2,.5)
A_4	(.6,.1)	(.3,.1)	(.1,.4)	(.7,.2)	(.5,.2)	(.3,.3)

Step 2: Intuitionistic Fuzzy decision matrix of cost criteria

C_2	
(.5,.3)	
(.7,.1)	
(.4,.3)	
(.3,.1)	

Step 3: Use concentration and Dilation operator in the cost criteria for cost minimization

C_2	
(.25,.51)	
(.49,.19)	
(.16,.52)	
(.09,.19)	

Step 4: Intuitionistic Fuzzy Decision matrix

	C_1	C2	C3	C4	C5	C_6
A_1	(.6,.3)	(.25,.51)	(.4,.3)	(.5,.4)	(.9, 0)	(.2,.5)
A_2	(.6,.3)	(.49,.19)	(.4,.1)	(.5,.3)	(.7,.2)	(.2,.4)
A_3	(.4,.2)	(.16,.52)	(.4,.3)	(.6,.2)	(.8,.1)	(.2,.5)
A_4	(.6,.1)	(.09,.19)	(.1,.4)	(.7,.2)	(.5,.2)	(.3,.3)

Step 5: Weight of criteria in intuitionistic fuzzy form

	C_1	C2	C3	C4	C5	C6
W_i	(.2,.4)	(.2,.2)	(.1,.5)	(.15,.5)	(.25,.3)	(.1,.3)

Step 6: Weighted intuitionistic fuzzy decision matrix

	C2	C_1	C3	C4	C5	C6
A_1	(.05,.6)	(.12,.58)	(.04,.65)	(.075,.7)	(.22,.3)	(.2,.65)
A_2	(.098,.35)	(.12,.58)	(.04,.55)	(.075,.65)	(.17,.44)	(.02,.58)
A_3	(.032,.61)	(.08,.52)	(.04,.65)	(.09,.6)	(.2,.37)	(.02,.65)
A_4	(.018,.35)	(.12,.46)	(.01,.7)	(.105,.6)	(.12,.44)	(.03,.51)

Step 6: Score of Alternatives

$S_1 = (.43, .030) = .4$,

$S_2 = (.67, .00057) = .6694$,

$S_3 = (.83, .0001) = .8299$,

$S_4 = (.89, .0000) = .89$

Ranking is $A_4 > A_3 > A_2 > A_1$

3.3. APPLICATION WITH IF TOPSIS METHOD

Step 1: Intuitionistic Fuzzy Decision Matrix

	C_1	C2	C3	C4	C5	C6
A1	(.6,.3)	(.5,.3)	(.4,.3)	(.5,.4)	(.9, 0)	(.2,.5)
A2	(.6,.3)	(.7,.1)	(.4,.1)	(.5,.3)	(.7,.2)	(.2,.4)
A3	(.4,.2)	(.4,.3)	(.4,.3)	(.6,.2)	(.8,.1)	(.2,.5)
A4	(.6,.1)	(.3,.1)	(.1,.4)	(.7,.2)	(.5,.2)	(.3,.3)

Step 2: Weight of Criteria in the form of Intuitionistic Fuzzy Set

	C_1	C2	C3	C4	C5	C6
W_i	(.2,.4)	(.2,.2)	(.1,.5)	(.15,.5)	(.25,.3)	(.1,.3)

Step 3: The Evaluation Value of Supplier

	C_1	C2	C3	C4	C5	C6
A_1	(.12,.58)	(.1,.44)	(.04,.51)	(.075,.7)	(.225,.3)	(.02,.65)
A_2	(.12,.58)	(.14,.28)	(.04,.55)	(.075,.65)	(.175,.44)	(.02,.58)
A_3	(.08,.52)	(.08,.44)	(.04,.65)	(.09, 6)	(.2,.37)	(.02,.65)
A_4	(.12,.46)	(.06,.28)	(.01,.7)	(.105,.6)	(.125,.44)	(.03,.51)

Step 4: Positive ideal solution and negative ideal solution

	C_1	C2	C3	C4	C5	C6
S^+	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
S^-	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)

Step 5: Calculate the distance to the Intuitionistic fuzzy ideal points and the relative access degree.

	d_i	d_t	k_i	Ranking
A_1	.6450	.3737	.3688	3
A_2	.6528	.3838	.3702	2
A_3	.6659	.3605	.3512	4
A_4	.6634	.3974	.3746	1

3.4. APPLICATION WITH IF WPM METHOD

Step 1: Intuitionistic Fuzzy Decision Matrix

	C_1	C2	C3	C4	C5	C6
A1	(.6,.3)	(.5,.3)	(.4,.3)	(.5,.4)	(.9, 0)	(.2,.5)
A2	(.6,.3)	(.7,.1)	(.4,.1)	(.5,.3)	(.7,.2)	(.2,.4)
A3	(.4,.2)	(.4,.3)	(.4,.3)	(.6,.2)	(.8,.1)	(.2,.5)
A4	(.6,.1)	(.3,.1)	(.1,.4)	(.7,.2)	(.5,.2)	(.3,.3)

Step: 2 Weight of Criteria in the form of Intuitionistic Fuzzy Set

	C_1	C2	C3	C4	C5	C6
W_i	(.2,.4)	(.2,.2)	(.1,.5)	(.15,.5)	(.25,.3)	(.1,.3)

Step 3: Defuzzified weight of Criteria

	C_1	C2	C3	C4	C5	C6
W_i	.173	.21	.13	.141	.2	.173

Step 4: Final Score of Alternative

$A_1 = (.43, .00000017) = .4299$

$A_2 = (.462, .00000017) = .4619$

$A_3 = (.4173, .00000017) = .4172$

$A_4 = (.351, .00000017) = .3509$

3.5. APPLICATION WITH LEXICOGRAPHIC METHOD

Step 1:We observe from IFS data that maximum membership value is assigned to fifth criteria
i.e. service.So choosing C5 as the most important criteria among other criterions.
Step 2:Looking at alternative vendors we pick one with Maximum membership and minimum non-membership.
Step 3:Vendor 1 seems to agree with step 2.
Step 4:The scores with respect to Vendors A_1,A_2,A_3,A_4 are
S(A1)=(.9,0)=.9
S(A2)=(.7,.2)=.5
S(A3)=(.8,.1)=.7
S(A4)=(.5,.2)=.3
So $A_1 > A_3 > A_2 > A_4$.

CONCLUSIONS

Five MADM methods have been implemented to determine which of four vendors is best suited for order allocation from a company or firm. Our findings reveal that ELECTRE, IFSAW, IFTOPSIS prefer vendor 4 (Table 3)as the best vendor. It appears quality and service of vendor 4 were the main reason for selection. The IF WPM yielded results that were too extreme to be taken into account because of the use of weights as exponents in the mathematical formulation. In lexicographic method too the ranking was diversified because this method utilizes only a small part of the available information in making a final choice. This study reveals a general view of MADM methods under IFS environment.To sum up the study reveals that some methods are better then the others but none are perfectly accurate for ranking of alternatives.

Table 3:Final ranking of vendors by various methods

RANKING ORDER OF VENDORS	IF ELECTRE	IFSAW	IF TOPSIS	IF LEXICOGRAPHIC METHOD	IFWPM
1	A4	A4	A4	A1	A2
2	A2	A3	A2	A3	A1
3	A3	A2	A1	A2	A3
4	A1	A1	A3	A4	A4

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