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The mathematical formulation of the problem of determining the horizontal well productivity

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Abstract
Estimation of well productivity is important in the design of horizontal wells. In order to avoid overestimation of horizontal well productivity it is necessary to take into account frictional pressure losses along completion. The motivation for this work is to find general solution to the problem of determining of the horizontal well productivity using mathematical apparatus such as partial discretization, method of variation, and apply it to some synthetic fields to calculate flowrates.

Keywords: Horizontal well, Frictional pressure losses, Well performance

Problem formulation

Pressure losses along a horizontal section of a well may lead to significant reduction on the overall well's production capacity. They are made up of three components aligned with the laws of conservation of mass, momentum and energy. Mainly, pressure losses are associated with friction:

$$\frac{dP}{dL} = \left(\frac{dP}{dL}\right)_{gravity} + \left(\frac{dP}{dL}\right)_{acceleration} + \left(\frac{dP}{dL}\right)_{friction} \quad (1)$$

Many researchers have studied different aspects of pressure losses along well completion due to friction [1], [2], [3]. Hereafter we refer only to those works which we consider most important.

In order to estimate pressure distribution along producing horizontal well Birchenko et al. consider two problems [4]:

- Inflow from the reservoir to the horizontal well;
- Fluid flow along horizontal well.

With the following assumptions:

- Reservoir is homogeneous;
- Reservoir fluid is incompressible;
- Flow is steady or pseudo-steady state, subject to Darcy law;
- Friction factor *f* is constant along the completion interval;
- Pressure drop due to acceleration is small compared to that of friction;
- Dependence of fluid's viscosity upon pressure can be neglected.

If all the above conditions are met a problem of turbulent flow along horizontal section of the well can be reduced to a second order non-linear ordinary differential equation:

$$y''(x) = h \cdot y^2(x), \quad (2)$$

where h *is a horizontal well number, and* $= \begin{cases} h_q \text{ is for rate constrained well}; \\ h_p \text{ is for pressure constrained well}. \end{cases}$

With two sets of boundary conditions

$$y_q(0) = 0, y_q(1) = 1 \text{ or } y_p(0) = 0, \frac{dy_p(1)}{dx} = 1.$$

In fact, fluid inflow from the reservoir is proportional to the pressure drop across external boundary of reservoir andwell [4]:

$$\frac{dq}{dl} = J_s(P_l - P(l)). \quad (3)$$

In turn, frictional pressure losses along horizontal section of the well are identified by Darcy-Weisbach equation:

$$\frac{dP}{dl} = -\frac{C_f \rho f_a B^2}{D^5} q^2(l). \quad (4)$$

Then the system of equations (3) and (4) can be reduced to a single non-linear ordinary differential equation of the second order [4]:

$$\frac{d^2q}{dl^2} = \frac{J_s C_f \rho f_a B^2}{D^5} q^2. \quad (5)$$

Next we introduce dimensionless distance from "toe" of the horizontal well$x = l/L$ and dimensionless flowrate for rate constrained wells:

$y_q(x) = q(Lx)/q_w$.

Here

$\frac{dy}{dx} = \frac{1}{q_W}\frac{dq}{dl}\frac{dl}{dx} = \frac{L}{q_W}\frac{dq}{dl}$,

$\frac{d^2y}{dx^2} = \frac{L}{q_w}\frac{d^2q}{dl^2}\frac{dl}{dx} = \frac{L^2}{q_w}\frac{d^2q}{dl^2}$.

Then equation (5) takes the following form:

$\frac{d^2y}{dx^2} = h_q y^2(x)ory''(x) = h_q \cdot y^2(x)$,

where$h_q = \frac{C_f \rho f_a B^2 J_s L^2 q_W}{D^5}$.

Here [4]: C_f – conversion coefficient, ρ - fluid density, f_a – an average value of the friction factor along completion interval, B – formation volume factor, J_s – specific productivity index, L – horizontal section length, q_w – production rate $q(L)$, D - completion internal diameter, ΔP_w – drawdown at the heel, $P_l - P_w$.

Note, many researchers agree that pressure drop due to acceleration in horizontal wells is usually small compared to that due to friction. However, quantitative analysis confirming this assumption is indeed almost inaccessible. It is also dissembled about pressure drop due to gravity [4].

Solution

Suggesting that equation (1) should be considered as a second order linear ordinary differential equation with variable coefficients:

$$\begin{cases} y''(x) = 2p(x)y'(x) + q(x)y(x) + h(x)y^2(x), \\ x \in [a,b] y(a) = \alpha, y(b) = \beta \text{ or } y(a) = \alpha, y'(b) = \beta \end{cases} \quad (6)$$

The problem thus stated cannot be solved. In order to obtain an analytic solution we generalize equation (2) in the following manner:

$$\begin{cases} y''(x) = 2py'(x) + qy(x) + hy^2(x), x \in [0,1] \\ \textit{with} \text{boundary conditions } y(0) = 0, y(1) = 1, \\ \quad y(0) = 0, y'(1) = 1. \end{cases} \quad (7)$$

i.e. instead of the equation (6) we consider second order ordinary non-linear differential equations with constant coefficients.

We discretize equation (7) by the method of partial discretization [5]. For this we use the theorem of partition of unity and take into account generalized functions properties. Then the equation $y''(x) - 2py'(x) - qy(x) = h \cdot y^2(x) \cdot 1$ can be written as:

$$y''(x) - 2py'(x) - qy(x) = \\ = +\frac{1}{2}\sum_k (x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - x_{k+1})]. \quad (8)$$

Where $1 \approx \frac{1}{2}\sum_k (x_k + x_{k+1})[\delta(x - x_k) - \delta(x - x_{k+1})]$, and used property of generalized functions $a(x)\delta(x - x_0) = a(x_0)\delta(x - x_0)$.

Now we find a general solution of a homogeneous equation:$y''(x) - 2py'(x) - qy(x) = 0$. The roots of the characteristic equation $\lambda_{1/2} = p \pm \sqrt{p^2 + q}$.Three scenarios will be considered:

I. If $p^2 + q = -\omega^2$, then $y_{hom} = (C_1 \cos\omega x + C_2 \sin\omega x)e^{px}$

We apply a method of variation of constants to find a particular solution of equation (8) under condition that$p^2 + q < 0$, and in the form: $y_{part} = C_1(x)y_1(x) + C_2(x)y_2(x)$, where $y_1(x) = e^{px}\cos\omega x, y_2(x) = e^{px}\sin\omega x$, and $C_1(x)$, $C_2(x)$ are differentiable functions to be determined.

Hence, according to the method of variation of constants:

$$\begin{cases} C_1'(x)y_1(x) + C_2'(x)y_2(x) = 0, \\ C_1'(x)y_1'(x) + C_2'(x)y_2'(x) = \Phi(x), \end{cases}$$

where$\Phi(x)$ is a right hand side of equation (8).

$\Delta = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} = y_1(x)y_2'(x) - y_2(x)y_1'(x)$,

$\Delta_{C_1'(x)} = \begin{vmatrix} 0 & y_2(x) \\ \Phi(x) & y_2'(x) \end{vmatrix} = -\Phi(x)y_2(x)$,

$\Delta_{C_2'(x)} = \begin{vmatrix} y_1(x) & 0 \\ y_1'(x) & \Phi(x) \end{vmatrix} = -\Phi(x)y_1(x)$.

Let

simplifyΔ: $\Delta = y_1(x)y_2'(x) - y_2(x)y_1'(x) = e^{px}\cos\omega x(pe^{px}\sin\omega x + \omega e^{px}\cos\omega x) - e^{px}\sin\omega x(pe^{px}\cos\omega x - \omega e^{px}\sin\omega x) = e^{2px}\omega\cos^2\omega x + e^{2px}\omega\sin^2\omega x = \omega e^{2px}$.

By Cramer's rule

$$C_1'(x) = \frac{\Delta_{C_1'(x)}}{\Delta} = -\frac{\Phi(x)y_2(x)}{\omega e^{2px}} = -\frac{\Phi(x)e^{px}\sin\omega x}{\omega e^{2px}} \\ = -e^{-px}\frac{\sin\omega x}{\omega}\Phi(x),$$

$$C_2'(x) = \frac{\Delta_{C_2'(x)}}{\Delta} = \frac{\Phi(x)y_1(x)}{\omega e^{2px}} = \frac{\Phi(x)e^{px}\cos\omega x}{\omega e^{2px}} \\ = e^{-px}\frac{\cos\omega x}{\omega}\Phi(x).$$

Then

$C_1(x) = C_1 - \frac{1}{\omega}\int e^{-px}\sin\omega x \frac{h}{2}\sum_k (x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - x_{k+1})]dx == C_1 - \frac{h}{2\omega}\sum_k (x_k + x_{k+1})[e^{-px_k}\sin\omega x_k y^2(x_k)H(x - x_k) - e^{-px_{k+1}}\sin\omega x_{k+1} y^2(x_{k+1})H(x - -x_{k+1})]$.

Here

$H(x) = \begin{cases} 1, x > 0 \ \textit{Heaviside function, and } H'(x) = \delta(x). \\ 0, x < 0 \end{cases}$

$C_2(x) = C_2 + \frac{1}{\omega}\int e^{-px}\cos\omega x \frac{h}{2}\sum_k (x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - x_{k+1})]dx == C_2 + \frac{h}{2\omega}\sum_k (x_k + x_{k+1})[e^{-px_k}\cos\omega x_k y^2(x_k)H(x - x_k) - e^{-px_{k+1}}\cos\omega x_{k+1} y^2(x_{k+1})H(x - -x_{k+1})]$.

Substituting found functions $C_1(x), C_2(x)$at $y = C_1(x)y_1(x) + C_2(x)y_2(x)$, we obtain the general solution of nonhomogeneous equation (8).

$$y(x) = (C_1\cos\omega x + C_2\sin\omega x)e^{px} \\ - \frac{h}{2\omega}e^{px}\Bigg\{\cos\omega x \sum_k (x_k + x_{k+1}) \cdot \\ \cdot [e^{-px_k}\sin\omega x_k y^2(x_k)H(x - x_k) \\ - e^{-px_{k+1}}\sin\omega x_{k+1} y^2(x_{k+1})H(x - x_{k+1})] \\ - \\ -\sin\omega x \sum_k (x_k + x_{k+1})[e^{-px_k}\cos\omega x_k y^2(x_k)H(x - x_k) - \\ -e^{-px_{k+1}}\cos\omega x_{k+1} y^2(x_{k+1})H(x - x_{k+1})]\Bigg\}.$$

We divide interval [0,1] into n equal parts $x_1, x_2, \ldots, x_{n-1}, x_n, x_{n+1}$. +0 will be taken as x_1i.e. $x_1 = +0$, then $x_{n+1} = 1$ and $\Delta x = \frac{x_{k+1} - x_k}{n} = \frac{1}{\omega}, k = 0,1, \ldots, n + 1$.

We use initial condition at "toe" of the horizontal well to determine C_1, C_2 constants.

$y(0) = 0, y'(0) = y'_o$ (9)

$y(0) = C_1 = 0$. To find C_2 constant $y'(x)$ should be found first:

$y'(x) = C'_1(x)y_1(x) + C'_2(x)y_2(x) + C_1(x)y'_1(x) + C_2(x)y'_2(x)$,

where $y'_1(x) = e^{px}(p\cos\omega x - \omega\sin\omega x), y'_2(x) = e^{px}(p\sin\omega x + \omega\cos\omega x)$.

$C'_1(x) = -e^{-px}\frac{\sin\omega x}{\omega}\Phi(x), C'_2(x) = e^{-px}\frac{\cos\omega x}{\omega}\Phi(x)$,

$\Phi(x) = \frac{h}{2}\sum_k(x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - x_{k+1})]$.

Then $y'(0) = C_2(0)y'_2(0) = C_2(0)\omega = C_2\omega \equiv y'_o$. From here $C_2 = \frac{1}{\omega}y'_o$, because $\Phi(0) = 0, C'_1(0) = 0, C'_2(0) = 0$.

Therefore the solution of equation (8) satisfying the initial conditions (9) and the condition $p^2 + q = -\omega^2$ can be represented as:

$y(x) = \{y'_o\sin\omega x - \frac{h}{2}[\cos\omega x\sum_k(x_k + x_{k+1})\cdot[e^{-px_k}\sin\omega x_k y^2(x_k)H(x - x_k) - e^{-px_{k+1}}\sin\omega x_{k+1}y^2(x_{k+1})H(x - x_{k+1})) - \sin\omega x\sum_k(x_k + x_{k+1})(e^{-px_k}\cos\omega x_k y^2(x_k)H(x - x_k) - e^{-px_{k+1}}\cos\omega x_{k+1}y^2(x_{k+1})H(x - x_{k+1}))]]\}\frac{1}{\omega}e^{px}$.

II. If the roots $\lambda_{1/2} = p \pm \sqrt{p^2 + q}$ of the characteristic equation are real-valued then the general solution of the homogeneous equation can be written as:

$y_{hom}(x) = C_1e^{\left(p-\sqrt{p^2+q}\right)x} + C_2e^{\left(p+\sqrt{p^2+q}\right)x}$.

Again we apply a method of variation of constants to find a particular solution of equation (8) under condition that $p^2 + q > 0$, and in the form: $y_{part} = C_1(x)y_1(x) + C_2(x)y_2(x)$. Assuming that $C_1(x), C_2(x)$ are differentiable functions, and defined from the system of algebraic equation by Cramer's method:

$$\begin{cases} C'_1(x)y_1(x) + C'_2(x)y_2(x) = 0, \\ C'_1(x)y'_1(x) + C'_2(x)y'_2(x) = \Phi(x), \end{cases}$$

where $y_1(x) = e^{\left(p-\sqrt{p^2+q}\right)x}, y_2(x) = e^{\left(p+\sqrt{p^2+q}\right)x}$, and $\Phi(x)$ is a right hand side of equation (8). Then

$\Delta = y_1(x)y'_2(x) - y_2(x)y'_1(x) = 2\sqrt{p^2 + q}e^{2px}$,

$\Delta_{C'_1(x)} = -\Phi(x)y_2(x), \Delta_{C'_2(x)} = \Phi(x)y_1(x)$.

By Cramer's rule:

$$C'_1(x) = -\frac{1}{2\sqrt{p^2 + q}}e^{-\left(p-\sqrt{p^2+q}\right)x}\Phi(x),$$

$$C'_2(x) = \frac{1}{2\sqrt{p^2 + q}}e^{-\left(p+\sqrt{p^2+q}\right)x}\Phi(x).$$

Integrating obtained equations we have:

$C_1(x) = C_1 - \frac{1}{2\sqrt{p^2+q}}\frac{h}{2}\int e^{-\left(p-\sqrt{p^2+q}\right)x}\cdot\sum_k(x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - -x_{k+1})]dx = C_1 - \frac{h}{4\sqrt{p^2+q}}\sum_k(x_k + x_{k+1})\left[e^{-(p-\sqrt{p^2+q})x_k}y^2(x_k)H(x - x_k) - e^{-(p-\sqrt{p^2+q})x_{k+1}}y^2(x_{k+1})H(x - -x_{k+1})\right]$.

$C_2(x) = C_2 + \frac{1}{2\sqrt{p^2+q}}\frac{h}{2}\int e^{-\left(p+\sqrt{p^2+q}\right)x}\cdot\sum_k(x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - -x_{k+1})]dx = C_2 + \frac{h}{4\sqrt{p^2+q}}\sum_k(x_k + x_{k+1})\left[e^{-(p+\sqrt{p^2+q})x_k}y^2(x_k)H(x - x_k) - e^{-(p+\sqrt{p^2+q})x_{k+1}}y^2(x_{k+1})H(x - -x_{k+1})\right]$.

Substituting found functions $C_1(x), C_2(x)$ at $y = C_1(x)y_1(x) + C_2(x)y_2(x)$, we obtain the general solution of nonhomogeneous equation (8).

$y(x) = C_1e^{\left(p-\sqrt{p^2+q}\right)x} + C_2e^{\left(p+\sqrt{p^2+q}\right)x} - \frac{h}{4\sqrt{p^2+q}}\Big\{e^{\left(p-\sqrt{p^2+q}\right)x}\sum_k(x_k + x_{k+1})\left[e^{-(p-\sqrt{p^2+q})x_k}y^2(x_k)H(x - x_k) - e^{-(p-\sqrt{p^2+q})x_{k+1}}y^2(x_{k+1})H(x - x_{k+1})\right] - e^{\left(p+\sqrt{p^2+q}\right)x}\sum_k(x_k + x_{k+1})\left[e^{-(p+\sqrt{p^2+q})x_k}y^2(x_k)H(x - x_k) - e^{-(p+\sqrt{p^2+q})x_{k+1}}y^2(x_{k+1})H(x - x_{k+1})\right]\Big\}$.

We construct a particular solution satisfying given initial conditions based on the general solution: $y(0) = 0, y'(0) = y'_o$.

$y(0) = C_1 + C_2 = 0$.

We differentiate the general solution to generate a second equation

$$y'(x) = C'_1(x)y_1(x) + C'_2(x)y_2(x) + C_1(x)y'_1(x) + C_2(x)y'_2(x),$$

where $y'_1(x) = \left(p - \sqrt{p^2 + q}\right)e^{\left(p-\sqrt{p^2+q}\right)x}$,

$y'_2(x) = \left(p + \sqrt{p^2 + q}\right)e^{\left(p+\sqrt{p^2+q}\right)x}$.

Then

$y'(0) = C_1(0)y'_1(0) + C_2(0)y'_2(0) = C_1\left(p - \sqrt{p^2 + q}\right) + C_2\left(p + \sqrt{p^2 + q}\right) \equiv y'_o$.

Thus we obtain a system of algebraic equations to find C_1 and C_2:

$$\begin{cases} C_1 + C_2 = 0 \\ \left(p - \sqrt{p^2 + q}\right)C_1 + \left(p + \sqrt{p^2 + q}\right)C_2 = y'_0. \end{cases}$$

Therefore $C_1 = -\frac{y'_0}{2\sqrt{p^2+q}}, C_2 = \frac{y'_0}{2\sqrt{p^2+q}}$.

Finally

$$y(x) = \frac{y'_0}{\sqrt{p^2+q}}e^{px}\mathrm{sh}\sqrt{p^2 + q}x - \frac{h}{4\sqrt{p^2+q}}\Big\{e^{\left(p-\sqrt{p^2+q}\right)x}\sum_k(x_k + x_{k+1})\left[e^{-(p-\sqrt{p^2+q})x_k}y^2(x_k)H(x - -x_k) - e^{-(p-\sqrt{p^2+q})x_{k+1}}y^2(x_{k+1})H(x - x_{k+1})\right] - e^{\left(p+\sqrt{p^2+q}\right)x}\sum_k(x_k + x_{k+1})\left[e^{-(p+\sqrt{p^2+q})x_k}y^2(x_k)H(x - x_k) - e^{-(p+\sqrt{p^2+q})x_{k+1}}y^2(x_{k+1})H(x - x_{k+1})\right]\Big\}. \quad (11)$$

III. If the roots of the characteristic equation are equal, the general solution of the homogenous equation can be written as:

$y_{hom} = C_1e^{px} + xC_2e^{px} = (C_1 + xC_2)e^{px}$.

Again a method of variation of constants is applied to find a particular solution of equation (8) under condition that $p^2 + q = 0$, and in the form:

$y_{part} = C_1(x)y_1(x) + C_2(x)y_2(x)$, where $y_1(x) = e^{px}, y_2(x) = xe^{px}$. According to the method, $C_1(x)$, $C_2(x)$ are found from the following system:

$$\begin{cases} C_1'(x)y_1(x) + C_2'(x)y_2(x) = 0, \\ C_1'(x)y_1'(x) + C_2'(x)y_2'(x) = \Phi(x), \end{cases}$$

where $y_1'(x) = pe^{px}, y_2'(x) = (1+xp)e^{px}, \Phi(x)$ is a right hand side of equation (8). Then

$\Delta = y_1(x)y_2'(x) - y_2(x)y_1'(x) = e^{2px}$,

$\Delta_{C_1'(x)} = -\Phi(x)y_2(x), \Delta_{C_2'(x)} = \Phi(x)y_1(x)$.

By Cramer's rule:

$C_1'(x) = -xe^{-px}\Phi(x)$,

$C_2'(x) = e^{-px}\Phi(x)$.

Integrating obtained equations we have:

$$C_1(x) = C_1 - \int xe^{-px}\Phi(x)dx = C_1 - \frac{h}{2}\int xe^{-px}\sum_k(x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - x_{k+1})]dx = C_1 - \frac{h}{2}\sum_k(x_k + x_{k+1})[x_k e^{-px_k}y^2(x_k)H(x - x_k) - x_{k+1}e^{-px_{k+1}}y^2(x_{k+1})H(x - -x_{k+1})].$$

$$C_2(x) = C_2 + \int e^{-px}\Phi(x)dx = C_2 + \frac{h}{2}\int e^{-px}\sum_k(x_k + x_{k+1})[y^2(x_k)\delta(x - x_k) - y^2(x_{k+1})\delta(x - -x_{k+1})]dx = C_2 + \frac{h}{2}\sum_k(x_k + x_{k+1})[e^{-px_k}y^2(x_k)H(x - x_k) - e^{-px_{k+1}}y^2(x_{k+1})H(x - x_{k+1})].$$

Substituting found functions $C_1(x), C_2(x)$at $y = C_1(x)y_1(x) + C_2(x)y_2(x)$, we obtain the general solution of nonhomogeneous equation (8)

$$y(x) = (C_1 + C_2x)e^{px} - \frac{-h}{2}\{e^{px}\sum_k(x_k + x_{k+1})[x_k e^{-px_k}y^2(x_k)H(x - x_k) - x_{k+1}e^{-px_{k+1}}y^2(x_{k+1})H(x - x_{k+1})] - -xe^{px}\sum_k(x_k + x_{k+1})[e^{-px_k}y^2(x_k)H(x - x_k) - e^{-px_{k+1}}y^2(x_{k+1})H(x - x_{k+1})]\}.$$

We construct a particular solution satisfying given initial conditions based on the general solution: $y(0) = 0, y'(0) = y_0'$.

At $x = 0, C_1 = y(0) = 0$. We differentiate the general solution to find second constant C_2:

$y'(x) = C_1'(x)y_1(x) + C_2'(x)y_2(x) + C_1(x)y_1'(x) + C_2(x)y_2'(x)$,

where $y_1(0) = 1$, $y_2(0) = 0$, $y_1'(0) = p, y_2'(0) = 1$, $C_1'(0) = 0, C_2'(0) = \Phi(0) = 0$.

Then $y'(0) = C_2(0)y_2'(0) = C_2 \equiv y_0'$.

Therefore

$$y(x) = y_0'xe^{px} - \frac{h}{2}\{e^{px}\sum_k(x_k + x_{k+1})[x_k e^{-px_k}y^2(x_k)H(x - x_k) - x_{k+1}e^{-px_{k+1}}y^2(x_{k+1})H(x - -x_{k+1})] - xe^{px}\sum_k(x_k + x_{k+1})[e^{-px_k}y^2(x_k)H(x - x_k) - e^{-px_{k+1}}y^2(x_{k+1})H(x - x_{k+1})]\}. \quad (12)$$

Equation (12) represents the general solution of Cauchy problem (8), (9) in caseof $p^2 + q = 0$.

Below are graphs of flowrates of some synthetic fields.

$y''(x) = 2py'(x) - qy(x) = hy^2(x)$,

$y(0) = 0, y(1) = 1, p = 0{,}1, q = -0.05$.

$p^2 + q = -0.004, \omega = 0.2, h = 1$.

I. $p = 0.2$, $q = 0.05$, $p^2 + q = 0.09$, $\sqrt{p^2 + q} = 0.3$, $L = 1$.

II. $p = q = 0, h = 1$.

In these examples, in order to plot a graph of $y(x)$ function (Fig.1) next condition $y(1) = 1$ is used at first in the solutions (10), (11), and (12). This condition is necessary to determine y_0'.

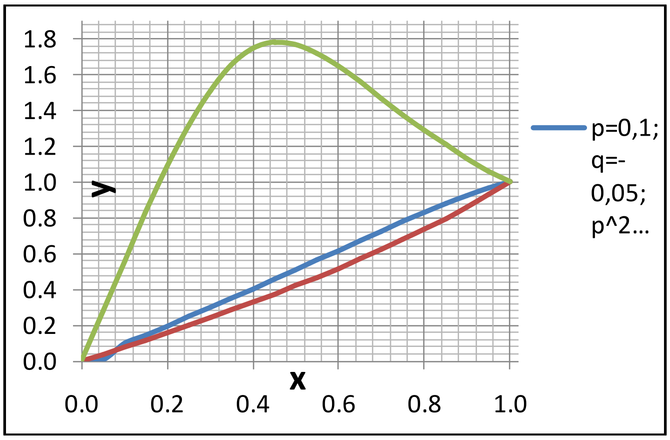

Figure 1. Horizontal well productivity based on synthetic data

Conclusion

In this paper we considered three aspects of pressure losses in horizontal wells – due to friction, gravity and acceleration. From all the work it is clear that mathematical tools are very powerful applications for engineering problems.

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