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The Performance analysis of Queueing System with general conditions Heterogeneous Servers Subject to Catastrophe Intensity

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Abstract:
The aim of this paper is to obtain the performance analysis of Queueing System with general conditions Heterogeneous Servers Subject to Catastrophe Intensity. The occurrence of a catastrophe makes the system empty instantly but the system takes its own time to be ready to accept new customers. The usual discipline first come first served (FCFS) is used with a more general condition. We obtain explicit expressions for the steady-state probabilities at arbitrary epoch using displacement operator method. The steady-state solution and some performance measures of the system have also been provided.

Keywords: Markovian queueing system, Catastrophe, Heterogeneous Servers, steady state solution,

I. INTRODUCTION

Queuing models with catastrophes gained considerable interest during the last few decades due to their applications in the analysis of computer and communication systems where catastrophes induced by external effects have an important influence on costs and performance from an economic viewpoint. Whenever a catastrophe occurs at the system, all the customers present are forced to abandon the system immediately, the server gets inoperative instantaneously, and the server is ready for service when a new customer arrives. The modeling and analysis of queuing systems with catastrophes may be used to study the migration processes with catastrophes and computer networks with virus infections or a reset order.

Queueing systems with catastrophes have been investigated by many researchers (Chao[1], Chen and Renshaw [2], Di Crescenzo et al. [3] and Boudali and Economou [4]). The catastrophes occur as negative customers to the system and its characteristic is to annihilate all the customers in the system and the momentary deactivation of the service facilities till a new arrival of customers. The catastrophes might arrive either from outside the system or from some other service station. In a queueing system, whenever catastrophe occurs, it may be thought of as a clearing mechanism which causes all the jobs in the system to be lost. If a job infected with virus in computer systems, it carries virus to other processors deactivating files and perhaps the system itself. It has enormous applications in a broad area especially in computer communication, industries, biosciences and population genetics. In real-world catastrophes appear in various situations in practice, for example, in the production sector, in the service sector, in the health care sector, in population genetics, in the transportation sector, in the telecommunication industry, etc. In most of the above cases, there is some sort of compensation for the jobs. Thus, the economic analysis of queueing systems with catastrophes who are forced to vacate the system pretend to be of concern from an applications point of view. Some articles on continuous-time queueing systems with catastrophes can be found in Kumar and Arivudainambi [5], Kumar and Madheswari [6], Kumar et al. [7]. The strategic behavior and social optimization in case of heterogeneous customers with Markovian vacation queues has been discussed in Guo and Hassin [8].

The study on multi-server queueing systems in general presumes the servers to be homogeneous. The heterogeneous service rates have many practical aspects in modeling real systems that permit customers to meet different qualities of service. For example, communications network supporting communication channels of various transmission rates, nodes in wireless systems serving different mobile users, nodes in telecommunications network
with links of various capacities, servers formed with different processors as a consequence of system updates, multiprogramming computer system which spools its output for printing on a set of printers of different speeds, or scheduling jobs on functionally equivalent processors of a local computer network, manual assembly formed with different workers with the average task completion time differing from person to person, machines undergoing
a process of rapid and constant technological renewal and depreciation, the transportation of goods with different abilities and capacities, etc. involve heterogeneous servers. The firms must give attention to the quality and service performance when designing and carrying out their operations

as these are requirements in customer perceptions. In a heterogeneous environment, resources are autonomous, distributed, dense, and dynamic, hence they should be effectively scheduled so that maximum utilization of the resources is possible. As a result, heterogeneous multi-server queues can be used to obtain more insight into these systems and thus make them more manageable. But literature on this class is limited to the servers having homogeneous service rates as it simplifies the analysis. For more details on this topic, see Larsen and Agrawala [9] and Lin and Kumar [10].

The analysis of two heterogeneous servers queue subject to catastrophes in continuous time has been carried out by Kumar et al. [11]. To evaluate system performance measures,discrete-time queueing models are better suited than their continuous-time counterparts for studying slotted digital computer communication systems, including mobile and broad integrated services digital networks (B-ISDN). It is more accurate and efficient than their continuous-time counterparts to analyze and design digital transmitting systems. Moreover, the modelling of discrete-time queues is more involved and rather different from the analysis applied for the corresponding continuous-time queueing models. The advantage of analyzing a discrete-time queue is that one can get the continuous-time results from it as a limiting case but the converse is not true. Comprehensive discussion of variouskind of discrete-time queueing models can be found in Hunter [12], Gravey and H´ebuterne [13], Bruneel and Kim [14], Takagi [15], Woodward [16]. The discrete-time *Geo/Geo/*1 queue with negative customers and disasters has been studied in Atencia and Moreno [17]. Multi-server discrete-time queueing systems *Geo/Geo/c* have been reported and Artalejo and Hernandez-Lerma [18]. Discrete-time two heterogeneous servers bulk-service infinite buffer queueing system has been discussed in Goswami and Samanta [19].

In the present paper, we investigate the analytical solution of Queueing System with general conditions Heterogeneous Servers Subject to Catastrophe Intensity. The occurrence of a catastrophe makes the system empty instantly but the system takes its own time to be ready to accept new customers. But in this paper we have considered discrete uniform distribution and modified binomial distribution.

The rest of this paper is organized as follows. In the next section, the description of the queueing model and its analysis for the steady-state probabilities at arbitrary epoch is carried out in Section 2. Numerical results to demonstrate the effect of the catastrophe on the behavior of the customers and on the various performance measures of the system are presented in Section 3. Section 4 concludes the paper.

II. MATHEMATICAL DESCRIPTION OF THE MODEL

The queueing model investigated in this chapter is based on the following assumptions:-

(i) The customers arrive in the system one by one in accordance with a Poisson Process in a single queue with rate > 0. λ

(ii) There are two servers: server 1 and server 2. The service times of the customers are independently identically exponentially distributed with rates and respectively. 1 μ 2 μ

(iii) A customer who arrives, when there are zero customers in the system, joins the server 1with probability p and the server 2 with probability 1- p.

(iv) The queue discipline is first- come- first- served.

(v) Initially, there are zero customers in the system.

(vi) When the system is not empty, catastrophes occur according to a Poisson process with rate ξ and intensity Cr,.It depends upon the intensity of the catastrophe that how many customers are destroyed instantaneously. C_r, (r=1,2,3…N), $\sum_{r=1}^{N} C_r = 1$.

(vii) The capacity of the system is limited to N. i.e., if at any instant there are N customers in the system, then the customers arriving in the duration for which the system remains in state N are not permitted to join the queue and considered lost for the system with probability one.

(viii) In case the catastrophe leaves only one customer in the system then the probability of his being served by server 1 is p and that of server 2 is (1 - p).

Define

$P_n(t)$ = The probability that there are n customers in the system at time t.

$P_{1,0}(t)$ = The probability that there is one customer in the system and he is being served by the server 1.

P(t)= The probability that there is one customer in the system and he is being served by the server 2.

The differential-difference equations governing the system are,

$$P_0^{'}(t)=-\lambda P_0(t)+\mu_1 P_{1,0}(t)+\mu_2 P_{0,1}(t)+\xi\sum_{n=1}^{N}\sum_{r=n}^{N}c_r P_n(t) \qquad (1)$$

$$P_{1,0}^{'}(t)=-(\lambda+\mu_1+\xi)P_{1,0}(t)+\lambda p P_0(t)+\mu_2 P_2(t)+\xi p\sum_{r=1}^{N-1}P_{(1+r)}(t) \qquad (2)$$

$$P_{0,1}^{'}(t)=-(\lambda+\mu_2+\xi)P_{0,1}(t)+\lambda(1-p)P_0(t)+\mu_1 P_2(t)+\xi(1-p)\sum_{r=1}^{N-1}c_r P_{(1+r)}(t) \qquad (3)$$

$$P_2^{'}(t)=-(\lambda+\mu_1+\mu_2+\xi)P_2(t)+\lambda p P_{1,0}(t)+\lambda p P_{0,1}(t)+(\mu_1+\mu_2)P_3(t)+\xi\sum_{r=1}^{N-1}c_r P_{(2+r)}(t) \qquad (4)$$

$$P_n^{'}(t)=-(\lambda+\mu_1+\mu_2+\xi)P_n(t)+\lambda P_{(n-1)}(t)+(\mu_1+\mu_2)P_{(n+1)}(t)+\xi\sum_{r=1}^{N-n}c_r P_{(n+r)}(t), n=3,4,5,..N-1 \qquad (5)$$

$$P_N^{'}(t)=-(\mu_1+\mu_2+\xi)P_N(t)+\lambda P_{(N-1)}(t) \qquad (6)$$

Taking, Laplace Transform of equations (1) to (6) with respect to “t”, We have

$$sP_0^{*}(s)=1-\lambda P_0^{*}(s)+\mu_1 P_{1,0}^{*}(s)+\mu_2 P_{0,1}^{*}(s)+\xi\sum_{n=1}^{N}\sum_{r=n}^{N}c_r P_n^{*}(s) \qquad (7)$$

$$sP_{1,0}^{*}(s)=-(\lambda+\mu_1+\xi)P_{1,0}^{*}(s)+\lambda p P_0^{*}(s)+\mu_2 P_2^{*}(s)+\xi p\sum_{r=1}^{N-1}P_{[1+r]}^{*}(s) \qquad (8)$$

$$sP_{0,1}^{*}(s)=-(\lambda+\mu_1+\xi)P_{0,1}^{*}(s)+\lambda(1-p)P_0^{*}(s)+\mu_1 P_2^{*}(s)+\xi(1-p)\sum_{r=1}^{N-1}P_{[1+r]}^{*}(s) \qquad (9)$$

$$sP_2^*(s)=-(\lambda+\mu_1+\mu_2+\xi)P_2^*(s)+\lambda P_{1,0}^*(s)+\lambda P_{0,1}^*(s)+(\mu_1+\mu_2)P_3^*(s)+\xi\sum_{r=1}^{N-2}c_rP_{[2+r]}^*(s) \quad (10)$$

$$sP_n^*(s)=-(\lambda+\mu_1+\mu_2+\xi)P_n^*(s)+\lambda P_{(n-1)}^*(s)+(\mu_1+\mu_2)P_{(n+1)}^*(s)+\xi\sum_{r=1}^{N-n}c_rP_{[n+r]}^*(s) \quad (11)$$

$$sP_N^*(s)=-(\mu_1+\mu_2+\xi)P_N^*(s)+\lambda P_{(N-1)}^*(s) \quad (12)$$

Where $P_n^*(s)=\int_0^\infty e^{-st}P_n(t)dt$,

And $P_0(0)=1$

Solving equations (11) and (12) recursively, we have

$$P_n^*(s)=\rho^{-N}\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{1_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^n\rho^{L_i+n}D_i\right\}P_N^*(s)$$

$$n=3,4,5,,,N-1 \quad (13)$$

$$\rho=\left(\frac{\lambda}{s+\mu_1+\mu_2+\xi}\right)\ \eta=\prod_{j=1}^{i}\left\{\frac{s+\xi\left(1-\sum_{r_j=1}^{k_j}c_{r_j}\right)}{s+\mu_1+\mu_2+\xi}\right\}^{1_{(i-j)}-1_{(i-(j-1))}}$$

[k] is an integral function.

$$\prod_j^i=1\ and\ \sum_j^i=0\ for\ i<j,k_0=0,$$

$$A_m=\frac{N-n-(i-m)1_0+\sum_{n=1}^{i-m-1}1_a-k_m1_{(i-m)}-\sum_{b=1}^{m-1}(k_{(m-b)}-k_{(m-(b-1))})1_{(i+b-m)}}{1_0-1_{(i-m)}}$$

$$L_m=\sum_{j=1}^{i}(1_{(i-j)}-1_{(i-(j-1))})k_j, 1_j=\begin{cases}0\ if\ \ j=1\\ 1_j\ if\ \ 1\le j<i\end{cases}, D_i=\prod_{j=1}^{i}\begin{pmatrix}N-n-L_i-1_{(i-(j-1))}\\ 1_{(i-j)}-1_{(i-(j-1))}\end{pmatrix}$$

Now the probabilities $P_0^*(s), P_{0,1}^*(s), P_{1,0}^*(s)\ and\ P_2^*(s)$ Remain to be found. For this, We consider the equations (7) –(10) after simplification. We have,

$$P_0^*(s)=\frac{(B_1\rho^{-N}P_N^*(s))+\left(\frac{(s+\xi)}{s}(\lambda Q_1-RR_1R_2)\right)}{G} \quad (14)$$

$$P_{1,0}^*(s)=\frac{(B_2\rho^{-N}P_N^*(s))+\frac{\lambda(s+\xi)}{s}(\lambda U_1-pRR_2)}{G} \quad (15)$$

$$P_{0,1}^*(s)=\frac{(B_3\rho^{-N}P_N^*(s))-\frac{\lambda(s+\xi)}{s}(\lambda U_1-(1-p)RR_1)}{G} \quad (16)$$

$$P_2^*(s)=\frac{(B_4\rho^{-N}P_N^*(s))-\left(\frac{\lambda^2(s+\xi)T_1}{s}\right)}{G} \quad (17)$$

In terms unknown $P^*_N(s)$ and the constraints.

$$B_1=\left\{\begin{array}{l}\left(\lambda\xi(c_1+c_2)Q_1+(\mu_1+\mu_2)Q_2-R_1R_2(R(c_1+c_2)+c_1(\mu_1+\mu_2))\right)P1_3^*(s)-\\ \left(\lambda Q_1-\lambda RR_1R_2\right)\xi\left(\rho^N\sum_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_3^*(s)\right)-\\ \xi\left(Q_2-c_1R_1R_2\right)\left(\rho^NC_{N-2}+\sum_{r=1}^{N-3}C_rP1_{2+r}^*(s)\right)+\\ \xi\begin{pmatrix}\lambda(1-p)S_2(\mu_2-\mu_1)+\lambda pS_1(\mu_1-\mu_2)\\ -R_2pL_1-R_1(1-p)L_2\end{pmatrix}\left(\rho^NC_{N-1}+\sum_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)\end{array}\right\}$$

$$B_2=\left\{\begin{array}{l}\left(\left(-\left(\lambda^2\xi(c_1+c_2)Q_1+\lambda\mu_2(\mu_1+\mu_2)\right)\right)U_1+R_2\begin{pmatrix}\lambda p(R(c_1+c_2)+c_1(\mu_1+\mu_2))\\ -(s+\lambda+\xi)(\mu_1+\mu_2)S_2\end{pmatrix}\right)P1_3^*(s)+\\ \left(\left(\left(-\lambda^2U_1+\lambda pRR_2\right)\xi\left(\rho^N\sum_{r=1}^{N-1}c_r+((c_1+c_2))\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right)\right)+\\ (-\lambda\xi\mu_2U_1+\lambda p\xi c_1R_2-(s+\lambda+\xi)\xi R_2S_2)\left(\rho^NC_{N-2}+\sum_{r=1}^{N-3}c_rP1_{2+r}^*(s)\right)\\ +(\lambda\xi(s+\lambda+\xi)U_1-(s+\lambda+\xi)\xi pRR_2)\left(\rho^NC_{N-1}+\sum_{r=1}^{N-2}c_rP1_{1+r}^*(s)\right)\end{array}\right\}$$

$$B_3=\left\{\begin{array}{l}\left(\left(\left(\lambda^2\xi(c_1+c_2)+\lambda\mu_1(\mu_1+\mu_2)\right)\right)U_1+R_1\begin{pmatrix}\lambda(1-p)(R(c_1+c_2)+c_1(\mu_1+\mu_2))\\ -(s+\lambda+\xi)(\mu_1+\mu_2)S_1\end{pmatrix}\right)P1_3^*(s)+\\ \left(\left(\lambda^2U_1+\lambda(1-p)RR_1\right)\xi\left(\rho^N\sum_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum_{N=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right)\right)+\\ \left(\lambda\xi\mu_1U_1+\lambda(1-p)\xi c_1R_1-(s+\lambda+\xi)\xi R_1S_1\right)\left(\rho^NC_{N-2}+\sum_{r=1}^{N-3}C_rP1_{2+r}^*(s)\right)\\ +(-\lambda\xi(s+\lambda+\xi)U_1-(s+\lambda+\xi)\xi(1-p)RR_1)\left(\rho^NC_{N-1}+\sum_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)\end{array}\right\}$$

$$B_4=\left\{\begin{array}{l}\left(\lambda^2\xi(c_1+c_2)T_1+\lambda\mu_1(\mu_1+\mu_2)T_2-(s+\lambda+\xi)(\mu_1+\mu_2)R_1R_2\right)P1_3^*(s)\\ +\lambda^2T_1\xi\left(\rho^N\sum_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum_{N=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right)+\\ \xi(\lambda T_2-(s+\lambda+\xi)\xi R_1R_2)\left(\rho^NC_{N-2}+\sum_{r=1}^{N-3}C_rP1_{2+r}^*(s)\right)\\ \lambda\xi(s+\lambda+\xi)T_1\left(\rho^NC_{N-2}+\sum_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)\end{array}\right\}$$

$R=(s+\lambda+\mu_1+\mu_2+\xi)\quad R_1=(s+\lambda+\mu_1+\xi), R_2=(s+\lambda+\mu_2+\xi), S_1=(\mu_1+\xi(1-p)c_1),$

$S_2=(\mu_2+\xi pc_1), L_1=(R\mu_1-\lambda\xi c_1), L_2=(R\mu_2-\lambda\xi c_1), T_1=(pR_2+(1-p)R_1),$

$T_2=(\mu_1 pR_2+\mu_2(1-p)R_1), Q_1=(S_2R_2+S_1R_1), Q_2=(\mu_1S_2R_2+\mu_2S_1R_1), U_1=(pS_1-(1-p)S_2),$

Where

$$P1_n^*(s)=\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod_{j=1}^{i-1}\Big(\sum_{1_j=(i-j)}^{1_{j-1}-1}\Big)\prod_{m=0}^{i-1}\Big(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\Big)\eta\rho^{L_1+n}D_i\right\}$$

$$P_N^*(s)=\frac{(G-(s+\xi)(\lambda Q_1-RR_1R_2-(R+\lambda)\lambda T_1))}{s\rho^{-N}\left\{\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{m=0}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}}$$

After using (18) equation (14)-(17) becomes,

$$P_0^*(s)=\frac{\left\{\begin{array}{l}\left\{\begin{array}{l}(\lambda\xi(c_1+c_2)Q_1+(\mu_1+\mu_2)Q_2-R_1R_2(R(c_1+c_2)+c_1(\mu_1+\mu_2)))P1_3^*(s)-\\ (\lambda Q_1-\lambda RR_1R_2)\xi\left(\rho^N\sum\limits_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum\limits_{N=4}^{N-1}+\sum\limits_{r=3}^{N-2}\sum\limits_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right)-\\ \lambda(Q_2-c_1R_1R_2)\left(\rho^NC_{N-2}+\sum\limits_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)+\\ \left(\begin{array}{l}\lambda(1-p)S_2(\mu_2-\mu_1)+\lambda pS_1(\mu_1-\mu_2)\\ -R_2pL_1-R_1(1-p)L_2\end{array}\right)\left(\rho^NC_{N-1}+\sum\limits_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)\end{array}\right\}\\ ((G-(s+\xi)(\lambda Q_1-RR_1R_2-(R+\lambda)\lambda T_1)))+((s+\xi)(\lambda Q_1-RR_1R_2))\\ \left\{\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{j=1}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}\end{array}\right\}}{s\rho^{-N}\left\{\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{j=1}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}}$$

$$P_{1,0}^*(s)=\frac{\begin{array}{l}\left\{\begin{array}{l}\left(\left(-\left(\lambda^2\xi(c_1+c_2)Q_1+\lambda\mu_2(\mu_1+\mu_2)\right)\right)U_1+R_2\left(\begin{array}{l}\lambda p(R(c_1+c_2)+c_1(\mu_1+\mu_2))\\ -(s+\lambda+\xi)(\mu_1+\mu_2)S_2\end{array}\right)\right)P1_3^*(s)+\\ \left(\left(-\lambda^2U_1+\lambda pRR_2\right)\xi\left(\rho^N\sum\limits_{r=1}^{N-1}c_r+\left((c_1+c_2)\right)\sum\limits_{n=4}^{N-1}+\sum\limits_{r=3}^{N-2}\sum\limits_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right)+\\ (-\lambda\xi\mu_2U_1+\lambda p\xi c_1R_2-(s+\lambda+\xi)\xi R_2S_2)\left(\rho^NC_{N-2}+\sum\limits_{r=1}^{N-3}c_rP1_{2+r}^*(s)\right)\\ +(\lambda\xi(s+\lambda+\xi)U_1-(s+\lambda+\xi)\xi pRR_2)\left(\rho^NC_{N-1}+\sum\limits_{r=1}^{N-2}c_rP1_{1+r}^*(s)\right)\end{array}\right\}\\ ((G-(s+\xi)\left(\lambda Q_1+RRR_2-(R+\lambda)\lambda T_1\right)))+\lambda(S+\xi)(\lambda U_1-pRR_2)*\\ \left\{\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{j=1}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}\end{array}}{\left\{\begin{array}{l}sG(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{j=1}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}} \quad (20)$$

$$P_{0,1}^*(s)=\frac{\begin{array}{l}\left\{\begin{array}{l}\left(\left(\lambda^2\xi(c_1+c_2)+\lambda\mu_1(\mu_1+\mu_2)\right)\right)U_1+R_1\left(\begin{array}{l}\lambda(1-p)(R(c_1+c_2)+c_1(\mu_1+\mu_2))\\ -(s+\lambda+\xi)(\mu_1+\mu_2)S_1\end{array}\right)\\ P1_3^*(s)+\left(\left(\lambda^2U_1+\lambda(1-p)RR_1\right)\xi\left(\rho^N\sum\limits_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum\limits_{N=4}^{N-1}+\sum\limits_{r=3}^{N-2}\sum\limits_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right)\right)+\\ (\lambda\xi\mu_1U_1+\lambda(1-p)\xi c_1R_1-(s+\lambda+\xi)\xi R_1S_1)\left(\rho^NC_{N-2}+\sum\limits_{r=1}^{N-3}c_rP1_{2+r}^*(s)\right)\\ +(-\lambda\xi(s+\lambda+\xi)U_1-(s+\lambda+\xi)\xi(1-p)RR_1)\left(\rho^NC_{N-1}+\sum\limits_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)\end{array}\right\}\\ ((G-(s+\xi)\left(\lambda Q_1+RR_1R_2-(R+\lambda)\lambda T_1\right)))-\lambda(S+\xi)(\lambda U_1-(1-p)RR_1)\\ \left\{\begin{array}{l}sG(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{j=1}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}\end{array}}{sG\left\{\begin{array}{l}sG(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n+\sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum\limits_{1_0=i}^{\left[\frac{2(N-n)-i(i-1)}{4}\right]}\prod\limits_{j=1}^{i-1}\left(\sum\limits_{1_j=(i-j)}^{1_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\rho^{L_1+n}D_i\right\}\end{array}\right\}} \quad (21)$$

Similarly we can find $P_2^*(s)$. Hence we can find all the probabilities.

III. PERFORMANCE MEASURE:

There are several system performance measures of the discussed queueing system, such as the expected number of jobs in the system, the expected number of jobs in the queue, the probability that an arriving job is expected to join the queue, the probability that the system has n (n = 1; 2) busy servers, the expected number of busy servers, the mean busy period of the system, etc. We obtain numerically measures of performance of this model by using simulation technique. The simulation analysis of the queueing model under investigation is carried out by using a computer program written in C language. In tables 1-5, the simulation results are obtained by assuming that the catastrophic intensity follows the modified binomial distribution.

Table: 1 catastrophic intensity Effect of change in mean inter catastrophic time ($1/\xi$) Mean inter arrival time = 2 minutes, Mean service time of server 1= 6 minutes, mean service time of server 2 = 8 minutes, simulation length= 480, N=5

Mean inter catastrophic time	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
101	12.2138	3.0654	0.7954	0.7882
102	9.7799	0.2531	0.1107	0.1097
103	11.6128	2.1070	0.5458	0.5415
104	9.3843	1.0284	0.3111	0.3113
105	10.2497	1.3395	0.3909	0.3812
106	9.9494	0.8560	0.2587	0.2556
107	10.4430	1.0812	0.3006	0.2922
108	9.8635	1.5633	0.4620	0.4479
109	10.1851	1.0888	0.3306	0.3355
110	12.1654	1.5348	0.4485	0.4424

Table: 2 Binomially distributed catastrophic intensity, Effect of change in mean inter arrival time ($1/\lambda$), Mean service time of server 1 = 6 minutes, Mean service time of server 2 = 8 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean inter arrival time	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
1	11.8935	4.2028	0.9806	0.9806
2	9.0571	0.1052	0.0499	0.0505
3	6.8118	0.8650	0.3777	0.3920
4	8.7793	0.4342	0.1777	0.1792
5	4.0348	0.3452	0.2630	0.2693
6	3.1294	0.0000	0.0051	0.0089
7	2.3051	0.1043	0.1419	0.1451
8	2.2160	0.0800	0.0720	0.0745
9	2.2716	0.0629	0.2946	0.2335
10	4.4700	0.0922	0.0890	0.0896

Table: 3 Effect of change in mean service time of server 1 ($1/\mu_1$), Mean inter arrival time = 2 minutes, Mean service time of server 2 = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean service time of server 1	Average delay in queue	Average No. in queue	Server 1 tilization	Server 2 tilization
1	0.5793	0.1374	0.2253	0.3426
2	3.1418	0.9103	0.4159	0.4500
3	4.7255	1.1797	0.5258	0.5459
4	4.4515	0.0916	0.0847	0.0970
5	7.3563	0.8923	0.3362	0.3395
6	8.9504	1.7259	0.5161	0.5192
7	8.6378	0.7640	0.2275	0.2256
8	8.9829	1.2061	0.3489	0.3417
9	11.8682	3.4734	0.9425	0.9284
10	10.8180	1.8999	0.5136	0.5116

Table: 4 Effect of change in mean service time of server 2 ($1/\mu_2$), Mean inter arrival time = 2 minutes, Mean service time of server 1 = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5.

Mean service time of server 2	Average delay in queue	Average No. in queue	Server 1 tilization	Server 2 tilization
1	0.6727	0.0640	0.0599	0.0438
2	1.5360	0.5795	0.6849	0.5215
3	4.0723	0.2473	0.1597	0.1395
4	5.2788	0.5434	0.2217	0.2210
5	7.3563	0.8923	0.3362	0.3395
6	7.6520	1.0865	0.3643	0.3662
7	8.1255	1.3601	0.4388	0.4436
8	12.1733	3.6002	0.9382	0.9542
9	10.1551	3.1477	0.9384	0.9256
10	8.6107	0.8195	0.2621	0.2627

Table: 5catastrophic intensity Effect of change in mean inter catastrophic time ($1/\xi$) Mean inter arrival time = 2 minutes, Mean service time of server 1= 6 minutes, mean service time of server 2 = 8 minutes, simulation length= 480, N=5

Mean inter catastrophi c time	Averag e delay in queue	Averag e No. in queue	Server 1 tilizatio n	Server 2 Utilizatio n
101	12.8427	3.6938	0.9433	0.9434
102	9.7839	0.1349	0.0783	0.0781
103	12.0068	2.3648	0.6054	0.6008
104	9.6638	1.0964	0.3287	0.3294
105	10.5273	1.5314	0.4582	0.4469
106	10.3545	1.1666	0.3428	0.3408
107	10.9170	1.1578	0.3221	0.3248
108	10.1586	1.9845	0.5836	0.5662
109	10.5369	0.9082	0.2831	0.2846
110	12.1190	1.1724	0.3512	0.3458

Conclusion:

In this paper we obtained the solution of Queueing System with general conditions Heterogeneous Servers Subject to Catastrophe Intensity. Explicit expressions for the steady-state probabilities are derived. Some performance measures of effectiveness of the system is calculated. The system size probability is calculated.

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