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Modelling Of The Movement Of Object In The Flat Dynamic Environment With Use Of Global Algorithm On The Basis Of The Sufficient Condition Of Lack Of Collisions

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Abstract
The proof of the statement about crossing by moving object of a dimensional circle of the moving obstacle if at the time of an object trajectory circle contact it is in a contact point is provided in article. Radius of a buffer zone round an obstacle which contact object won't lead to its collision with an obstacle is calculated. The found sufficient condition of lack of collision is applied when modeling the movement of the operated object to the set purpose in the plane containing mobile "prohibited zones". The algorithm of calculation of an object trajectory, optimum on time, between initial and final points excluding its hit in "prohibited zones" is constructed. The program allowing to make machine experiments both for stationary and for a non-stationary working environment is developed.

Keywords: Modeling of the movement of object, program, trajectory, obstacle, dimensional circle, lack of crossing, sufficient conditions, dynamic environment.

1. INTRODUCTION
Now possibilities of the computer equipment on carrying out numerical modeling of real processes allow to extend the sphere of use of mobile robots to many sectors of human activity [1-3]. Any practical use of the similar mobile automated devices inevitably faces a problem of their proper response to change of environment [4-6]. Thus, development of the algorithms providing the free movement of objects to the set purpose in the working environment containing dynamic hindrances to the movement is represented actual.
In work [7] the mathematical behavior model of mobile object for a problem of finding of algorithm of management of the flat movement of the robot, optimum on time modelled by a material point in a working environment with the motionless and moving obstacles which are set by circles is constructed. Actually the circle is under construction so that to cover all points of a projection of an obstacle to the plane. Border of such circle is the dimensional circles[8].
It is considered that obstacles (and, therefore, the centers of dimensional circles) can rectilinearly move with a constant speed. Initial position of the center, radius and a vector of speed of each obstacle are considered as the known. In the course of the movement the object shouldn't cross dimensional circles which limit the "forbidden" areas occupied by obstacles at the moment of time.

The purpose of this work is clarification of conditions of lack of crossing of the set circles which laws of the movement are known, object which needs to be moved from the initial point in final for the minimum time, development of algorithm of calculation of an optimum trajectory and creation on its basis of the program of modeling of mutual movement of object (the mobile robot) and obstacles.

2. TECHNIQUE OF THE ANALYSIS OF CONDITIONS OF CROSSING
We will consider the flat rectilinear movement of a material point (we will designate it a letter A). We will consider that the point Amoves along a coordinate axis OX to a positive side. Let the obstacle with a dimensional circle of radius r move rectilinearly and evenly:

$$\left.\begin{aligned} x_c &= x_{c0} + V_x \cdot t \\ y_c &= y_{c0} + V_y \cdot t \end{aligned}\right\}.$$

Here x_c, y_c – coordinates of the center of a circle; t – time; x_{c0}, y_{c0} – coordinates of the center of a circle point in time $t = 0$; V_x, V_y – components of a vector of speed of the center of a circle along axes of OX, OY. Without restriction of a community it is possible to accept $V_y > 0$.
Let the circle cross a straight line OH after a start of motion. We will designate $t_{con\,1}$, $t_{con\,2}$ – the contact moments a straight line circle OX in points $x_{con1}(t_{con1})$, $x_{con2}(t_{con2})$. We will consider that $t_{con\,1} < t_{con\,2}$. It is obvious that

$$y_c(t_{con1}) = -r, \ \ y_c(t_{con2}) = r$$

Weset $t = t_{con1} + \tau, \ \ \tau > 0$. Then

$$\left.\begin{aligned} x_c &= x_{con1} + V_x \cdot \tau \\ y_c &= -r + V_y \cdot \tau \end{aligned}\right\}$$

The circle point of intersection with an axis OX having a larger coordinate in a time t is set by equality:

$x_2 = x_c + \sqrt{r^2 - y_c^2} = x_{con1} + V_x \cdot \tau + \sqrt{2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2}$. (1)

Point with less coordinate defined by the formula:

$x_1 = x_c - \sqrt{r^2 - y_c^2} = x_{con1} + V_x \cdot \tau - \sqrt{2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2}$. (1')

We will consider coordinate x_A of a point A in the small vicinity of a point x_{con1}. The law of its movement can be presented in the form of the following decomposition:

$$x_A(\tau) = x_{con1} + v \cdot \tau + a \cdot \frac{\tau^2}{2} + o(\tau^2), \quad (2)$$

where v – speed, a – acceleration of a point A in a point x_{con1}, and $v > 0$.

We will show that there are such values [tau], at which $x_1(\tau) \le x_A(\tau) \le x_2(\tau)$. It will also mean that the point A will cross a circle. At first we will consider right of the specified inequalities $x_A(\tau) \le x_2(\tau)$. Having substituted in it (1) and (2), we will receive:

$$x_{con1} + v \cdot \tau + a \cdot \frac{\tau^2}{2} + o(\tau^2) \le x_{con1} + V_x \cdot \tau + \sqrt{2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2}. \quad (3)$$

We will consider the worst conditions of performance of an inequality (3), when $v > V_x,\ a > 0$. We will copy (3) in a look:

$$(v - V_x)\tau + a \cdot \frac{\tau^2}{2} + o(\tau^2) \le \sqrt{2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2}. \quad (4)$$

Under the specified conditions (4) it is equivalent to an inequality

$$(v - V_x)^2 \cdot \tau^2 + (v - V_x)a \cdot \tau^3 + o(\tau^3) \le 2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2. \quad (5)$$

As $\tau > 0$, we will reduce (5) by [tau], we will receive:

$$V_y^2 \cdot \tau + (v - V_x)^2 \cdot \tau + (v - V_x)a \cdot \tau^2 + o(\tau^2) \le 2V_y \cdot r = const. \quad (6)$$

The left part of an inequality (6) aspires to zero at $\tau \to 0$. Thus, it is possible to find such rather small [tau] that the left part will be less than any set positive number, in particular $2V_y r$.

Having considered an inequality $x_1(\tau) \le x_A(\tau)$, at similar reasonings we will receive:

$$x_{con1} + V_x \cdot \tau - \sqrt{2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2} \le x_{con1} + v \cdot \tau + a \cdot \frac{\tau^2}{2} + o(\tau^2), \quad (3')$$

$$(v - V_x)\tau + a \cdot \frac{\tau^2}{2} + o(\tau^2) \ge -\sqrt{2V_y \cdot r \cdot \tau - V_y^2 \cdot \tau^2}. \quad (4')$$

Under conditions $v > V_x,\ a > 0$ the formula (4') is carried out automatically. Therefore $x_1(\tau) \le x_A(\tau) \le x_2(\tau)$, and the point A will surely cross a circle.

Similarly the case, when is considered

$y_c(t_{con2}) = r,\ t = t_{con2} - \tau,\ \tau > 0$.

The statement proved above can be formulated in the form of the following lemma.

Lemma (about crossing). Let the point A move along some straight line on the plane, its coordinates are infinitely differentiable functions of time, and let this straight line be crossed by the circle which is evenly moving in the same plane. If at the time of a contact a circle of this straight line the point A is in a contact point, the trajectory of a point A crosses a circle.

The consequence given below is more suitable for practical use.

Consequence. Hit of object (robot) in a contact point an obstacle of a straight section of its trajectory at the time of a contact is inadmissible.

3. MAIN PART

According to a lemma, when finding object at the time of a contact its subsequent trajectory will pass in a contact point in the prohibited zone occupied by an obstacle as will cross its dimensional circle for some time t_f. We will find height of a segment of this zone [delta] in which there will be an object.

We will consider two systems of coordinates. Let the initial point O systems of coordinates of OXY coincide with a point of the first contact a dimensional circle of a trajectory of object (a point A) which, as before, moves along a coordinate axis OX to a positive side. The beginning O' systems of coordinates of O'X'Y' coincides with the center of a moving circle, and couples of axes OX, *O'X'* and*OY*, *O'Y' is* the same direction. We will consider that the point A at the time of a contact is in a point O. We will designate point coordinates in system of coordinates of OXY (x, y), in system of coordinates of O'X'Y' – (x', y'). If component of a vector of speed of the center of a circle along an axis OX (V_x) it is equal to zero,then $x' = x,\ y' = y - y_0$, where $(0,\ y_0)$ – coordinates of the center of a dimensional circle.

We will consider a special case of uniform motion of a point A with a speed v_x. Thus its movement in systems of coordinates of OXY, O'X'Y' is described by the following equations:

$$x(t) = x'(t) = v_x t,\quad y(t) = 0,\quad y'(t) = -y_0(t),\quad y_0(t) = -r + V_y t,\quad V_y > 0. \quad (7)$$

Here V_y – a component of a vector of speed of the center of a circle along OY axis, and counting of time is conducted with the moment of the first contact by a dimensional circle of a trajectory of the robot. If $V_x \ne 0$, under v_x it is necessary to understand the speed of a point A concerning the system of coordinates connected with a circle.

Coordinates of a point A at the time of crossing of a dimensional circle have to satisfy to the equation

$$(x'(t))^2 + (y'(t))^2 = r^2. \quad (8)$$

Having substituted (7) in (8), it is possible to find time $t = t_f$ during which the point A will be in area, a limited dimensional circle:

$$t_f = \frac{2rV_y}{(V_x)^2 + (V_y)^2}. \quad (9)$$

During t_f obstacle will pass distance

$$\Delta = V_y \cdot t_f = \frac{2r(V_y)^2}{(V_x)^2 + (V_y)^2} = \frac{2r}{1 + \frac{(V_x)^2}{(V_y)^2}}. \quad (10)$$

Thus, for a complete elimination of collision of object with an obstacle when modeling it is necessary to consider not the real radius r of a dimensional circle of an obstacle, and the "buffer" radius $r_{buf} = r + \Delta$,which corresponds to the circle covering "a buffer zone" of an obstacle, dangerous from the point of view of collision. If to enter coefficient $k = \frac{v_x}{V_y}$, then

$$r_{buf} = \frac{3 + k^2}{1 + k^2} \cdot r. \quad (11)$$

In case of the accelerated movement corresponding to a formula (2), all reasonings concerning finding of "buffer" radius remain, but for finding of time t_f it is necessary to solve the cubic equation following from a ratio (8). In this case it is possible to understand the average speed v_x of a point as speed A in time t_f.

The received results were used when developing algorithm of the computer program allowing to display on the display screen the movement of the robot in the working environment containing obstacles. The program allows carrying out machine experiments both for stationary, and for a non-stationary working environment. Thus in the program opportunity to vary location of initial and final points of the robot, and also number, situation and speed of obstacles is given. The program provides representation of the studied process in dynamics, that is allows to observe directly the movement of obstacles and the robot taking into account accelerations and braking of the last.

At the beginning of modeling the trajectory in the set working environment pays off, and then the movement of the robot and if it is necessary, obstacles, coming to an end at hit of the robot in a final point begins. Calculation of a trajectory begins with the analysis like a working environment (obstacles are absent, the environment is stationary, the environment is not stationary). In the absence of obstacles in working space the movement of the robot is carried out on the straight line connecting initial and final points taking into account its dispersal and braking.

If set to the stationary obstacle blocking to the robot rectilinear access to a final point is set, at first the direction of its round is defined by the robot. Thus the optimum trajectory of the robot will consist of three parts two of which lie on the tangents to a circle passing through initial and final points the third passes on the corresponding arch of a circle. At calculations was accepted that the movement of the robot on an arch happens to constant angular speed.The moments of switching of the managing director of influence are calculated hit in a final point took place for the minimum time, and the robot thus could keep on an arch of a circle and not slide off it under the influence of centrifugal force. If the motionless obstacle doesn't stop the straight line connecting initial and final points of the robot, calculation of a trajectory happens the same as in the absence of obstacles in a working environment.

In case of a non-stationary working environment, that is in the presence of moving obstacles, the directions and speeds of their movement are analyzed. If necessary it is timed delays of a start of motion of the robot (in case in the course of the movement without delay the robot has to get to the space area limited to an obstacle) for each of potentially dangerous obstacles. Time of a start of motion of the robot allowing to avoid collisions with all obstacles which are available in a working environment is defined with it.

It should be noted that the algorithm applied in the program is among global in which full calculation of a trajectory between initial and final points of the robot prior to its movement [9] is carried out. As the working environment is necessary determined, that is the robot possesses full information on geometry and dynamics of environment, on an arrangement, a form and the law of the movement of each obstacle, at first in the program there is a planning of a trajectory [10], and modeling of the movement of object on it is carried out then.

4. CONCLUSION

Proven provisionscan be used at creation of algorithms of modeling of the flat operated movement of objects (for example, robots) in a various environment in the presence of the obstacles creating hindrances to movement. The working environment of the mobile robot can be both stationary, and dynamic in which location of obstacles changes [11] eventually. Besides, in the environment there can be no reliable information about quantity, an arrangement, a form and laws of the movement of obstacles, and in the course of modeling new obstacles with unknown characteristics can be added.

As the mobile robot isn't a material point, from the practical point of view in a lemma and the subsequent calculations it is necessary to consider a contact and crossing of dimensional circles of the robot and an obstacle. This remark needs to be considered at further improvements of algorithm.

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