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Synthesis And Analysis Of Algorithm For Tracking Of Pilot Component Phase Of L1oc Glonass Signal Processed On The Subcarrier Frequencies

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Abstract
The paper is devoted to the problem on synthesis of the optimal pilot component phase tracking algorithm of L1OC advanced signal of the GLONASS satellite navigation system with processing on the subcarrier frequencies. The authors present a brief description of the advanced open access signal with L1OC GLONASS code division, which includes the pilot and information components. The pilot component of the signal has a BOC(1,1) modulation (binary offset carrier), while the information component has a BPSK(1) modulation (binary phase shift keying). The authors consider the mathematical formulation of the problem on synthesis of an algorithm for tracking the phase of the signal, which extracts the information only from the L1OC pilot signal component. The authors present the equations describing the evolution of the posteriori probability density of the estimated state vector, showing the phase change of the signal in the space of states, as well as the equations of quasi-optimal filtering of the given state vector that is based on a Gaussian approximation of the posterior probability density. A representation of the equations of quasi-optimal filtering is presented in the form of a tracking system, comprising of a phase discriminator, which smoothes filter, and a reference signal generator. Paper presents a block diagram of the synthesized phase discriminator. It is shown that the phase discriminator with the processing of the L1OC pilot signal component on subcarrier frequencies has a structure, different from that of the classical phase discriminator processed on the carrier frequency. The calculation of the statistical characteristics of the correlators, used in the subcarrier frequencies processing algorithms, is carried out. The authors obtained the expressions for discriminatory and fluctuating characteristics of the synthesized phase discriminator and presented the results from the analysis of concerned characteristics, which suggest that discriminatory characteristic has a point of stable equilibrium. The results of the comparative analysis between the characteristics of the synthesized phase discriminator and the similar characteristics of the classic phase discriminator are presented; the latter relates to the phase tracking system over the $L1OC$ pilot signal component on the carrier frequency. The authors describe averaging technique for the dispersion equations of the synthesized quasi-optimal filtering algorithm, and present corresponding averaged dispersion equations, which do not depend on the random input signals and assessments, and allow one to build a system for tracking the phase of a signal with constant coefficients (stationary tracking system). The authors present computer simulation results of the synthesized tracking system. Various models of the signal phase change are considered, including those in a form of a random process and a highly dynamic quasi deterministic process involving intense spurts over the line of sighting. The authors also consider the options for simulating a quasi-optimal tracking system for a phase of a signal with variable and constant parameters.

Keywords
Satellite navigation, tracking system, modulation, subcarrier frequencies, statistical characteristics, accuracy, optimal filtering, phase sampler.

1. Introduction

Satellite radio navigation systems (SRNS) are increasingly used in various fields of human activity: in everyday life, in the ground, rail, air and marine transport, in construction, geodesy and cartography, etc. This fact is responsible for their rapid development and advancement. One of the ways for such an improvement is the use of more advanced radiofrequency signals, securing enhancement of the accuracy of navigational sighting. The SRNS GPS [1, 2], Galileo [3], BeiDou [4] and GLONASS [5] are operated using the subcarrier frequency modulated signals. Such modulation is called the BOC (Binary Offset Carrier) modulation. In SRNS GLONASS new open access signals with code division within the frequency ranges $L1$, $L2$ are two-component signals including the pilot and information components, whose combination into a single navigation signal is carried out by the bit time-division multiplexing method [6]. The information component of a given signal has a standard binary phase modulation (BPSK) with a repetition frequency of the ranging code characters of 1.023 MHz, while the pilot component is modulated on the subcarrier frequencies BOC(1,1) by base frequency of 1.023 MHz. Tracking of the BOC-modulated signals has a number of features related to the fact that the relevant correlation function has the envelope of several extreme points. Therefore, the development of algorithms for receiving and processing of BOC-modulated signals becomes an urgent problem. This research area is studied in a large number of papers. Thus, in [7] the problem of BOC-modulated signal detection is considered and the algorithm is described, in which the BOC-modulated signal is regarded as the sum of the two BPSK-modulated signals on the subcarrier frequencies, located symmetrically relative to the carrier frequency. Paper [8] presents the results of concerned approach to detect BOC(10,5) and BOC (14,2) modulated signals, which showed their high efficiency. In [9-

10] the idea to present BOC-modulated signal as two BPSK-modulated signals on the subcarrier frequencies is used for the tasks related to tracking of signal's delay and phase. However, the proposed processing algorithms are heuristic, and are not resulted from rigorous synthesis. Works [11-18] consider various algorithm options for tracking the delay of BOC-modulated signal though do not consider algorithms for tracking phase of a signal. Faithful development of an optimal algorithm for tracking the BOC-modulated signal in the coherent mode, processed on the carrier frequency with the use of an additional variable method is considered in [19]. Paper [20] presents the developed quasi-optimal algorithm for tracking the delay of a signal processed on subcarrier frequencies under incoherent signal processing at the receiver. A synthesis and analysis of the coherent tracking algorithm for the delay of the pilot component of L1OC GLONASS signal, when processing on subcarrier frequencies, is presented in [21]. This algorithm uses a reference signal correlators in the upper and lower subcarrier frequencies. The use of such correlators in standard signal phase tracking algorithms [5, 6, 22, 23] is impossible. It is therefore necessary to use other algorithms for phase tracking of a signal processed on subcarrier frequencies. This paper presents the synthesis and analysis of the quasi-optimal algorithm for tracking of the pilot component phase of L1OC GLONASS signal processed on subcarrier frequencies.

2. Synthesis problem statement

To synthesize the optimal (quasi-optimal) tracking system, we will use the theory of optimal filtering [24], which involves statistical description of processes and observations. We assume that the receiver input consists of additive mixture of the navigation signal $s(t)$ and the internal noise of the receiver $n(t)$, which is believed to be white Gaussian noise with one-sided power spectral density N_0.

Under the navigation signal consider L1OC GLONASS signal with code division, which is a two-component signal including the pilot and information components [5, 6]. The pilot component $s_p(t)$ has a BOC(1,1) modulation with a base frequency f_b=1.023 MHz and is designed to measure the delay, phase, and Doppler frequency shift. Information component $s_d(t)$ has a BPSK(1) modulation and carries digital information (navigation message). Combining the two components into a single signal is carried out based on the bit time-division multiplexing.

We assume that the receiver samples the input process over time, so that the processing system receives the discrete time implementation $t_{k,i}$ (see Fig. 1).

$$y_{k,i} = s(t_{k,i}, \varphi_{k,i}) + n_{k,i}, \qquad (1)$$

where $t_{k,i} = kT + iT_d$; $T = NT_d$ is the discrete processing step in the tracking system contour; T_d - is the sampling period in ADC; $\varphi_{k,i}$ - is the phase of the signal to be evaluated; $n_{k,i}$ - is the discrete white Gaussian noise with variance $\sigma_n^2 = \frac{N_0}{2T_d}$, where N_0 - is the one-sided power spectral density of the internal receiver noise.

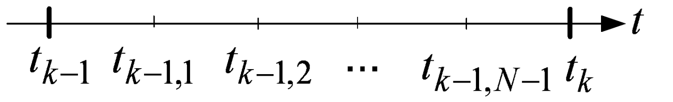

Figure 1. Time indexing scheme.

In this paper we use a dual time indexing necessary to correctly take into account the sample inputs $y_{k,i}$ processing in the correlator and a rarer data handling (with time step T) in the smoothing network, when synthesizing tracking system. We will transform filtering process evaluations φ_k into sampling instants t_k, $k = 1, 2, ...$, such that $t_k - t_{k-1} = NT_d = T$, $t_{k-1,N} = t_k$, $t_{k-1,0} = t_{k-1}$.

The paper deals with synthesis of the tracking system, which is designed to retrieve information only from the pilot component of L1OC signal. To carry out such a synthesis, L1OC signal $s(t_{k,i}, \varphi_{k,i})$ in (1) can be written as

$$s(t_{k,i}, \varphi_{k,i}) = s_p(t_{k,i}, \varphi_{k,i}) \otimes s_d(t_{k,i}, \varphi_{k,i}), \qquad (2)$$

where the symbol $\otimes$ denotes the operation of the bit time-division multiplexing;

$$s_p(t_{k,i}, \varphi_{k,i}) = Ah_{c,p}(t_{k,i} - \tau_k)\, h_{ds}(t_{k,i} - \tau_k) \cos(\omega_0 t_{k,i} + \varphi_{k,i})$$

— the pilot component of the signal

$$s_d(t_{k,i}, \tau_k) = Ah_{c,d}(t_{k,i} - \tau_k)\, h_{DI}(t_{k,i} - \tau_k) \cos(\omega_0 t_{k,i} + \varphi_{k,i})$$

— the information component of the signal

A - is the signal amplitude; τ_k - is the signal delay; $h_{c,p}(t_{k,i})$, $h_{c,d}(t_{k,i})$ - are the modulation functions by ranging codes of the pilot and information components; $h_{ds}(t_{k,i}) = \mathrm{sign}(\sin(2\pi f_s))$ - is the modulation function by digital sinusoid, where f_s - is the sinusoid frequency; $h_{DI}(t_{k,i})$ - is the modulation function by digital information (navigation message).

Note that the information about signal phase $\varphi_{k,i}$ is contained in both the pilot and the information signal components $s(t_{k,i}, \varphi_{k,i})$, since both components are formed coherently on board of the navigation satellite. Therefore we can formulate and solve the problem on synthesis of the tracking system with the processing of the two mentioned components of the signal. However, the subject of this paper is the synthesis of the tracking system, in which information on the phase of the signal is extracted only from the pilot component $s_p(t_{k,i}, \varphi_{k,i})$, because it is this component of the navigation signal which is primarily intended for navigation measurements.

To perform the synthesis of optimal phase filtering algorithm over the pilot component, let represent the time-multiplexed signal (2) as the sum of two signals:

$$s\left(t_{k,i},\varphi_{k,i}\right)=\tilde{s}_p\left(t_{k,i},\lambda_k\right)+\tilde{s}_d\left(t_{k,i}\right), \quad (3)$$

$$\tilde{s}_p\left(t_{k,i},\varphi_{k,i}\right)=A\tilde{h}_{c,p}\left(t_{k,i}-\tau_k\right)h_{ds}\left(t_{k,i}-\tau_k\right)\cos\left(\omega_0 t_{k,i}+\varphi_{k,i}\right), \quad (4)$$

$$\tilde{s}_d\left(t_{k,i}\right)=A\tilde{h}_{c,d}\left(t_{k,i}-\tau_k\right)h_{DI}\left(t_{k,i}-\tau_k\right)\cos\left(\omega_0 t_{k,i}+\varphi_{k,i}\right).$$

where $\tilde{h}_{c,d}\left(t_{k,i}\right)$ - is the sequence obtained from the $h_{c,d}\left(t_{k,i}\right)$ sequence by representing each character in the form of two semicharacters. The value of the first semicharacter is equal to the value of the corresponding character $h_{c,d}\left(t_{k,i}\right)$, while the value of the second semicharacter is set to zero; $\tilde{h}_{c,p}\left(t_{k,i}\right)$ - is the sequence obtained from the $h_{c,p}\left(t_{k,i}\right)$ sequence by presenting each ranging code character in the form of two semicharacters. The value of the first semicharacter is assumed to be zero, whereas the value of the second one is equal to the corresponding character $h_{c,p}\left(t_{k,i}\right)$.

When writing (3), we formally assume that the information component does not depend on the estimated parameter φ_k.

Assuming that the estimated parameter is a phase of the signal (3), observations (1) can be written as

$$y_{k,i}=\tilde{s}_p\left(t_{k,i},\varphi_{k,i}\right)+\tilde{s}_d\left(t_{k,i}\right)+n_{k,i}.$$

Let specify the model of a signal $\tilde{s}_p\left(t_{k,i},\varphi_{k,i}\right)$ for the problem of synthesis of optimal signal phase tracking system. First, we assume that a signal delay τ is known. Complete signal phase $\omega_0 t_{k-1,i}+\varphi_{k-1,i}$ can be represented as

$$\omega_0 t_{k-1,i}+\varphi_{k-1,i}=\omega_0 t_{k-1,i}+\varphi_{k-1}+\left(i-1\right)T_d\omega_{D,k-1},$$

where $\omega_{D,k-1}$ - is the Doppler frequency shift, i.e. we assume that within the range $\left[t_{k-1,1},t_k\right]$ the phase varies linearly

$$\varphi_{k-1,i}=\varphi_{k-1}+\left(i-1\right)T_d\omega_{D,k-1},$$

while during the transition from instant $t_{k,N}=t_{k+1,0}$ to instant $t_{k+1,1}$ the value of the state vector $\mathbf{x}_k=\left|\varphi_k \quad \omega_{D,k} \quad \nu_k\right|^{\mathrm{T}}$ changes according to the equations

$$\varphi_k=\varphi_{k-1}+T\omega_{D,k-1},\ \omega_{D,k}=\omega_{D,k-1}+T\nu_{k-1},$$
$$\nu_k=\nu_{k-1}+\xi_{k-1}, \quad (5)$$

where ξ_{k-1} - is the discrete white Gaussian noise with zero mathematical expectation and variance D_ξ.

Equations (5) can be written in vector form:

$$\mathbf{x}_k=\mathbf{F}\mathbf{x}_{k-1}+\mathbf{G}\boldsymbol{\xi}_{k-1},\ \varphi_k=\mathbf{c}\mathbf{x}_k,$$

where $\mathbf{c}=\left|1 \quad 0 \quad 0\right|$, $\mathbf{F}=\begin{vmatrix}1 & T & 0\\ 0 & 1 & T\\ 0 & 0 & 1\end{vmatrix}$, $\mathbf{G}=\begin{vmatrix}0\\0\\1\end{vmatrix}$.

3. Synthesis of the quasi-optimal phase filtering algorithm by the pilot component of L1OC signal

For the synthesis of the quasioptimal phase filtering algorithm we will use the theory of optimal filtering [24]. In accordance with this theory, it is necessary to find the posterior probability density $p\left(\mathbf{x}_k\middle|Y_0^k\right)$, where Y_0^k - is the set of observations (3) from the initial instant $t_0=0$ to the current instant t_k. Most applications use a Gaussian approximation of the posterior probability density $p\left(\mathbf{x}_k\middle|Y_0^k\right)$, whose parameters are mathematical expectation (process evaluation)

$$\hat{\mathbf{x}}_k=\int_{\Omega_x}\mathbf{x}_k p\left(\mathbf{x}_k\middle|Y_0^k\right)d\mathbf{x}_k,$$

and variance matrix (variance matrix of filtering errors)

$$\mathbf{D}_{\mathbf{x},k}=\int_{\Omega_x}\left(\mathbf{x}_k-\hat{\mathbf{x}}_k\right)\left(\mathbf{x}_k-\hat{\mathbf{x}}_k\right)^T p\left(\mathbf{x}_k\middle|Y_0^k\right)d\mathbf{x}_k,$$

where "T" - denotes the conjugation sign.

Posteriori probability density $p\left(\mathbf{x}_k\middle|Y_0^k\right)$ is described by the recurrence equations [24]:

$$p\left(\mathbf{x}_k\middle|Y_0^k\right)=cp\left(\mathbf{x}_k\middle|Y_0^{k-1}\right)p\left(Y_{k-1,1}^k\middle|\mathbf{x}_k\right),$$

$$p\left(\mathbf{x}_k\middle|Y_0^{k-1}\right)=\int_{-\infty}^{\infty}p\left(\mathbf{x}_{k-1}\middle|Y_0^{k-1}\right)p\left(\mathbf{x}_k\middle|\mathbf{x}_{k-1}\right)d\mathbf{x}_{k-1},$$

where $Y_{k-1,1}^k$ - is the set of observations $y_{k-1,1},y_{k-1,2},...,y_{k-1,N}$ within the interval of $\left[t_{k-1,1},t_k\right]$ with duration T; $p\left(\mathbf{x}_k\middle|\mathbf{x}_{k-1}\right)$ - is the probability density of a random transition of the process $\mathbf{x}$ from the state $\mathbf{x}_{k-1}$ to the state $\mathbf{x}_k$.

In the concerned problem, within the time interval $\left[t_{k-1,1},t_k\right]$ observations $Y_{k-1,1}^k$ depend on the value of the estimated state vector $\mathbf{x}_{k-1}=\left|\varphi_{k-1} \quad \omega_{D,k-1} \quad \nu_{k-1}\right|^{\mathrm{T}}$, corresponding to the instant t_{k-1}. Therefore, if we are interested in assessing the current state of the vector $\mathbf{x}_{k-1}$ according to the observations Y_0^k, it is necessary to consider the posteriori probability density $p\left(\mathbf{x}_{k-1}\middle|Y_0^k\right)$, for which we can write the equations similar to those noted above:

$$p\left(\mathbf{x}_{k-1}\middle|Y_0^k\right)=cp\left(\mathbf{x}_{k-1}\middle|Y_0^{k-1}\right)p\left(Y_{k-1,1}^k\middle|\mathbf{x}_{k-1}\right),$$

$$p\left(\mathbf{x}_{k-1}\middle|Y_0^{k-1}\right)=\int_{-\infty}^{\infty}p\left(\mathbf{x}_{k-2}\middle|Y_0^{k-1}\right)p\left(\mathbf{x}_{k-1}\middle|\mathbf{x}_{k-2}\right)d\mathbf{x}_{k-2}.$$

Thus it follows that by processing all of the observations Y_0^k available up to the current point of observation Y_0^k at an instant t_k we will form a state vector estimate $\hat{\mathbf{x}}_{k-1}$, corresponding to the state vector $\mathbf{x}_{k-1}$ at an instant t_{k-1}.

The equations describing change in the estimate $\hat{\mathbf{x}}_{k-1}$ and filtering errors variance matrix $\mathbf{D}_{\mathbf{x},k-1}$ are called optimal filtering equations [5, 23, 24]. When using a Gaussian approximation of the posterior probability density, filtering algorithm should be called more accurately as quasi-optimal.

For the concerned problem statement the quasi-optimal filtering algorithm of vector $\mathbf{x}_{k-1}$ is given by the following equations [24]:

$$\hat{\mathbf{x}}_{k-1} = \tilde{\mathbf{x}}_{k-1} + \mathbf{D}_{\mathbf{x},k-1}\left(\frac{\partial \tilde{F}_k(\tilde{\mathbf{x}}_{k-1})}{\partial \mathbf{x}}\right)^T, \; \tilde{\mathbf{x}}_{k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-2}, \quad (6)$$

$$\tilde{\mathbf{D}}_{\mathbf{x},k-1} = \mathbf{F}\mathbf{D}_{\mathbf{x},k-2}\mathbf{F}^T + \mathbf{G}D_{\xi}\mathbf{G}^T,$$

$$\mathbf{D}^{-1}_{\mathbf{x},k-1} = \tilde{\mathbf{D}}^{-1}_{\mathbf{x},k-1} - \frac{\partial}{\partial \mathbf{x}}\left(\frac{\partial \tilde{F}_k(\tilde{\mathbf{x}}_{k-1})}{\partial \mathbf{x}}\right)^T, \quad (7)$$

where $\hat{\mathbf{x}}_{k-1}$ - is the assessment of process to be filtered; $\tilde{\mathbf{x}}_{k-1}$ - is the extrapolated assessment of the process; $\mathbf{D}_{\mathbf{x},k-1}$ - is the filtering errors variance matrix; $\tilde{\mathbf{D}}_{\mathbf{x},k-1}$ - is the extrapolation errors variance matrix;

$$\tilde{F}_k(\mathbf{x}_{k-1}) = \frac{1}{\sigma_n^2}\sum_{i=1}^{N} y_{k-1,i}\left(\tilde{s}_p(t_{k-1,i},\mathbf{x}_{k-1}) + \tilde{s}_d(t_{k-1,i})\right). \quad (8)$$

Since $\tilde{s}_d(t_{k-1,i})$ does not depend on the estimated state vector $\mathbf{x}_{k-1}$, then for further consideration, involving differentiation of $\tilde{F}_k(\mathbf{x}_{k-1})$ function with respect to $\mathbf{x}_{k-1}$, we consider a simplified implementation

$$\tilde{F}_k(\mathbf{x}_{k-1}) \approx \frac{1}{\sigma_n^2}\sum_{i=1}^{N} y_{k-1,i}\tilde{s}_p(t_{k-1,i},\mathbf{x}_{k-1}). \quad (9)$$

We transform the right-hand side of expression (9)

$$\frac{1}{\sigma_n^2}\sum_{l=1}^{N_p} y_{k-1,l}\tilde{s}_p(t_{k-1,l},\mathbf{x}_{k-1}) = \frac{A}{\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\,h_{ds}(t_{k-1,l}-\tau_{k-1})\cos(\omega_0 t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}) =$$

$$= \frac{A}{\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\,h_{ds}(t_{k-1,l}-\tau_{k-1})\cos(\omega_0 t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}). \quad (10)$$

This correlation can be interpreted as the correlation processing of input samples $y_{k-1,l}$ with the reference signal $\tilde{s}_p(t_{k-1,l},\mathbf{x}_{k-1})$, which should be formed at the point of extrapolated estimate $\mathbf{x}_{k-1} \rightarrow \tilde{\mathbf{x}}_k$ that involves a workup on the carrier frequency ω_0.

Thus, this expression does not contain explicitly the subcarrier frequencies that would allow processing on these frequencies. In order to ensure such a processing, consider signal function $\tilde{s}_p(t_{k,i},\varphi_{k-1})$ (4), and replace at a first approximation sign $\sin(\Omega t_{k-1,l})$ to $\sin(\Omega t_{k-1,l})$ the same way as it was done in [21]:

$$\tilde{s}_p(t_{k-1,l},\mathbf{x}_{k-1}) = A\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\sin(\Omega(t_{k-1,l}-\tau_{k-1}))\cos(\omega_0 t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}). \quad (11)$$

We introduce the variables

$\omega_{sc1} = \omega_0 + \Omega$, $\omega_{sc2} = \omega_0 - \Omega$,

and write (11) in the following form

$$\tilde{s}_p(t_{k-1,l},\mathbf{x}_{k-1}) = 0,5A\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1}) \times$$

$$\times\left(\sin(\omega_{sc1}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}-\Omega\tau_{k-1}) - \sin(\omega_{sc2}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}+\Omega\tau_{k-1})\right) \quad (12)$$

Taking into account (11), we can write (10) in the form

$$\frac{1}{\sigma_n^2}\sum_{l=1}^{N_p} y_{k-1,l}\tilde{s}_p(t_{k-1,l},\mathbf{x}_{k-1}) = \frac{A}{2\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\sin(\omega_{sc1}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}-\Omega\tau_{k-1}) -$$

$$-\frac{A}{2\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\sin(\omega_{sc2}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}+\Omega\tau_{k-1}). \quad (13)$$

This implementation already contains a correlation processing on the subcarrier frequencies ω_{sc1} and ω_{sc2} that will be used for further specification of processing algorithms in the tracking system.

Using trigonometric transformations, represent (13) in the form

$$\frac{1}{\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{s}_p(t_{k-1,l},\mathbf{x}_{k-1}) = \left(Q_{sc1,k}(\varphi_{k-1}) - Q_{sc2,k}(\varphi_{k-1})\right)\cos(\Omega\tau_{k-1}) -$$

$$-\left(I_{sc1,k}(\varphi_{k-1}) + I_{sc2,k}(\varphi_{k-1})\right)\sin(\Omega\tau_{k-1}),$$

where

$$I_{sc1,k}(\varphi_{k-1}) = \frac{A}{2\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\cos(\omega_{sc1}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}),$$

$$Q_{sc1,k}(\varphi_{k-1}) = \frac{A}{2\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\sin(\omega_{sc1}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}),$$

$$I_{sc2,k}(\varphi_{k-1}) = \frac{A}{2\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\cos(\omega_{sc2}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}),$$

$$Q_{sc2,k}(\varphi_{k-1}) = \frac{A}{2\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{h}_{c,p}(t_{k-1,l}-\tau_{k-1})\sin(\omega_{sc2}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}). \quad (14)$$

Formulas (14) describe correlators processing the input signal on the subcarrier frequencies ω_{sc1} and ω_{sc2}. In the filtering algorithm (6) we should use correlators (14), where the reference signals are taken with the extrapolated value of the $\tilde{\varphi}_{k-1}$ phase evaluation and the Doppler frequency shift $\tilde{\omega}_{D,k-1}$, i.e. in (14) we must make the following replacement $\varphi_{k-1} \rightarrow \tilde{\varphi}_{k-1}$, $\omega_{D,k-1} \rightarrow \tilde{\omega}_{D,k-1}$.

The equations of quasi-optimal filtering (6) are sometimes referred to as "extended Kalman filter". The form of such implementation is not quite adequate to meet the presentations of classical tracking system [5, 23], where one can distinguish a sampler and a flattening filter. The sampler is referred to a device, whose output process carries information about the mismatch between the true value of the signal parameter, which is under the tracking, and its estimated value, which is formed in the tracking system. In concerned problem, a parameter, for which tracking is carried out, is the signal phase. Therefore, we consider the phase sampler φ, which is generally described by the expression [5, 24]:

$$u_{d\varphi,k} = \left.\frac{\partial \tilde{F}_k(\mathbf{x}_{k-1})}{\partial \varphi}\right|_{\mathbf{x}_{k-1}=\tilde{\mathbf{x}}_{k-1}}.$$

Given that in the concerned problem the function $\tilde{F}_k(\mathbf{x}_{k-1})$ is defined by the formula (9), for which we obtained the implementation (14), we write the expression for the phase sampler in the form:

$$u_{d\varphi,k} = \frac{\partial}{\partial\varphi}\left[\frac{1}{\sigma_n^2}\sum_{l=1}^{N} y_{k-1,l}\tilde{s}_p\ \ t_{k-1,l}, \mathbf{x}_{k-1}\ \ \right]\Bigg|_{\mathbf{x}_{k-1}=\tilde{\mathbf{x}}_{k-1}} =$$

$$= I_{sc1,k}\ \tilde{\varphi}_{k-1}\ - I_{sc2,k}\ \tilde{\varphi}_{k-1}\ \cos\ \Omega\tau_{k-1}\ +\ Q_{sc1,k}\ \tilde{\varphi}_{k-1}\ + Q_{sc2,k}\ \tilde{\varphi}_{k-1}\ \sin\ \Omega\tau_{k-1}\ . \qquad (15)$$

Formula (15) describes the structure of the phase sampler, which is shown in Fig. 2.

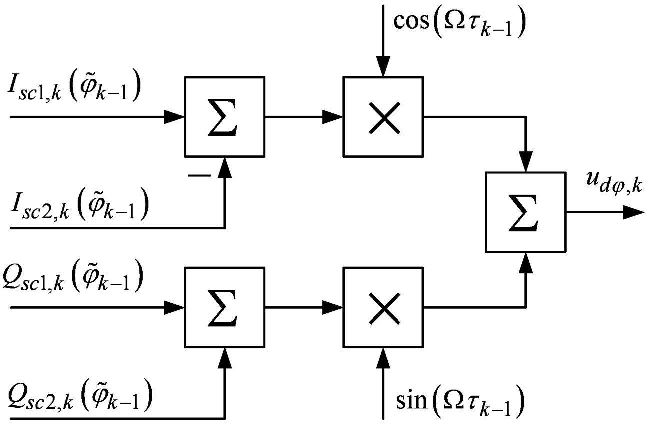

Figure 2. Phase sampler structure when processing on the subcarrier frequencies.

Note that the structure of the phase sampler (14) differs substantially from the known phase samplers [5, 24]. This is due to the fact that in this approach the correlators are formed on the two subcarrier frequencies, rather than on a single carrier frequency, as it is done in the standard approach.

The optimal filtering equations (6) include a derivative $\frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_{k-1}}{\partial\mathbf{x}}$, for which we can write the relation:

$$\frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_{k-1}}{\partial\mathbf{x}} = \frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_{k-1}}{\partial\varphi}\frac{\partial\varphi}{\partial\mathbf{x}} = \frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_{k-1}}{\partial\varphi}\mathbf{c} = u_{d\varphi,k}\mathbf{c}\ . \qquad (16)$$

Taking into account (15), equation (6) takes the form

$$\hat{\mathbf{x}}_{k-1} = \tilde{\mathbf{x}}_{k-1} + \mathbf{D}_{\mathbf{x},k-1}\mathbf{c}^{\mathrm{T}}u_{d\varphi,k}\ .$$

This equation describes a tracking system with the third order astaticism, and it is similar to the equation of the classical system, tracking the phase of the signal [5, 23]. The only difference consists in the formulas of reference formation $u_{d\varphi,k}$ at the phase sampler output.

If at the instant t_k the consumer is interested in estimating the state vector $\mathbf{x}_k$, corresponding to the same point in time, then in accordance with the provisions of the optimal filtering theory [24], the corresponding estimate is given by

$$\tilde{\mathbf{x}}_k = \mathbf{F}\hat{\mathbf{x}}_{k-1}\ .$$

At that, the error variance of the extrapolated estimate $\tilde{\mathbf{x}}_k$ is determined by extrapolation errors variance matrix $\tilde{\mathbf{D}}_{\mathbf{x},k-1}$ (7).

4. Statistical analysis of the phase sampler

The main feature of the synthesized filtering algorithm consists in a phase sampler (15). Therefore, let analyze its statistical characteristics.

Calculate the mathematical expectations of the correlation sums (14) carrying out the following replacement $\varphi_{k-1} \to \tilde{\varphi}_{k-1}$, $\omega_{D,k-1} \to \tilde{\omega}_{D,k-1}$ ..

$$E\left[I_{sc1,k}\right] = \bar{I}_{sc1,k} = \frac{A^2T}{4N_0}\sin\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 - \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ =$$

$$= \frac{q_{c/n_0}T}{2}\sin\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 - \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ ,$$

$$E\left[Q_{sc1,k}\right] = \bar{Q}_{sc1,k} = \frac{A^2T}{4N_0}\cos\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 - \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ =$$

$$= \frac{q_{c/n_0}T}{2}\cos\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 - \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ ,$$

$$E\left[I_{sc2,k}\right] = \bar{I}_{sc2,k} = \frac{-A^2T}{4N_0}\sin\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 + \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ =$$

$$= \frac{-q_{c/n_0}T}{2}\sin\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 + \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ ,$$

$$E\left[Q_{sc2,k}\right] = \bar{Q}_{sc2,k} = \frac{-A^2T}{4N_0}\cos\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 + \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2\ \ =$$

$$= \frac{-q_{c/n_0}T}{2}\cos\ \ \varepsilon_{\varphi,k} + \varepsilon_{\omega,k}T/2 + \Omega\tau_{k-1}\ \ \mathrm{sinc}\ \ \varepsilon_{\omega,k}T/2$$

where $\mathrm{sinc}\ \ x\ = \frac{\sin\ \ x}{x}$,

$\varepsilon_{\varphi,k} = \varphi_{k-1} - \tilde{\varphi}_{k-1}$ - is the phase tracking error;

$\varepsilon_{\omega,k} = \omega_{k-1} - \tilde{\omega}_{k-1}$ - is the tracking error of Doppler frequency shift;

$q_{c/n_0} = \frac{A^2}{2N_0} = \frac{P_c}{N_0}$ - is the ratio of a signal power to the spectral density of the noise power.

Calculate the discriminatory characteristic of phase sampler (14), which is understood as the dependence of the mathematical expectation $E\left[u_{d\varphi,k}\right]$ from the tracking error ε_φ.

$$U_{d,\varphi}\ \ \varepsilon_\varphi\ \ = E\left[u_{d\varphi,k}\right] = q_{c/n_0}T\sin\ \ \varepsilon_{\varphi,k}\ \ .$$

The steepness of the discriminatory characteristic $U_{d,\varphi}\ \ \varepsilon_\varphi$ is determined by the expression

$$S_{d,\varphi} = \frac{\partial U_{d,\varphi}\ \ \varepsilon_\varphi}{\partial\varepsilon_\varphi} = q_{c/n_0}T\ .$$

Calculate the fluctuation characteristic of the phase sampler (15). To do this, we obtain an expression for the sample variance at the correlators output (14):

$$D_{fl,I} = M\left[\ \ I_{\text{п}1,k} - \left[I_{\text{п}1,k}\right]\ \ ^2\right] =$$

$$=\left(\frac{A}{N_0}\right)^2 M\left(\sum_{l=1}^{N_p} n_{k-1,l}\tilde{h}_{c,p}\ \ t_{k-1,l}-\tau_{k-1}\ \ \cos\ \omega_{sc1}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}\ \ T_d\right)^2=\frac{A^2T}{8N_0}=\frac{q_{c/n_0}T}{4}$$

.

For the fluctuation characteristic of the phase sampler, we write the correlation

$$D_{fl,d\varphi}=M\left[\ u_{d\varphi,k}-U_{d,\varphi}\ \ ^2\right]\Big|_{\varepsilon_\varphi=0}=$$

$$=\ D_{fl,I_{sc1}}+D_{fl,I_{sc2}}\ \cos^2\ \Omega\tau_{k-1}\ +\ D_{fl,Q_{sc1}}+D_{fl,Q_{sc2}}\ \sin^2\ \Omega\tau_{k-1}\ =2D_{fl,I_{sc1}}=\frac{q_{c/n_0}T}{2}. \qquad (17)$$

When writing (17) we took into account that $D_{fl,I_{sc1}}=D_{fl,I_{sc2}}=D_{fl,Q_{sc1}}=D_{fl,Q_{sc2}}$.

An important characteristic of the tracking systems performance quality is the noise variance of the phase sampler, reduced to the measured parameter [5], which is denoted as $D_{ef,\varphi}$. This characteristic is uniquely connected with the lower Cramer-Rao bound (CRB) for the variance of the signal parameter estimate, i.e. it determines the potential accuracy of the estimate of the corresponding parameter [24]. For the concerned phase sampler, we write

$$D_{ef,\varphi}=\frac{D_{fl,d\varphi}}{S_{d,\varphi}^2}=\frac{1}{2q_{c/n_0}T}. \qquad (18)$$

Let compare the given variance with a similar variance for the phase sampler, corresponding to the algorithm for processing on the carrier frequency. This type of sampler is obtained by differentiating (10) with respect to φ_{k-1}:

$$\tilde{u}_{d\varphi,k}=\frac{\partial}{\partial\varphi}\left[\frac{1}{\sigma_n^2}\sum_{l=1}^{N}y_{k-1,l}\tilde{s}_p\ \ t_{k-1,l},\mathbf{x}_{k-1}\ \ \right]\Bigg|_{\mathbf{x}_{k-1}=\tilde{\mathbf{x}}_{k-1}}=$$

$$=\frac{-A}{\sigma_n^2}\sum_{l=1}^{N}y_{k-1,l}\tilde{h}_{c,p}\ \ t_{k-1,l}-\tau_{k-1}\ \ h_{ds}\ \ t_{k,i}-\tau_k\ \ \sin\ \omega_{sc1}t_{k-1,l}+\varphi_{k-1}+lT_d\omega_{D,k-1}\ .$$

The variance of the noise component $\tilde{u}_{d\varphi,k}$ is determined by the expression $\tilde{D}_{fl,d\varphi}=q_{c/n_0}T$, while the steepness of the discriminator curve is $\tilde{S}_{d,\varphi}=q_{c/n_0}T$ (taking into account that the $L1OC$ signal is a two-component signal with the bit time-division multiplexing). Therefore, the noise variance, reduced to the measured parameter, for the concerned phase sampler is given by the expression:

$$\tilde{D}_{ef,\varphi}=\frac{\tilde{D}_{fl,d\varphi}}{\tilde{S}_{d,\varphi}^2}=\frac{1}{q_{c/n_0}T}.$$

From a comparison of the given expression with (18) it follows that the reduced noise variance $\tilde{D}_{ef,\varphi}$ is twice as large as the variance $D_{ef,\varphi}$. This fact suggests that the potential tracking accuracy over a signal phase in the system processed on subcarrier frequencies is higher than in the system processed on the carrier frequency. This is due to the fact that in the derivation of the processing algorithm on subcarrier frequencies the digital sinusoid is replaced by a normal sinusoid. This leads to the fact that the input noise of the receiver is modulated by smoother decaying function. This results in a decrease of the noise power at such processing.

Statistical characteristics of the phase sampler can be used to simplify the dispersion equations (7). The essence of this simplification is to replace the derivative $\frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_k}{\partial\mathbf{x}}\right)^T$ by its mean value $E\left[\frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_k}{\partial\mathbf{x}}\right)^T\right]$. Carrying out this averaging, we obtain

$$E\left[\frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial\tilde{F}_k\ \ \tilde{\mathbf{x}}_k}{\partial\mathbf{x}}\right)^T\right]=E\left[\frac{\partial}{\partial\mathbf{x}}\ u_{d\varphi,k}\mathbf{c}\ ^T\right]=\frac{\partial}{\partial\mathbf{x}}\ E\left[u_{d\varphi,k}\right]\mathbf{c}\ ^T=-q_{c/n_0}T\mathbf{c}^T\mathbf{c}.$$

Substituting this expression to the dispersion equations (7), we write the simplified equations:

$$\tilde{\mathbf{D}}_{\mathbf{x},k-1}=\mathbf{F}\mathbf{D}_{\mathbf{x},k-2}\mathbf{F}^T+\mathbf{G}D_\xi\mathbf{G}^T,$$

$$\mathbf{D}_{\mathbf{x},k-1}^{-1}=\tilde{\mathbf{D}}_{\mathbf{x},k-1}^{-1}+q_{c/n_0}T\mathbf{c}^T\mathbf{c}. \qquad (19)$$

These dispersion equations will then be used during simulation modelling of the synthesized tracking system.

5. Simulation of synthesized tracking system

To verify the functionality and performance of the synthesized system for tracking the phase of pilot component of L1OC GLONASS signal, processed on subcarrier frequencies, we conducted its computer simulation in a MATLAB simulation modeling environment. The simulation was performed using the method of statistical equivalents under the following conditions: $\tilde{q}_{c/n_0}=\log\ q_{c/n_0}\ =45$ dBHz; T = 8 ms.

First, consider the case when the phase φ_k is described by equation (5), and the tracking system is described by unsteady equations (6) and (19). Let D_ξ = 2.88 rad^2/c^4. Figure 3 shows the implementation of the phase tracking error resulting from the simulation. The solid red lines in Fig. 3 show the limits of the confidence interval $\pm3\sigma_\varphi$, where σ_φ - is the mean-square error of phase tracking, calculated by the Riccati equation (19).

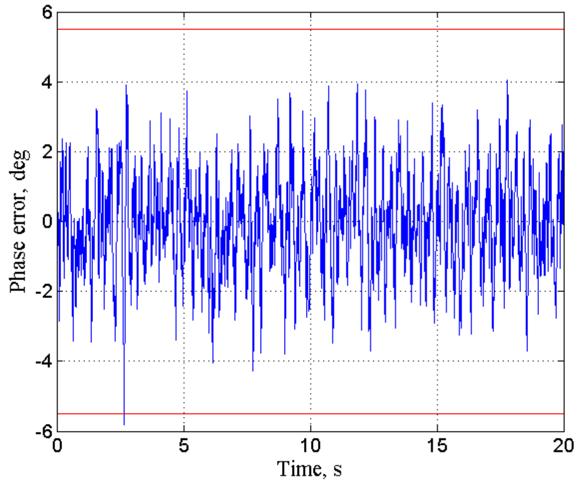

Figure 3. Implementation of the signal phase tracking error at D_ξ = 2.88 rad^2/c^4.

Figure 4 shows the dependence of the phase filtering error variance $\mathbf{Dx}\ 1,1$ on time, calculated by the Riccati equation (19).

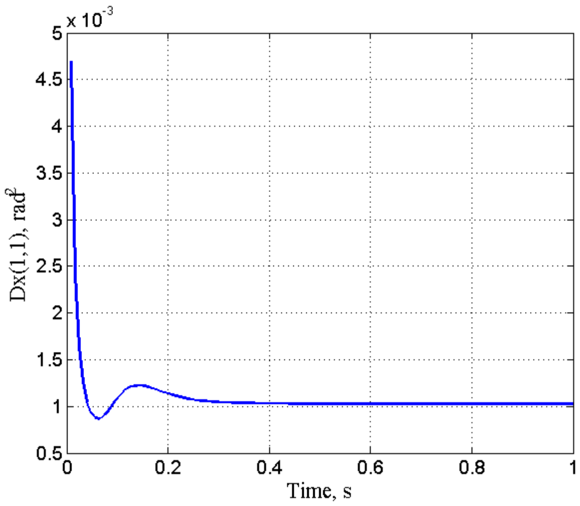

Figure 4. Diagram of the phase filtering error variance.

From Fig.4 it follows that after 0.3s phase filtering error variance reaches a steady-state value, while signal phase tracking system becomes stationary. Therefore, in the steady-state mode, one may talk of the frequency passband of the tracking system. It can be shown [5] that in this mode, the weight ratios $\mathbf{K} = |K_1 \quad K_2 \quad K_3|^T = \mathbf{D}_{\mathbf{x},k-1}\mathbf{c}^T S_{d,\varphi}$ in the tracking system are related by the following correlations:

$$K_1/T = 2\ K_3/T\ ^{1/3}, K_2/T = 2\ K_3/T\ ^{2/3}, \tag{20}$$

and the frequency passband of the tracking system is determined by the formula $\Delta f_{PLL} = 5\ K_3/T\ ^{1/3}/6$.

Calculation by this formula gives the following meaning of the tracking system frequency passband for the concerned case Δf_{PLL} = 11.6 Hz.

For the stationary tracking region, root mean square of phase tracking error, calculated by the implementation, illustrated in Fig. 3, is equal to $\sigma_{\varphi,calc}$ = 1.6 deg.

Consider a more intense change of signal phase, assuming that D_ξ = 288 rad^2/c^4. Figure 5 shows the implementation of the phase tracking error, resulting from the simulation. The solid red lines in Fig. 5 show the limits of the confidence interval $\pm 3\sigma_\varphi$, where σ_φ - is the root mean square of the phase tracking error, calculated by the Riccati equation (19).

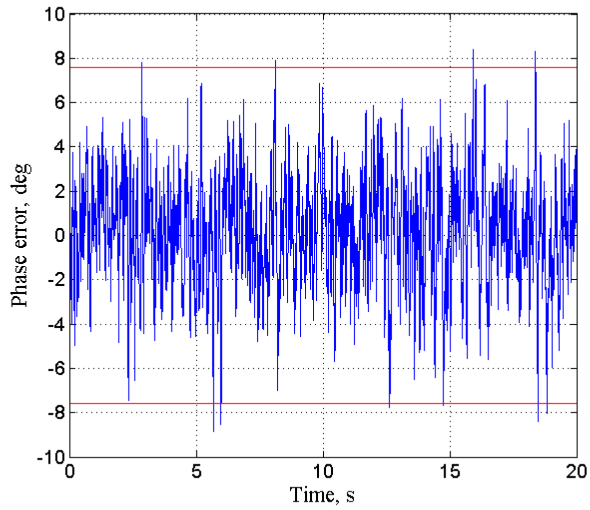

Figure 5. Implementation of the signal phase tracking error at D_ξ = 288 rad^2/s^4.

In steady state conditions, the tracking system frequency passband is Δf_{PLL} = 23.9 Hz, while the root mean square of the phase tracking error, calculated by this implementation, is $\sigma_{\varphi,calc}$ = 2.54 deg.

Comparing these results with the similar results obtained above for the poor dynamic pattern of the signal phase, we see that for the strong dynamic pattern of the signal phase the frequency passband of the optimal tracking system at steady state mode has increased by 2 times, whereas the mean square error of phase tracking has increased by 1.6 times.

In practice, the actual change in the signal phase cannot be described by stochastic equations (5), since has more regular character. Consider the limiting case, where the phase of the signal varies according to a deterministic law. We assume that the change in the phase of the signal in time is determined by the dynamic pattern of the consumer movement with jerk along the line of sighting (where jerk refers to derivative of acceleration with respect to time), shown in Fig. 6.

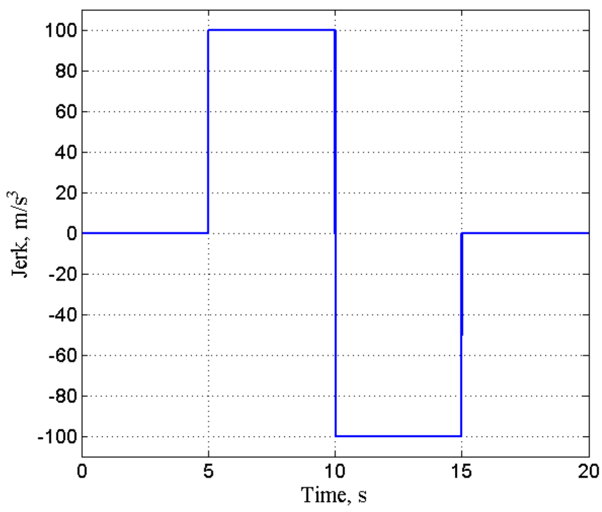

Figure 6. Change of jerk along the line of sighting.

Consider the performance mode of the tracking system (6) with constant weight ratios corresponding to the steady state mode. Define the frequency passband of the tracking system over the signal phase Δf_{PLL} = 20 Hz, and determine the weight ratios of the tracking system by (20).

Implementation of the signal phase tracking error, obtained in consequence of the simulation, is shown in Fig.7.

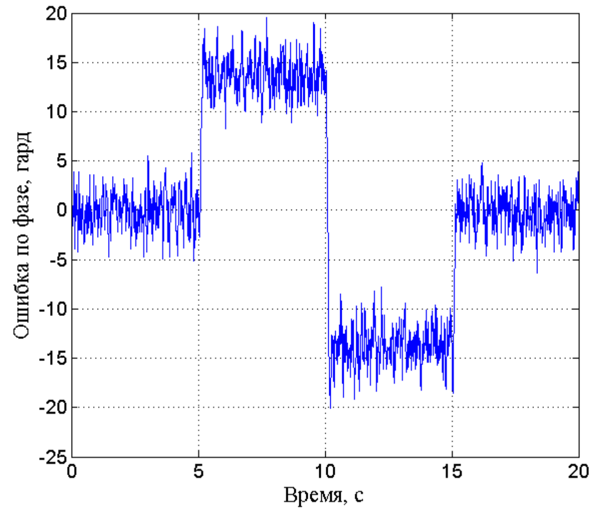

Figure 7. The implementation of the signal phase tracking error at consumer jerks.

Figure 7 shows that the tracking system successfully monitors changes in signal phase. Dynamic tracking error in the maneuvering areas (jerk) is about 14 degrees. The value of root mean square of the fluctuation error is about 1.9 degrees. The theoretical value of the dynamic tracking error in the tracking system with astaticism of third order under the influence of jerk $J = 100$ m/c^3 is equal to $\varepsilon_{\varphi,st} = 13.8$ degrees. Thus, the simulation results are in good agreement with the theoretical (expected) results.

6. Conclusions

In this paper, quasi-optimal algorithm for phase filtering of the pilot component of L1OC signal, processed on subcarrier frequencies, is synthesized using the theory of optimal filtering of information processes. Equations are obtained for optimal filtering, as well as their implementation in the form of a tracking system, including a phase sampler and a flattening filter. It is shown that the phase sampler with processing of the pilot component of the L1OC signal on subcarrier frequencies has a structure different from that of the classical phase sampler with processing on the carrier frequency. The author presents the functional block diagram of such sampler, as well as the calculated discriminatory and fluctuation characteristics of the synthesized phase sampler. It is shown that the discriminator curve has a point of stable equilibrium, while the noise variance of the phase sampler, reduced to the parameter measured, is two times less than the similar value obtained for the tracking system, following the phase of a pilot component of *L1OC* signal on the carrier frequency. Author presents the results of simulation modeling of synthesized tracking system. It is shown that in the optimal tracking system at a signal/noise ratio of 45 dBHz, mean square error of signal phase estimation is about 1.6 degrees for slow-moving consumer and about 2.5 degrees for fast-moving consumer.

When driving with a jerk of 100 m/s^3, tracking system with a frequency passband of 20 Hz provides a stable phase tracking signal, ensuring a constant component of the error equal to about 14 degrees, and the fluctuation component of the error of about 1.9 degrees.

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