International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 14 (2015) pp 34316-34323

Order Reduction Of Linear Interval System Using Differential Evolution Algorithm

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Abstract
This paper presents a new method for reduction of higher order linear continuous time interval system into lower order model. This method is based on Kharitonov's theorem, and most popular conventional method i.e. Stability equation method and the error minimization by a soft computing technique i.e. Differential Evolution. The reduced order denominator of lower order model is obtained by using Kharitonov's polynomials and stability equation method, while the numerator is obtained using Differential Evolution algorithm. By this method, the obtained lower order interval model from higher order interval system retains stability and the steady-state value. The proposed method is illustrated with the help of typical numerical example considered from the literature.

Keywords- Model order reduction; Interval System; Kharitonov's theorem; Stability equation method; Differential Evolution; Integral Square Error.

1. INTRODUCTION

In general the original system model is fairly complex and is of higher order. The understanding of the behavior of the system is difficult due to complexity. The analysis of higher order is both tedious and costly. Therefore the use of an order reduction makes it easier to implement analysis and design of control system. It has become necessary to use reduced order modeling techniques for the fundamental understanding of the system characteristics. Model order reduction (MOR) is a branch of systems and control theory, for reducing their complexity, while preserving their input-output behavior. Order reduction methods are broadly classified into two types namely frequency domain and time domain order reduction methods. The frequency domain order reduction methods are for transfer function model, whereas the time domain order reduction methods are for state space model. Several methods are available in the literature for the order reduction of linear continuous systems in time domain as well as frequency domain. The reduced order model obtained in the frequency domain gives better matching of the impulse response with its higher order system.

Some of the most popularly used frequency domain order reduction methods are Pade approximation and continued fraction method. These are computationally fast and being able to exactly match the maximum number of system parameters to the reduced model. But one of the disadvantage of these methods is stability of the reduced model is not guaranteed for stable higher order system. Effort has been devoted to developing stability preserving methods such as Routh stability criterion, Mihailov criterion, Hurwitz polynomial. The absolute stability of these methods achieved only by the cost of series loss of accuracy. Among these various model order reduction methods for stability preservation available in the literature, the stability equation method is one of the most popular techniques. The advantage of this method is that it preserves stability in the reduced model, if the original higher-order system is stable, and retains the first two time-moments of the system.

There are several methods available in the literature for order reduction, which are based on the minimization of the integral square error (ISE) criterion. In [13], [14] the values of the denominator coefficients of the low order system are determined by some stability preservation methods and then the numerator coefficients of the low order systems are determined by minimization of the ISE using optimization technique.

However many systems the coefficients are constants but uncertain within a finite range. Such systems are classified as interval systems. In [4] $\gamma - \delta$ Routh Approximation method for interval systems is proposed. The reduced model of interval system is unstable even the original higher order interval system is stable. An improvement is proposed in [5] to the $\gamma - \delta$ Routh approximation for interval systems using the Kharitonov's polynomials such that the resulting interval Routh approximant is robustly stable. To improve the effectiveness of model order reduction many mixed methods are proposed recently is [9], [10], [11] based on interval arithmetic. Thus the stability of the reduced order model obtained using interval arithmetic is not guaranteed, even if the original interval system is stable. A reduction technique for linear interval systems using Kharitonov's theorem are presented in [18-19] to generate stable reduced order interval models. In [20] recently a reduction technique for linear interval systems using Kharitonov's theorem and Routh approximation is presented to generate stable reduced order interval system.

Recently one of the most popular research fields has been "Evolutionary Techniques", inspired by the natural evolution of species. Evolutionary techniques have been successfully applied to solve numerous optimization problems. Differential Evolution was first proposed by Rainer Storn and Kenneth Price in 1996, it is a branch evolutionary algorithm. DE is a stochastic population based direct search algorithm. The advantages of DE are simplicity, accurate, reasonably fast and robust optimization method, therefore used to optimize real parameter, real valued function. The differential evolution (DE) algorithm can be used to find approximate solution non-differentiable, nonlinear and multi-modal objective functions. The main difference between DE from other Evolutionary Algorithms (EA) in the mutation and recombination phases. Another difference between DE and other EAs such as GA is DE has ability to search with floating point representation instead of binary representation as used in GA. DE employs a greedy selection. Also it has a minimum number of EA control parameters, which can be tuned effectively. The above method are available only for fixed systems only.

In this paper to overcome the above drawbacks a new method for model order reduction of interval systems is carried out by using Kharitonov's theorem, stability equation method and differential evolution by minimising using ISE method. The denominator of the reduced model is obtained by stability equation method and the numerator is obtained by minimising integral square error between transient response of original higher order system and the reduced order model pertaining to a unit step input. Thus the stability is guaranteed for the reduced order system by stability equation method if the original higher order system is stable and the responses matching between original higher order system and the reduced order system is by minimising ISE using DE.

2. PROBLEM FORMULATION

Consider a higher order continuous interval system by the transfer function

$$G_n(s)=\frac{N(s)}{D(s)}=\frac{[B_0^-,B_0^+]+[B_1^-,B_1^+]s+\ldots+[B_{n-1}^-,B_{n-1}^+]s^{n-1}}{[A_0^-,A_0^+]+[A_1^-,A_1^+]s+\ldots+[A_n^-,A_n^+]s^n}$$

Where $[A_i^-,A_i^+]$ for i=0,1….,n are denominator coefficients of $G_n(s)$ with A_i^- and A_i^+ as lower and upper bounds of interval $[A_i^-,A_i^+]$ respectively, and $[B_i^-,B_i^+]$ for i=0,1…,n-1 are numerator coefficients of $G_n(s)$ with B_i^- and B_i^+ as lower and upper bounds of interval $[B_i^-,B_i^+]$ respectively.

It is proposed to obtain a reduced order interval model of the form

$$G_r(s)=\frac{[b_0^-,b_0^+]+[b_1^-,b_1^+]s+\ldots+[b_{r-1}^-,b_{r-1}^+]s^{r-1}}{[a_0^-,a_0^+]+[a_1^-,a_1^+]s+\ldots+[a_r^-,a_r^+]s^r}$$

Where $[a_i^-,a_i^+]$ for i=0,1….,r are denominator coefficients of $G_r(s)$ with a_i^- and a_i^+ as lower and upper bounds of interval $[a_i^-,a_i^+]$ respectively, and $[b_i^-,b_i^+]$ for i=0,1…,r-1 are numerator coefficients of $G_r(s)$ with b_i^- and b_i^+ as lower and upper bounds of interval $[b_i^-,b_i^+]$ respectively.

3. PROPOSED METHOD

Kharitonov Theorem:

An interval polynomial family $p(s)=\sum_{i=0}^{n}[a_i^-,a_i^+]s^i$ with invariant degree is robustly stable if its four Kharitonov polynomials are stable.

According to the Kharitonov theorem, every interval polynomial *p(s)* is associated with four following fixed parameter polynomials called Kharitonov polynomials. They are defined as

$$p^1(s)=a_0^-+a_1^-s+a_2^+s^2+......+a_n^-s^n$$
$$p^2(s)=a_0^-+a_1^+s+a_2^+s^2+......+a_n^-s^n$$
$$p^3(s)=a_0^++a_1^-s+a_2^-s^2+......+a_n^+s^n$$
$$p^4(s)=a_0^++a_1^+s+a_2^-s^2+......+a_n^+s^n$$

The interval system is stable if and only if its four Kharitonov polynomials satisfies Routh Hurwitz stability criterion.

Reduction Procedure

Consider a family of real interval transfer function

$$G_n(s)=\frac{N(s)}{D(s)}=\frac{[B_0^-,B_0^+]+[B_1^-,B_1^+]s+\ldots+[B_{n-1}^-,B_{n-1}^+]s^{n-1}}{[A_0^-,A_0^+]+[A_1^-,A_1^+]s+\ldots+[A_n^-,A_n^+]s^{n-1}}$$

The four fixed Kharitonov's transfer functions associated with $G_n(s)$ are given as:

$$G_n^1(s)=\frac{N_n^1(s)}{D_n^1(s)}=\frac{B_0^-+B_1^-s+B_2^+s^2+\ldots+B_{n-1}^-s^{n-1}}{A_0^-+A_1^-s+A_2^+s^2+\ldots+A_n^-s^n}$$
$$=\frac{B_{10}^-+B_{11}^-s+B_{12}^+s^2+\ldots+B_{1(n-1)}^-s^{n-1}}{A_{10}^-+A_{11}^-s+A_{12}^+s^2+\ldots+A_{1n}^-s^n}$$

$$G_n^2(s)=\frac{N_n^2(s)}{D_n^2(s)}=\frac{B_0^-+B_1^+s+B_2^+s^2+\ldots+B_{n-1}^-s^{n-1}}{A_0^-+A_1^+s+A_2^+s^2+\ldots+A_n^-s^n}$$
$$=\frac{B_{20}^-+B_{21}^+s+B_{22}^+s^2+\ldots+B_{2(n-1)}^-s^{n-1}}{A_{20}^-+A_{21}^+s+A_{22}^+s^2+\ldots+A_{2n}^-s^n}$$

$$G_n^3(s)=\frac{N_n^3(s)}{D_n^3(s)}=\frac{B_0^++B_1^-s+B_2^-s^2+\ldots+B_{n-1}^+s^{n-1}}{A_0^++A_1^-s+A_2^-s^2+\ldots+A_n^+s^n}$$
$$=\frac{B_{30}^++B_{31}^-s+B_{32}^-s^2+\ldots+B_{3(n-1)}^+s^{n-1}}{A_{30}^++A_{31}^-s+A_{32}^-s^2+\ldots+A_{3n}^+s^n}$$

$$G_n^4(s) = \frac{N_n^4(s)}{D_n^4(s)} = \frac{B_0^+ + B_1^+ s + B_2^- s^2 + \ldots + B_{n-1}^+ s^{n-1}}{A_0^+ + A_1^+ s + A_2^- s^2 + \ldots + A_n^+ s^n}$$
$$= \frac{B_{40}^+ + B_{41}^+ s + B_{42}^- s^2 + \ldots + B_{4(n-1)}^+ s^{n-1}}{A_{40}^+ + A_{41}^+ s + A_{42}^- s^2 + \ldots + A_{4n}^+ s^n}$$

The above Kharitonov's transfer functions are, in general represented as,

$$G_n^I(s) = \frac{N_n^I(s)}{D_n^I(s)} = \frac{\sum_{j=0}^{n-1} B_{Ij} s^j}{\sum_{j=0}^{n} A_{Ij} s^j}$$

where I=1,2,3,4.

Step:1

Determination of the denominator coefficients of lower order system for first Kharitonov transfer function by stability equation method:

Stability Equation Method

For I=1

$$G_n^1(s) = \frac{N_n^1(s)}{D_n^1(s)} = \frac{B_{10}^- + B_{11}^- s + B_{12}^+ s^2 + \ldots + B_{1(n-1)}^- s^{n-1}}{A_{10}^- + A_{11}^- s + A_{12}^+ s^2 + \ldots + A_{1n}^- s^n} \quad (1)$$

For first Kharitonov transfer function the reduced order model is

$$G_r^1(s) = \frac{N_r^1(s)}{D_r^1(s)} = \frac{b_{10}^- + b_{11}^- s + b_{12}^+ s^2 + \ldots + b_{1(r-1)}^- s^{r-1}}{a_{10}^- + a_{11}^- s + a_{12}^+ s^2 + \ldots + a_{1r}^- s^r} \quad (2)$$

For stable first Kharitonov transfer function $G_n^1(s)$, the denominator $D_n^1(s)$ of the Higher Order System (HOS) into bifurcated into even and odd parts in the form of stability equations as

$$\left.\begin{aligned} D_e^n(s) &= A_{10}\prod_{i=1}^{m_1}\left(1+\frac{s^2}{z_i^2}\right) \\ D_o^n(s) &= A_{11}\prod_{i=1}^{m_2}\left(1+\frac{s^2}{p_i^2}\right) \end{aligned}\right\} \quad (3)$$

Where z_i is poles of even parts of the denominator polynomial and p_i is poles of odd parts of the denominator polynomial, m_1 and m_2 are the integer parts of n/2 and (n-1)/2 respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2 \ldots\ldots$

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in (3), the stability equations for r^{th} order LOS are obtained as:

$$\left.\begin{aligned} D_e^r(s) &= a_{10}\prod_{i=1}^{m_3}\left(1+\frac{s^2}{z_i^2}\right) \\ D_o^r(s) &= a_{11}\prod_{i=1}^{m_4}\left(1+\frac{s^2}{p_i^2}\right) \end{aligned}\right\} \quad (4)$$

Where m_3 and m_4 are the integer parts of r/2 and (r-1)/2, respectively.

Combining these reduced stability equations and therefore proper normalizing it, the r[th] order denominator of Lower Order System (LOS) is obtained as:

$$D_r^1(s) = D_e^r(s) + D_o^r(s) = \sum_{i=0}^{r} a_{1i} s^i \quad (5)$$

Therefore, the denominator polynomial in (2) is now known, which is given by

$$D_r^1(s) = a_{10} + a_{11}s + a_{12}s^2 + \ldots\ldots + a_{1(r-1)}s^{r-1} + a_r s^r \quad (6)$$

Step-2

There is a steady state error, this error is between the outputs of original and reduced systems. To avoid steady state error we match the steady state response by following relationship, to obtain correction factor 'h' a constant as follows:

$$\frac{B_0}{A_0} = h\frac{b_0}{a_0}$$

The final reduced order model is obtained by multiplying 'h' with numerator of the reduced model.

Step-3

Determination of the numerator coefficients of the reduced model by Differential Evolution (DE)

Differential Evolution

In this, DE is employed to minimize the objective function '*J*', which is the error between original higher order system and reduced order system and it is represented in the form

$$J = \int_0^\infty [\, y(t) - y_r(t)]^2\, dt$$

Mathematically, the integral square error can be represented as

$$J = \sum_{t=0}^{M} [\, y(t) - y_r(t)]^2$$

Where, *y(t)* is the unit step response of higher order and $y_r(t)$ is the unit step response lower order system at the t[th] instant in the time interval 0≤t≤N, where N is to be chosen.

The objective is to model lower order system which is closely approximate original system. The objective function is to minimize ISE by using DE.

Differential evolution (DE) is a stochastic, population based direct search optimization algorithm introduced by Storn and Price in 1996 [16]. DE works with two populations; old generation and new generation of the same population. *NP* is the size of the population is adjusted. The population consists of real valued vectors with dimension *D* that equals the number of design parameters/control variables. The population is randomly initialized within the initial parameter bounds. The three main operations carry optimization process: mutation, crossover and selection. In each generation, individuals of the current population become target vectors. For each target vector, the mutation operation produces a mutant vector, by adding the weighted difference between two randomly chosen vectors to a third vector. The crossover operation generates a new vector, called trial vector, by mixing the parameters of the mutant vector with those of the target vector. If the trial vector obtains a better fitness value than the target vector, then the trial vector replaces the target vector in the next generation.

Initialization

Define upper and lower bounds for each parameter.

$X_j^L \le X_{j,i,1} \le X_j^U$

Randomly select the initial parameter values uniformly on the intervals [X^L_j , X^U_j]

For example, the initial value of the jth parameter in the ith individual at the generation G=0 is generated by

$$X_{i,0}^j = X_{\min}^j + rand(0,1).(X_{\max}^j - X_{\min}^j) \quad (7)$$

j=1,2,...D

Where *NP* is the population size, *rand(0,1)* is a random number uniformly distributed between 0 and 1, *D* is the number of control variables.

Mutation

Mutation expands the search space. DE undergoes mutation operation after initialization. In mutation operation it produce mutant vector $V_{i,G}$, with respective to each individual $X_{i,G}$, so called target vector, in the current population via mutation strategy.

$$V_{i,G} = X_{i,G} + F(X_{best,G} - X_{i,G}) + F(X_{r_1^i,G} - X_{r_2^i,G})$$

For a given parameter vector $X_{i,G}$ randomly select two vectors $X_{r1,G}$ and $X_{r2,G}$ such that the indices r_1, r_2.

The mutation factor F is a constant from [0, 2].

$V_{i,G}$ is called the donor vector.

Crossover

Crossover incorporates successful solutions from the previous generation. After mutation, DE undergoes crossover. The trial vector $U_{i,G}$ is developed from the elements of the target vector, $X_{i,G}$, and the elements of the donor vector, $V_{i,G+1}$

$$u_{i,G}^j = \begin{cases} v_{i,G}^j & if\ (rand_j[0,1) \le CR)\ or\ (j = j_{rand}) \\ X_{i,G}^j & otherwise \end{cases} \quad (9)$$

j=1,2,....D

Elements of the donor vector enter the trial vector with probability CR(crossover rate) is set to [0,1].

Selection

The newly generated values of trail vectors exceed the corresponding upper and lower bounds, we initialize them randomly and uniformly within the pre-specified range.

$$X_{i,G+1}^j = \begin{cases} U_{i,G}^j & if\ f(U_{i,G}) \le f(X_{i,G}) \\ X_{i,G}^j & otherwise \end{cases} \quad (10)$$

The trail vector $X_{i,G}$ is compared with trail vector $U_{i,G}$ and the one with lowest function value is admitted to the next generation.

Therefore, the reduced r^{th} order four Kharitonov's transfer functions are obtained by using stability equation method for denominator and the numerator is obtained by minimizing integral square error using Differential Evolution and they are represented as

$$G_r^1(s) = \frac{b_{10} + b_{11}s + ... + b_{1(r-1)}s^{r-1}}{a_{10} + a_{11}s + ... + a_{1r}s^r}$$

$$G_2^2(s) = \frac{b_{20} + b_{21}s + ... + b_{2(r-1)}s^{r-1}}{a_{20} + a_{21}s + ... + a_{2r}s^r}$$

$$G_2^3(s) = \frac{b_{30} + b_{31}s + ... + b_{3(r-1)}s^{r-1}}{a_{30} + a_{31}s + ... + a_{3r}s^r}$$

$$G_2^4(s) = \frac{b_{40} + b_{41}s + ... + b_{4(r-1)}s^{r-1}}{a_{40} + a_{41}s + ... + a_{4r}s^r}$$

Finally the reduced order interval model is obtained by the following equation

$$G_r(s) = \frac{\sum_{j=0}^{k-1} [\min(b_{Ij}), \max(b_{Ij})] s^j}{\sum_{j=0}^{k} [\min(a_{Ij}), \max(a_{Ij})] s^j} \quad (11)$$

I=1,2,3,4.

NUMERICAL EXAMPLE

Consider a higher order interval system from literature[8]

$$G(s) = \frac{[1,1]s^2 + [1,2]s + [1,2]}{[1,1]s^4 + [3,4]s^3 + [4,4]s^2 + [5,8]s + [1,1]}$$

This higher order interval system can be represented as four fixed parameter Kharitonov transfer function they are given as

$$G_4^1(s) = \frac{s^2 + s + 1}{s^4 + 4s^3 + 4s^2 + 5s + 1}$$

$$G_4^2(s) = \frac{s^2 + 2s + 1}{s^4 + 3s^3 + 4s^2 + 8s + 1}$$

$$G_4^3(s) = \frac{s^2 + s + 2}{s^4 + 4s^3 + 4s^2 + 5s + 1}$$

$$G_4^4(s) = \frac{s^2 + 2s + 2}{s^4 + 3s^3 + 4s^2 + 8s + 1}$$

Step 1: Bifurcating the denominator of the above higher order 1st Kharitonov's transfer function in even and odd parts, we get the stability equations as:

$$D_e^4(s) = s^4 + 4s^2 + 1$$

$$D_o^4(s) = 4s^3 + 5s$$

$$D_e^2(s) = (s^2 + 0.26795)(s^2 + 3.73205)$$

$$D_o^2(s) = s(4s^2 + 5)$$

Now by discarding the factors with large magnitude of z_i^2 and p_i^2 in $D_e^n(s)$ and $D_o^n(s)$ respectively, the stability equations for the second-order reduced reduced model are given by:

$$D_e^2(s) = 3.73205(s^2 + 0.26795)$$

$$D_o^2(s) = 5s$$

$$D_2^1 = D_e^2(s) + D_o^2(s)$$

$$= 3.73205s^2 + 5s + 1$$

The reduced model is

$$G_r^1(s) = \frac{b_{11}s + b_{10}}{3.73205s^2 + 5s + 1}$$

Same as for remaining Kharitonov transfer function the reduced order transfer functions are:

$$G_r^2(s) = \frac{b_{21}s + b_{20}}{3.73205\,s^2 + 8s + 1}$$

$$G_r^3(s) = \frac{b_{31}s + b_{30}}{3.73205\,s^2 + 5s + 1}$$

$$G_r^4(s) = \frac{b_{41}s + b_{40}}{3.73205\,s^2 + 8s + 1}$$

Step 2: The numerator coefficients are obtained by minimizing integral square error using differential evolution

TABLE I. **TYPICAL PARAMETER USED BY DIFFERENTIAL EVOLUTION FOR 1ST KHARITONOV'S TRANSFER FUNCTION**

NAME	VALUE
Population size	50
CR	0.8
F	0.5
Parameter 1, min,max	0,5
Parameter 2,min, max	0,2
Maximum generation	10

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 1st Kharitonov's transfer function is

$$N_2^1(s) = 1.1266\,s + 1$$

TABLE II. **TYPICAL PARAMETER USED BY DIFFERENTIAL EVOLUTION FOR 2ND KHARITONOV'S TRANSFER FUNCTION**

NAME	VALUE
Population size	50
CR	0.8
F	0.5
Parameter 1, min,max	0,3
Parameter 2,min, max	0,2
Maximum generation	10

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 2nd Kharitonov's transfer function is

$$N_2^2(s) = 2.1216\,s + 1$$

TABLE III. **TYPICAL PARAMETER USED BY DIFFERENTIAL EVOLUTION FOR 3RD KHARITONOV'S TRANSFER FUNCTION**

NAME	VALUE
Population size	50
CR	0.8
F	0.5
Parameter 1, min,max	0,4
Parameter 2,min, max	0,3
Maximum generation	10

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 3rd Kharitonov's transfer function is

$$N_2^3(s) = 1.06496\,s + 2$$

TABLE IV. **TYPICAL PARAMETER USED BY DIFFERENTIAL EVOLUTION FOR 4TH KHARITONOV'S TRANSFER FUNCTION**

NAME	VALUE
Population size	50
CR	0.8
F	0.5
Parameter 1, min,max	0,4
Parameter 2,min, max	0,3
Maximum generation	10

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 4th Kharitonov's transfer function is

$$N_2^4(s) = 2.094849\,s + 2$$

The four reduced order Kharitonov's transfer function are

$$G_2^1(s) = \frac{1.1266s + 1}{3.73205s^2 + 5s + 1}$$

$$G_2^2(s) = \frac{2.1216s + 1}{3.73205s^2 + 8s + 1}$$

$$G_2^3(s) = \frac{1.06496s + 2}{3.73205s^2 + 5s + 1}$$

$$G_2^4(s) = \frac{2.094849s + 2}{3.73205s^2 + 8s + 1}$$

Therefore the reduced order interval system obtained by equation (11) is

$$G_2(s) = \frac{[1.06496, 2.1216]s + [1,2]}{[3.73205, 3.73205]s^2 + [5,8]s + [1,1]}$$

Comparison this with other method γ-δ method [4]

$$R_2(s) = \frac{[0.172421, 1]s + [0.10775, 1.6]}{[1,1]s^2 + [0.86207, 4]s + [0.10775, 0.8]}$$

TABLE V. **COMPARISION OF INTEGRAL SQUARE ERROR FOR REDUCED KHARTONOV'S TRANSFER MODEL**

NAME	PROPOSED MODEL (ISE VALUE)	B.BANDYOPADHYAY[4] (ISE VALUE)
1ST KHARITONOV'S TRANSFER FUNCTION	0.11126	2.8771
2ND KHARITONOV'S TRANSFER FUNCTION	0.88268	47.6446

3RD KHARITONOV'S TRANSFER FUNCTION	0.19965	53.9263
4TH KHARITONOV'S TRANSFER FUNCTION	0.67372	11.8177

Therefore the step responses comparison of 1st, 2nd, 3rd and 4th Kharitonov reduced order transfer functions obtained by the proposed method and the method given is [4] are shown in fig. 2, fig. 4, fig. 6 and fig. 8 respectively.

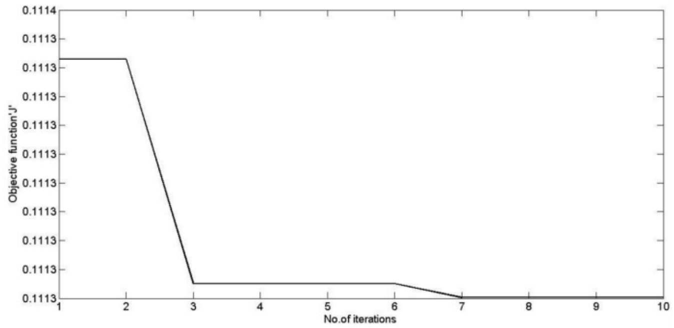

Fig. 1. **Convergence graph (1st kharitonov's TF)**

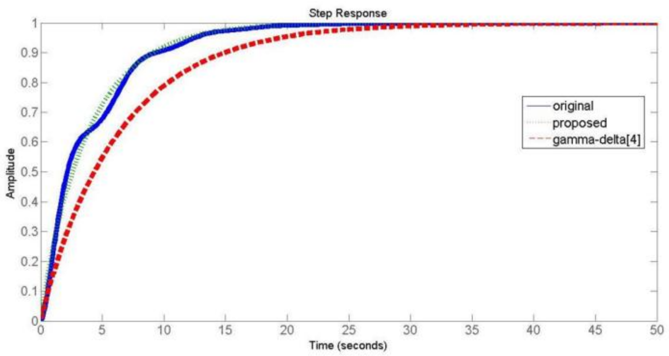

Fig. 2. **Step Response (1st Kharitonov's TF)**

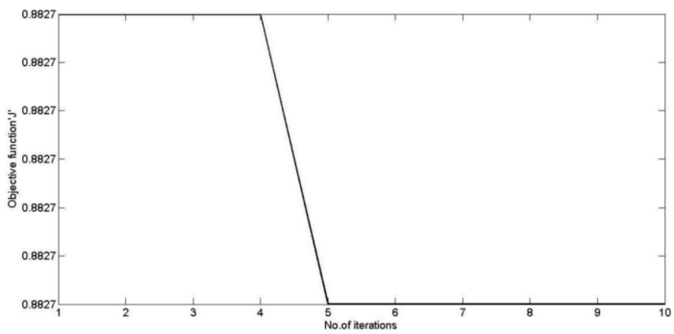

Fig. 3. **Convergence graph(2nd Kharitonov's TF)**

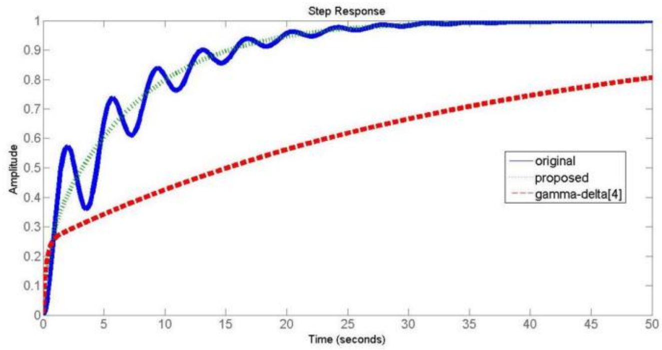

Fig. 4. **Step Response(2nd Kharitonov's TF)**

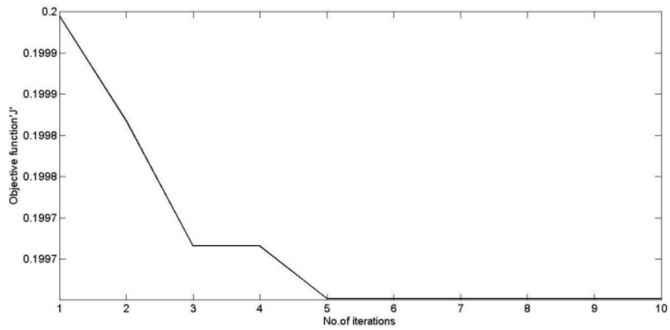

Fig. 5. **Convergence graph(3rd Kharitonov's TF)**

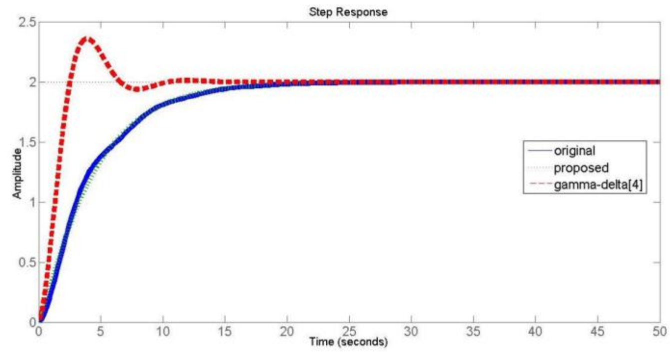

Fig. 6. **Step Response(3rd Kharitonov's TF)**

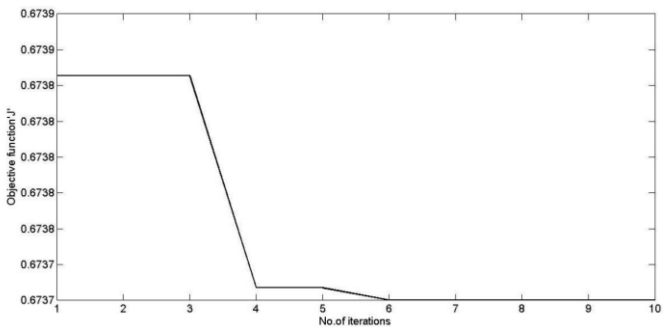

Fig. 7. **Convergence graph(4th Kharitonov's TF)**

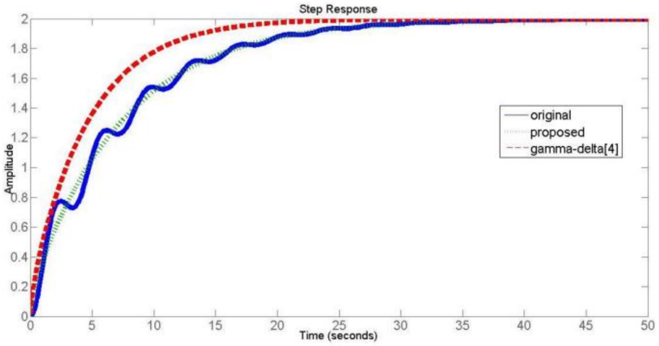

Fig. 8. **Step Response(4th Kharitonov's TF)**

It has been observed from fig.2, fig.4, fig.6 and fig.8 that the step response of four kharitonov transfer functions of higher order system and their corresponding reduced order transfer functions obtained by proposed method are closely matching. And thereby the reduced order interval model retains the stability. It is also observed from Table V that the ISEs of the four Kharitonov reduced order transfer functions obtained by the proposed method are less than the given in method [4].

CONCLUSION

In this paper, order reduction is done by combining the advantages of conventional method and a soft computing technique. The reduced order interval system is obtained using Kharitonov's polynomial and stability equation method for denominator coefficients, while numerator is obtained using Differential Evolution by minimising ISE. The use of Kharitonov's polynomial makes the reduced order interval system to retains stability while the reduced order model obtained by γ-δ method which makes use of interval arithmetic sometimes generate unstable reduce order model.
By this proposed method the reduced order denominator obtained using stability equation method preserves the stability, while the numerator is obtained using Differential Evolution by minimising ISE is used to give better matching of step response.

REFERENCES

[1] J.S.Rana, "Order reduction using Mihailov Criterion and Pade Approximations", International journal of innovations in engineering and technology,ISSN:2319-1058,ICAECE-2013.

[2] T.C. Chen, C.Y. Chang and K.W. Han, "Reduction of transfer functions by the stability equation method", Journal of Franklin Inst., Vol. 308, No. 4, pp. 389-404, 1979.

[3] S. Mukherjee, and R.N. Mishra, "Order reduction of linear systems using an error minimization technique", Journal of Franklin Inst., Vol. 323, No. 1, pp. 23-32, 1987.

[4] B. Bandyopadhyay , Avinash Upadhye and Osman Ismail, "γ − δ Routh Approximation for Interval Systems", IEEE Trans.on Automatic Control, Vol.42, no. 8, pp. 1126-1130, Aug. 1997.

[5] B. Bandyopadhyay, V. Sreeram, and Prashant Shingare, "Stable γ −δ Routh approximation of interval systems using Kharitonov polynomials", International journal of information and systems sciences, Vol. 4, no. 3, pp. 348-361, 2008.

[6] Barmish R.,1989, "A generalization of Kharitonovo's four-polynomial concept for Robust stability problems with linearly dependent coefficient perturbations", IEEE Trans.on Automatic Control,Vol.34,no.2.

[7] Kharitonov V. L., "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations", Differentsial'nye Uraveniya, Vol. 14, no. 11, pp. 2086-2088, 1978.

[8] Priya N., Dr. T.K. Sunil Kumar, "Low Order Controller Synthesis for Interval System", International Journal of Engineering Research Volume No.3, Issue No.2, pp : 62-65.

[9] D.Kranthi Kumar, S.K.Nagar and J.P.Tiwari, "Model order reduction of interval systems using Routh Approximations and Cauer Second form", International journal of engineering science and technology,Vol.3-No.2,ISSN 2229 5216, august 2011.

[10] D.Kranthi Kumar, S.K.Nagar and J.P.Tiwari, "Model order reduction of interval systems using Mihailov Criterion and Factor Division Method", International journal of computer applications, Vol.28-No.11,pp.4-8,August 2011.

[11] D.Kranthi Kumar, S.K.Nagar and J.P.Tiwari, "Model order reduction of interval systems using Mihailov Criterion and Cauer Second Form", International journal of computer application,Vol.32 No.6,pp.0975-8887,October 2011.

[12] S.N.Deepa, G. Sugumaran, "MPSO based model order formulation technique for SISO continuous systems", World Academy of Science, Engineering and Technology, Vol:5, 2011-03-25.

[13] G.Parmar, LifeMember SSI, AMIE, R.Prasad, and S.Mukherjee, FIE, "Order Reduction of linear dynamics systems using stability equation method and GA", International Journal of Electrical, Robotics, Vol:1, No.2, 2007

[14] J.S.Yadav, N.P.Patidar, J.Singhai, S.Panda and C.Ardil, "A combined conventional and differential evolution method for model order reduction", International Journal of Electrical, Robotics, Vol:5 N0:9,2011

[15] A.K.Qin, V.L. Huang, and P.N. Suganthan, "Differential Evolution Algorithm with Strategy Adaptation for Global Numerical Optimization", IEE.Trans. on Evolutionary computation, Vol.13, No.2, April 2009.

[16] Stron Rainer and Price Kennth, Differential Evolution , "A simple and efficient adaptive scheme for Global Optimization over continuous spaces", Journal of Global Optimization, Vol.11, pp. 341-359, 1997.

[17] Storn Rainer, "Differential Evolution for Continuous Function Optimization", http://www.icsi.berkeley.edu/- storn/code.html, 2005.

[18] Dr G.V.K.R Sastry ,M.Siva kumar, "High –order MIMO interval system reduction using direct routh approximation method", International Journal of Engineering Reasearch & Indus. Appls., vol .1,no.4,pp 45-54, 2008.

[19] Dr G.V.K.R Sastry ,M.Siva kumar, "Direct Routh Approximation method for linear SISO uncertain systems Reduction", International Journal of Applied Engineering Reasearch, vol .5, no.1, pp. 99-98,2010.

[20] M.Sivakumar, N.Vijaya Anand and R.Srinivas Rao, "Model Reduction of Linear Interval Systems Using Kharitonov's Polynomials", IEEE International conference on Energy Automation and Signal (ICEAS), 2011.

[21] Y.Shamash, "Stable reduced order models using Pade type approximations", IEEE Trans. On Automatic Control, Vol.19, pp.615-616.

[22] M.F.Hutton, and B.Friedland, "Routh approximations for reducing the order of linear time-invariant systems", IEEE Trans. Autom. Control, Vol 20,pp.329-337,1975

[23] D.Devi, P.Poongodi, "Order reduction of linear dynamic system using matlab programming method", International Conference on VLSI, Communication & Instrumentation (ICVCI) 2011.