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Delay properties of a transmission line model with non linear Miller loading

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***Abstract*—The theme of this paper is to study the effect of Miller type loading on a conventional transmission line with non-linear elements, capacitor and inductor. The purpose of this analysis is to investigate the use of non-linear elements, like varactor diodes or variable capacitor or an inductor on iron core material in the transmission line model in Miller form to produce negative group delay. The reason for choosing the Miller form loading is the practical simplicity of realizing these transmission line models on microstrip. Two different types of loadings are considered in this paper, one with the non-linear capacitor and other with the non-linear inductor. The gain for a unit cell of this model is obtained and the group delay and phase delay has been analyzed. After the analysis, is observed that the non-linear elements can produce the opposite delays in the circuit models.**

Keywords—Transmission line model; group delay; phase delay; Miller loading; non-linearity; group velocity; phase velocity

I. INTRODUCTION

There has been lot of advancement in material engineering where the properties of a medium can be altered with different types of orientations and combinations of other material. One simple method to come up with the negative refractive index material in microwave engineering is by periodic loading of a transmission line with resonators [1-6].In all these models and for purpose of analysis, the elements in the model are considered as linear due to the simplicity in analysis. But in reality all most all components exhibit some sort of non-linearity beyond its operating regions. However to meet the practical constraints, these non-linearities are modelled as linear components and analysis is carried out with some approximations. Sometimes these non-linear devices may not be much effective but sometimes even a small non-linearity in the device can produce adverse effects on stability and other problems like harmonic production and frequency conversion.

In microwave engineering, two wire transmission line theory [7] is the starting point in analyzing any type of circuit and it has been the heart of filter theory. With the advent of microstrip fabrication with very accurate and small tolerances, there has been a lot of research going on in realization of meta-material structures with the simple conventional transmission line theory and different types of components are realized in large number [1]. The meta-material has refractive index as negative which indicates that the phase velocity and group velocity are in opposite direction for a traveling wave on the transmission line [2]. The extensively used model for this realization is composite right hand left hand meta-material (CRLH-MTM) structure [3]. In all these models analysis, all the components are assumed to me linear and this theory has been well established. But when a component is being realized on microstrip with a very high accuracy, each component can be modelled as a non-linear element and the observations can be completely different from the linear case. This paper aims at studying the behavior of conventional transmission line model with a non-linear element loading.

II. PROBLEM FORMULATION

The aim of this paper is to study the effect of non-linearity on the group delay and phase delay of Miller type loaded transmission line. The conventional transmission line model is shown in Fig.1.It is a very familiar circuit with series element as inductor and shunt element as capacitor making as LC ladder network. This has been analyzed [7] and for this transmission line model, loading has been added by connecting the output of the unit cell with the input of the unit cell. This is Miller type loading and the element has been taken as a non-linear capacitor and non-linear inductor.

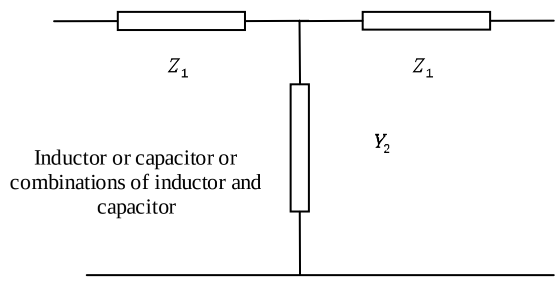

Fig.1 Conventional Transmission line model.

Fig.2 represents the Miller form loading of the unit cell of the conventional transmission line model.

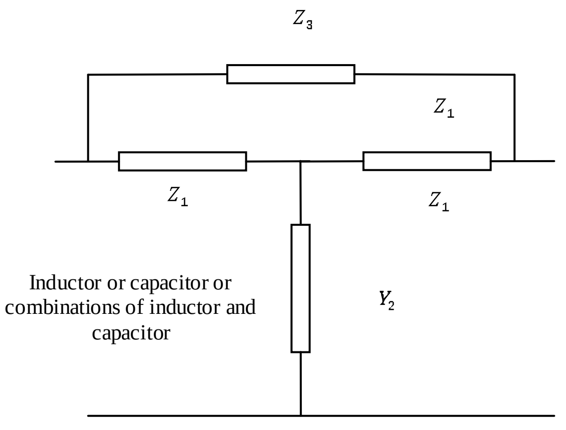

Fig.2 Novel Miller type loading of Transmission line model.

For an inductor on iron core, the relationship between magnetic flux and current passing through the loop are related as

$$i(t) = \frac{N\phi(t)}{L} + A\phi(t)^3 \quad (1)$$

where N is the number of turns of the coil, A is the cross sectional area of the core and L is the self inductance [8]. For a linear inductor the coefficient $A = 0$. Using the Faraday's law of induction along with Lenz's law, the induced voltage can be found out as

$$v(t) = -N\frac{d\phi(t)}{dt} \quad (2)$$

For a linear inductor the relation between induced voltage and current are related as

$$v(t) = L\frac{di}{dt} \quad (3)$$

For a non-linear device, it is little difficult to come up with a simple equation of $\phi(t)$ in terms of $i(t)$ not involving radical signs. If the values of A, N and L are known, then with the help of curve fitting it is easy to approximate the same function and $\phi(t)$ can be expressed as some function of $i(t)$. After observing this non-linearity, it is a reasonable choice to analyse the non-linear inductor with the following relationship between voltage and current as

$$\phi(t) = L_0 + L_1\, i(t) + L_2\, i^2(t) \quad (4)$$

which leads to

$$v(t) = \frac{d\phi}{dt} = \dot{\phi} = L_1\frac{di(t)}{dt} + L_2\, 2\, i(t)\, \frac{di(t)}{dt} \quad (5)$$

Using the same arguments to the non-linear capacitor, the same non-linearity can be assumed(even though the cause of it may not known precisely), the effects of this can be analysed to understand the behaviour of non-linear devices in circuits as a starting point. For a non-linear capacitor, these equations can be taken as

$$q(t) = C_0 + C_1\, v(t) + C_2\, v^2(t) \quad (6)$$

And

$$i(t) = \frac{dq(t)}{dt} = C_1\frac{dv(t)}{dt} + C_2\, 2\, v(t)\, \frac{dv(t)}{dt} \quad (7)$$

It is now required to study the effects of non-linearities in time domain and frequency domains to understand the behaviour with regards to phase velocity and group velocity.

III. MATHEMATICAL ANALYSIS

The circuit is analysed with the help state-space analysis using KCL and KVL [9]. These equations are higher order non-linear coupled equations which can be solved numerically. The current passing through the inductor and voltage drop across the capacitor are taken as independent variables and other currents and voltages are expressed in these variables. x_1 is the current passing through the left inductor and x_2 is the current passing through the right inductor while x_4 is the current passing through the non-linear inductor on the top. x_3 is the voltage drop across the shunt capacitor. Fig.3 represents the Miller loading with non-linear inductor and Fig.4 represents the capacitive loading. The governing equations for this circuit are as follows.

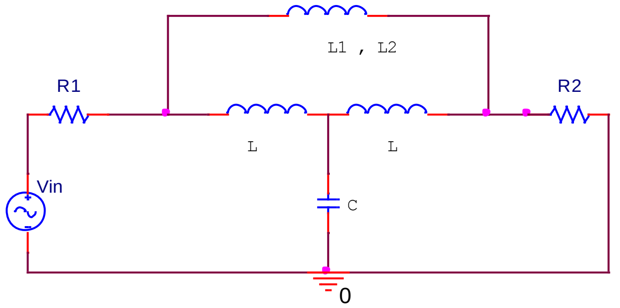

Fig.3 Miller type loading with non-linear inductor.

$$V_i = R_1(x_1 + x_4) + L\dot{x}_1 + x_3 \quad (8)$$

$$x_1 = C\ \dot{x}_3 + x_2 \quad (9)$$

$$x_3 = L\dot{x}_2 + R_2(x_2 + x_4) \quad (10)$$

$$L_A\dot{x}_4 = L(\dot{x}_1 + \dot{x}_2) \quad (11)$$

$$L_A = L_1 + L_2 x_4 \quad (12)$$

These equations are expressed as first order non-linear differential equations.

$$\dot{x}_1 = \frac{V_i - R_1(x_1+x_4) - x_3}{L} \quad (13)$$

$$\dot{x}_2 = \frac{x_3 - R_2(x_2+x_4)}{L} \quad (14)$$

$$\dot{x}_3 = \frac{(x_1 - x_2)}{C} \quad (15)$$

$$\dot{x}_4 = \frac{V_i - R_1(x_1+x_4) - R_2(x_2+x_4)}{L_A} \quad (16)$$

These equations are solved with the Runge-Kutta [10] fourth order method and obtained the observations in terms of voltages and currents. Similar types of equations are obtained for the capacitive loading which is shown in Fig.4. Matlab [11] is used to simulate these equations for numerical solutions.

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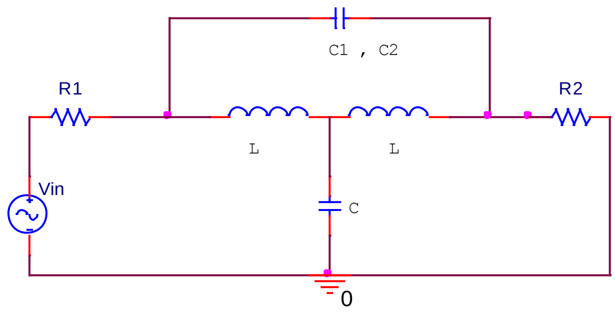

Fig.4 Miller type loading with non-linear capacitor.

A Gaussian voltage pulse is given as input and the voltages and currents are obtained in time domain. The ratio of output voltage (across R_2) to input volateg v_{in} is obtained by converting the time domain wave forms into frequency domain with the help of Fourier Transform theory. The phase response $\phi(\omega)$of the gain is used to find the phase delay and group delay[12] from the equations

$$\tau_p = -\frac{\phi(\omega)}{\omega} \quad (17)$$

And

$$\tau_g = -\frac{d\phi(\omega)}{d\omega} \quad (18)$$

IV. SIMULATION RESULTS

For the numerical purposes, the above mentioned two circuit models are analyzed with different non-linearities. Firstly with the forward wave supporting structure (with LC ladder network) being loaded with a non-linear inductor. Secondly the same structure with a non-linear capacitor loading is considered. The values of R_1 and R_2 are taken as 1Ω and $L = 0.1H$ and $C = 0.01F$. These values can be scaled in accordance with the input signal and frequency of interest and hence the choice of these values does not change the analysis observations.

Fig.5 represents the response of unit cell for a non-linear inductor. From the phase response of the unit cell, it is observed that the non-linear Miller type loading is producing a decreasing and increasing phase response, which indicates that there is a change of sign in the group delay from positive sign to negative sign and vice-versa.Fig.6 represents the phase delay and group delay from Fig.5. Blue curve represents the linear, green curve represents $L_2 = 0.1H$. Red curve represents$L_2 = 0.2H$ and cyan curve is for $L_2 = 0.3H$ with $L_1 = 0.1H$ for Fig.5 and Fig.6.

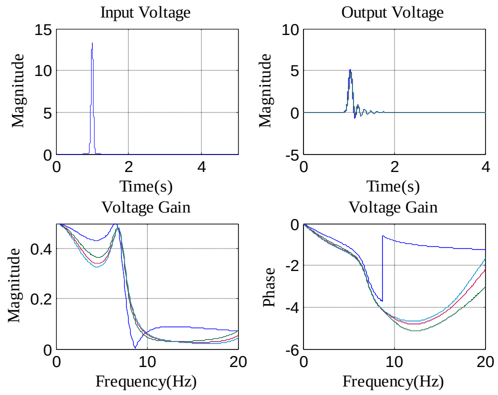

Fig.5 Unit cell characteristic with different non-linearities.

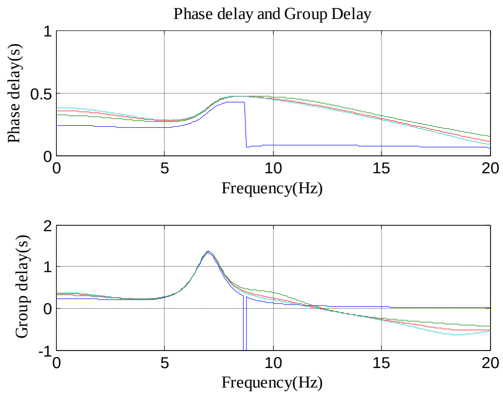

Fig.6 Phase delay and group delays for Miller non-linear inductor loading.

Fig.7 represents the response with non-linear capacitor loading. The response is observed for different ratios of non-linearity from high non-linearity to low non-linarity.Fig.7 represents the response for $C_2 = C_1/10$. Fig.8 represents the response for $C_2 = C_1/50$ and Fig.9 represents the response for $C_2 = C_1/200$.From the response it is clearly observed that the capacitive loading has more effects on the phase and gain response of the system. The gain of the system is having more ripples and the phase is changing from increasing to decreasing at multiple frequencies which indicates that the group velocity has been changed its sign from positive to negative and negative to positive validating that the non-linearities in the circuits can produce opposite delays. Blue curve represents $C_1 = 0.01F$. Green curve represents $C_1 = 0.015F$ and red curve is for $C_1 = 0.02F$ with different ratios of $C_2 and\ C_1$ for all Fig.8, Fig.9 and Fig.10.

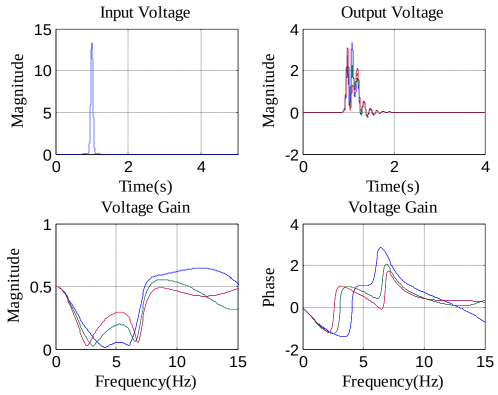

Fig.7 Unit cell characteristic with $C_2 = C_1/10\ F$.

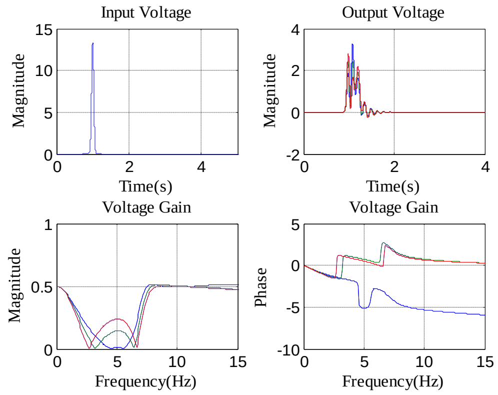

Fig.8 Unit cell characteristic with $C_2 = C_1/50\ F$.

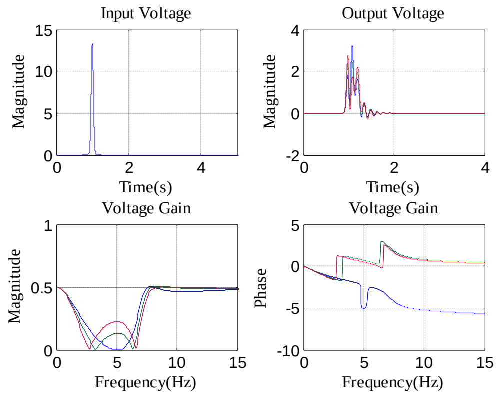

Fig.9 Unit cell characteristic with $C_2 = C_1/200\ F$.

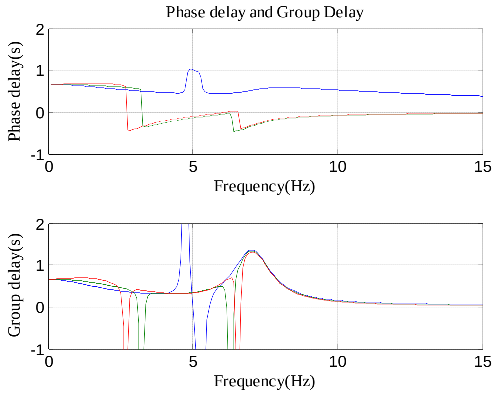

Fig.10 Phase delay and group delay for non-linear capacitive loading with $C_2 = C_1/10\ F$.

V. CONCLUSIONS

From the analysis, it is observed that there has been a possibility in obtaining the negative group delay in the transmission line model with the non-linearities. It is also observed that the phase reversal is happening at non zero gain of the circuit which is the main advantage with these non-linear circuits in case of negative group delay is required at reasonable gains of the system. For a smooth transition of group delay, conventional transmission line model with capacitive loading is a better option.

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